# AZIMUTHAL REDISTRIBUTTION OF BEAM IN THE FERMILAB MAIN RING 

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A technique is described by which the proton heam in the Fermilab main accelerator can be relocated from its original uniform distribution into a much smaller azimuthal region. This allows fast extraction of the entire beam in such a way as to allow single turn injection into a much smaller ring. The technique is applicable to generation, capture, and subsequent cooling of antiprotons. Experimental results are presented.

## Introduction

Various scenarios have been considered for production, recapture, and cooling of anti-protons at Fermilab. 1 It is usually considered necessary to capture the $\overline{\mathrm{p}} s$ either in the booster or in an accumulator ring approximately the size of the booster. In order to maximize the $\bar{p}$ flux in such a ring, it may be desirable to extract a large fraction of the main ring beam and deliver it to a $\bar{p}$ production target in a time equal to the rotation period of the small ring; i.e., short compared to the period of the main ring. (The circumference of the main ring is 13.25 times that of the booster.) In order to do this it will be necessary to coalesce a large fraction of the main ring beam into a small azimuthal region just previous to extraction. In this paper a procedure is described for introducing the required selective momentum spread using only the existing rf system ( $\mathrm{h}=1113$ ).

## Description of Beam Relocation Procedure

In this procedure the main ring ( $\mathrm{h}=1113$ ) is loaded with booster batches ( $\mathrm{h}=84$ ) in the normal manner and the beam is accelerated to 100 GeV where a "front porch" constant field is created for about two seconds. Only that fraction of the machine which will be coalesced need be filled with booster batches initially. During the transition from acceleration to front porch the rf voltage (bucket height) is adjusted adiabatically to a level which gives optimum bunch width and momentum spread. After the required level has been established (perhaps fifty milliseconds into the front porch) the rf is abruptly turned off, retuned as quickly as possible to a new frequency one harmonic number removed from the original frequency, and turned back on, perhaps at a different level. During acceleration from 8 GeV to 100 GeV the rf frequency changes from 52.8130 MHz to 53.1025 MHz so it is assured that the system frequency can be lowered by one harmonic number ( 47.7 kHz ) to 53.0548 MHz . The ( $\mathrm{h}-1$ ) rf voltage is

$$
\begin{equation*}
V(t)=V \sin (h-1) \Omega_{0} t \tag{1}
\end{equation*}
$$

where $h$ is the original harmonic number and $\Omega_{0}$ is the synchronous rotational angular frequency. The time of arrival of the $k$-th bunch at the effective accelerating gap location will be

$$
t_{k}=k / f_{r f}=\frac{k}{\hbar} T_{o}, \quad T_{0}=\begin{gathered}
\text { synchronous, rotational } \\
\text { period }
\end{gathered}
$$

so the voltage seen by the $k$-th bunch, immediately following reapplication of the rf voltage is

$$
\begin{equation*}
V_{k}=V \sin \left(\frac{-2 \pi k}{h}\right) \tag{2}
\end{equation*}
$$

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since $k$ ranges from zero to $h$ the voltage applied to the h bunch locations has initially the appearance of a voltage applied at harmonic number one. The important distinction between this ring voltage and a voltage at $h=1$ is that the bunches are constrained to remain within a phase space defined by new buckets which have (within $0.1 \%$ ) the same phase extent as the original buckets. The center of the $k$-th bunch is displaced from the center of the new bucket by an angle $\delta$ (rf radians) where $\delta=2 \pi \mathrm{k} / \mathrm{h}$. It is convenient to number the bumch locations $k,-556,-555 \ldots-1,0,1, \ldots$ 556. In Fig. 1a, a superposition of the 1112 buckets containing progressively displaced bunches is shown. The $\mathrm{k}=0$ bunch is at the center of its bucket while those locations $k= \pm 556$ are at unstable fixed points. A representation of all bunches would consist of 1113 overlapping bunches. The figure is representative only of selected bunch locations. With the application of the (h-1) rf voltage the bunches begin to execute coherent synchrotron oscillations from the initial positions. The synchrotron motion is allowed to continue until bunches with small displacements have executed about $3 / 8$ of one oscillation. This allows bunches with larger initial phase displacement but lower synchrotron frequency to reach a higher momentum deviation. The bunch distribution after this period of time is represented in Fig. 2a. The momentum deviation as a function of location number after $3 / 8$ of a "symchrotron period" is shown in Fig. 2. The distribution is significantly better than that which would result after only $1 / 4$ of a "synchrotron period." At the time when the momentum distribution has been optimized in this manner the rf is removed and the bunches are allowed to drift. While the bunches are drifting together they are also debunching because of their individual momentum spread. The drift time is limited by the time required for the zeroth bunch to spread into the required azimuthal space. The required "rf on" time is a few milliseconds while the drift time will be several seconds.


Fig. 1. Superposition of 1112 (h-1) buckets showing location of 1113 bunches, (a) immediately following harmonic jump, (b) after $3 / 8$ synchrotron period.


Fig. 2. Momentum distribution of 1113 bunches after $3 / 8$ synchrotron oscillation in $\mathrm{h}=1112$ buckets.

In the following discussion we use cannonical coordinates:
$W=E / \omega$ (eV-Sec) and $\phi$ (rf radians)
$\mathrm{E}_{\mathrm{S}}$ Total energy of a synchronous particle
$W_{\text {om }}$ Bucket height of original bucket, $\mathrm{h}=1113$
$W_{o b}$ Bunch height of beam bunch in original bucket
$W_{1 m}$ Bucket height of new bucket, $\mathrm{h}=1112$
$W_{1 b}$ Bunch height of beam in (h-1) bucket
S Longitudinal emittance of single bunch, (eV-Sec)
$\mathrm{T}_{0}$ Rotation period of main ring
F Fraction of main ring into which we coalesce
$T_{d}$ Debunching time of zeroth bunch into fraction F of ring
0 Azimuthal drift range of momentun displaced bunch
$\delta$ Initial phase displacement of a bunch from center of (h-1) bucket
$n=\frac{1}{\gamma^{2}} \frac{1}{\gamma_{t}^{2}} 20.00275$ at 100 GeV .
Differences in bucket heights and widths resulting from differences in harmonic numbers 1112 and 1113 are ignored.

We are constrained by the longitudinal emittance S. Before the harmonic jump

$$
\begin{align*}
& S=\pi W_{o b} \phi_{0}, \\
& W_{o b}=W_{o m} \sin \frac{\phi_{0}}{2}=\frac{W_{o m} \phi_{0}}{2} . \tag{3}
\end{align*}
$$

The bunch parameters, expressed in terms of the emittance $S$ and the bucket height $H_{m}$ are:

$$
\begin{equation*}
W_{o b}^{2}=\frac{S W_{o m}}{2 \pi} \text { and } \phi_{0}^{2}=\frac{2 S}{\pi W_{o m}} \tag{4}
\end{equation*}
$$

If the harmonic jump is done in a time short compared to a synchrotron period, the bunch parameters remain the same inmediately following the jump. If the bunches are not matched to the new buckets the zeroth (centered) bunch will begin to tumble within the bucket. After one-quarter of a phase oscillation the zeroth bunch height in the new bucket becomes

$$
\begin{equation*}
W_{1 b}=W_{1 m} \sin \frac{\phi_{0}}{2} \approx \frac{W_{1 m} \phi_{0}}{2}=W_{1 m}\left[\frac{S}{2 \pi W_{o m}}\right]^{\frac{1}{2}} . \tag{5}
\end{equation*}
$$

A bunch with its center displaced from the new bucket center by an angle $\delta$ will, after one quarter synchrotron period, have its center displaced in momentum to a value $M$,

$$
\begin{equation*}
M=W_{1 m} \sin \frac{\delta}{2} \tag{6}
\end{equation*}
$$

The momentum spread of a displaced bunch, after a quarter synchrotron period, is always smaller than that of the centered bunch. Therefore, the centered bunch will debunch more rapidly than a displaced bunch so the drift time allowed for a displaced bunch is limited by the time required for the centered bunch to spread into some fraction $F$ of the machine circumference. The bunch width $\Delta \mathrm{T}$ per unit time $\mathrm{T}_{\mathrm{d}}$ is

$$
\begin{equation*}
\frac{\Delta T}{\mathrm{~T}_{\mathrm{d}}}=\eta \frac{\Delta \mathrm{p}}{\mathrm{p}}=\eta \frac{\Delta \mathrm{E}}{E_{\mathrm{E}}}=\frac{\eta 2 u W_{\mathrm{b}}}{E_{\mathrm{s}}} \tag{7}
\end{equation*}
$$

or, if $\Delta T$ is expressed as some fraction $F$ of the rotation period $\mathrm{T}_{\mathrm{o}}$,

$$
\begin{equation*}
T_{d}=\frac{\mathrm{FE}_{\mathrm{S}}}{2 \eta \omega W_{b}}=\frac{\mathrm{FE}_{\mathrm{S}}\left(2 \pi W_{\mathrm{om}}\right)^{\frac{1}{2}} \mathrm{~T}_{0}}{2 \eta \omega W_{1 \mathrm{~m}} \mathrm{~S}^{\frac{1}{2}}} \tag{8}
\end{equation*}
$$

The total azimuthal angle gained by a synchronous particle in time $T_{d}$ is $\theta=2 \mathrm{~T}_{\mathrm{d}} / \mathrm{T}_{0}$ so the incremental angle gain in time $T_{d}$ by the center of a bunch displaced in momentum by $M$ (eq. 5) will be,

$$
\begin{equation*}
\frac{\Delta T}{T}=\frac{\Delta \theta}{\theta}=\frac{n \omega M}{E_{S}} \text { or } \Delta \theta=\frac{n \omega W_{1 m} 2 \pi T_{d} \sin \frac{\delta}{2}}{E_{S} T_{0}} \tag{9}
\end{equation*}
$$

combining (7) and (8)

$$
\begin{equation*}
\frac{\Delta \theta}{2}=\frac{(2 \pi)^{\frac{3}{2}} F}{4}\left|\frac{W_{o m}}{S}\right|^{\frac{k}{2}} \operatorname{Sin} \frac{\delta}{2} \tag{10}
\end{equation*}
$$

For proper coalescence we require drift angle $\theta$ to be equal to the initial displacement angle $\delta$ so eq. 10 is an expression of the maximum collection angle in terms of the initial bucket height and emittance. While the drift time, eq. 8 , is dependent upon the ( $\mathrm{h}-1$ ) bucket voltage, the maximum collection angle is not. Let $\theta / 2=\delta / 2=\Gamma$. We have

$$
\begin{equation*}
\frac{\operatorname{Sin} \Gamma}{\Gamma}=\frac{4}{(2 \pi)^{\frac{3}{2}} \mathrm{~F}}\left[\frac{S}{W_{o m}}\right]^{\frac{1}{2}}=\frac{0.254}{\mathrm{~F}}\left[\frac{\mathrm{~S}}{W_{o m}}\right]^{\frac{1}{2}} \tag{11}
\end{equation*}
$$

At 100 GeV the bucket height $W_{\mathrm{m}} \sim 4 \times 10^{-4} \mathrm{~V}^{\frac{1}{2}}$. For a ring voltage of $4 \mathrm{MV}, W_{m}=0.8 \mathrm{eV} \mathrm{Sec}$. The longitudinal emittance of the main ring beam is not well known, but if we agree to attempt to coalesce that part of the beam contained within an emittance of 0.1 eV Sec, eq. 11 yields

$$
\begin{equation*}
\frac{\sin \mathrm{F}}{\Gamma}=\frac{0.09}{\mathrm{~F}} . \tag{12}
\end{equation*}
$$

If, for example, $\Gamma=\pi / 3$, the resulting value of $F$ is 0.108 , meaning that beam can be collected from $\pm 120$ degrees into one ninth of the circumference subject to the emittance and bucket area conditions stated. If the (h-1) rf voltage is $3 \mathrm{MN}, \mathrm{W}_{1 \mathrm{~m}}=0.69 \mathrm{eV} \mathrm{Sec}$. The drift time, eq. 7 is 1.03 seconds. The momentum spread associated with this deviation is $\not \approx 2.1 \times 10^{-3}$ which is consistent with the momentum aperture of the main ring at 100 GeV .

## Experimental Results

Figure 3 shows the main ring rf cavity voltage signal at the start of a 2 second front porch. The rf is turned off for about $800 \mu \mathrm{Sec}$ then turned on at harnonic number 1112. Techniques for doing this are described elsewhere. ${ }^{2}$ The envelope during the following three mSec reflects the transient response of the tuning system. In the figure the rf voltage previous to turn-off is 2.75 MV , following ( $\mathrm{h}-1$ ) turrn-on it appears to have an average value of about 1.3 MV .

In the first experiment two booster batches of 82 bunches each were injected approx. 180 degrees apart. Figure 4 shows the evolution of the two batches during the 2 second front porch. Time progresses upward and the traces are separated by 0.2 seconds. One booster batch appears to broaden and move to slightly earlier time while the other batch appears to broaden less and to move more rapidly toward later time. A small fraction of the latter batch appears to depart and move toward earlier time at about the same rate with very little broadening. Gross features of the data, when extrapolated to the intersection point, appear to indicate that this beam distribution could be collected into about one-seventh of the azimuth in about 2.2 seconds. The data are consistent with the interpretation that the two batches were located on either side of the zero point with the batch which moves rapidly and broadens less being very close to the unstable fixed point. Some fraction of the left batch may actually be beyond the unstable fixed point, accounting for the small anount of beam noving away from the batch.

The results of a later experiment are shown in Fig. 5. The ring was filled with thirteen booster batches, visible on the bottom trace. The initial ring voltage was 4 MV and the average voltage following harmonic switch was about 2 M . The rf remained on for 5 mSec following the switch. There is clear evidence of coalescence of the beam at about 0.8 sec . Because of limited low frequency response of the bean detector used it is difficult to determine accurately the range into which the beam has coalesced, but it appears to be about three booster batch lengths, or about one quarter of the ring. No beam loss was observed during either of the beam redistributions described.

## Conclusion

Given the emittance, momentum aperture, and rf system capability of the Fermilab main ring, it appears promising that a reasonable fraction of the main ring beam can be coalesced into a small azimuthal region to allow extraction of a high intensity beam for a short duration for anti-proton generation.

## References

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2. C. Ankenbrandt, J. Griffin, R. Johnson, J. Lackey and K. Meisner. Longitudinal Motion of the Beam in the Fermilab Booster. IEEE Trans. Nucl. Sci. NS-24 1449, (1977).


Fig. 3. Sum of rf gap voltages before and after harmonic jump. Horizontal scale 1 ms per division. Vertical calibration about 1M per division.


Fig. 4. Coalescence of two booster batches orginally separated by 2180 degress. Tíme proceeds upward, traces scparated by 0.2 Sec .


Fig. 5. Coalescence of 13 booster batches.

