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SECOND ORDER EFFECTS IN THE SEXTUPOLE-CORRECTED SPS LATTICE

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Summary

Chromaticity correction is essential in the SPS accelerator both for normal operation and for high density single bunch acceleration and storage, required for the CERN proton-antiproton collider project. Linear correction is presently achieved with one set of 36 F sextupoles and another set of 36 D sextupoles, regularly placed around the machine circumference. This lumped sextupole distribution induces non-linear variations of the machine tune with $\Delta p/p$. These higher order effects which are also encountered in large electron machines cannot actually be corrected in the SPS, where they are well observable at least up to second order. Analytic calculation of the non-linear chromaticity, using Fourier expansions of the lattice functions, as well as computer simulation with the AGS program closely agree with the measurements performed at various energies. For the same reason, the transition energy depends upon the momentum deviation $\Delta p/p$ of the beam, which can lead to losses when beams with large $\Delta p/p$ are accelerated through transition, since no correct phase-jump timing can be found for all particles. Measurements of this effect in the presence of different chromatic corrections are presented and also compared with the predictions of the analytic calculations and of the computer evaluations with the AGS program.

Introduction

Some protons are always lost during the early part of the SPS acceleration cycle, i.e. from injection up to after transition. Part of these losses can be attributed to injection errors and to particles which are not trapped by the accelerating system, but the remaining losses are very sensitive to the fine adjustment of the machine sextupoles used for chromaticity correction. In this paper, we will show how the sextupole distribution is responsible for the non-linear chromaticities observed in the SPS. The formalism developed will also be used to estimate the momentum dependence of the transition timing which is responsible for a few percent beam loss around transition.

Non-linear chromaticities

Chromaticity measurements

The variations of the betatron tunes with the beam momentum deviation were carefully measured at various energies [1], by displacing the beam radially with the RF control system. Fig. 1 shows typical results obtained on the 10 GeV injection platform and on the 200 GeV intermediate flat top. Although there is a residual chromaticity for $\Delta p/p = 0$, due to an imperfect compensation, one clearly sees here the non-linear behaviour of the horizontal tune with momentum. This effect is more important at low energy, where the correction given by the machine sextupoles is relatively higher : because of remanent field and eddy current effects, the SPS chromaticities vary with the main field B, [2], according to :

$$\xi_{H,V} = \frac{p}{\nu}\cdot\frac{\Delta\nu}{\Delta_p} = a + b\,\frac{B_{inj}}{B} + c\,\frac{\dot{B}}{B} \qquad (1)$$

which was checked experimentally, [1], [3].

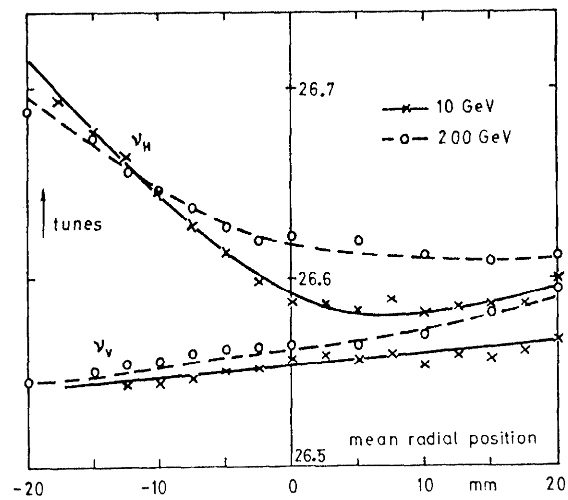

Fig. 1 SPS tunes versus momentum deviation ($\Delta p/p = .54\ \Delta R$ mm)

Analytic calculation of the chromaticity

The non-linear part of the chromaticity can be calculated by developing the Courant and Snyder equation to the desired order in momentum deviation $\delta = \Delta p/p$, [5], [6]. Up to the second order, the perturbed tune ν is given by :

$$\nu^2 = \nu_o^2 + \bar{a}\delta + \left[2\sum_{n>o}\frac{a_n^2}{n^2 - 4\nu_o^2} + \bar{b}\right]\delta^2 \qquad (2)$$

The a_n are the coefficients of the Fourier expansion of the lattice function $a(\phi)$, with respect to the normalized betatron phase angle ϕ :

$$a(\phi) = \nu_o^2\,\beta_o^2\,(S\alpha_{po} - K) = \bar{a} + \sum_{n\neq o} a_n e^{in\phi} \qquad (3)$$

where we have used the standard notations :

$$K = \frac{B'}{B\rho}\,, \quad S = \frac{B''}{B\rho}\,, \quad d\phi = \frac{ds}{\nu_o\beta_o} \qquad (4)$$

When the sextupoles, which give a chromatic correction $\Delta\xi$, are adjusted to compensate the natural chromaticity then $\bar{a} = 0$, and the second quadratic term $\bar{b}$ in (2) is given by, [1] :

$$\bar{b} = -2\,\nu_o^2\Delta\xi + \nu_o^4\sum_n\,(\beta_o^{5/2}S)_{-n}\,F_n^1 \qquad (5)$$

where the index n means the nth harmonic coefficient of the corresponding function, the subscript 0 refers to unperturbed quantities, and :

$$F^1 = \beta_o^{3/2}\,(K\alpha_{po} - \frac{S}{2}\,\alpha_{po}^2) \qquad (6)$$

Using the lattice functions calculated with the AGS computer program, [4], one can perform the Fourier analysis of the above functions and then clearly see the influence of each spacial harmonic of the sextupole distribution. Fig. 2 shows for instance the relative strength of the terms $|a_n^2/n^2 - 4\nu_o^2|$ of equation (2): the 54th harmonic is important as it is very close to $2\nu_o = 53.2$, but harmonics 36 and 72, which are

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further away from the resonance, still contribute a lot because of their greater intrinsic strength. Similarly the most important contribution to $\bar{b}$ (eq. 5) comes from the 36th harmonic. This $\bar{b}$ term can only be reduced by increasing the number of sextupoles - the actual number of 36 F and 36 D sextupoles is too near to the ν_0 value - whereas the sum term in (2) can be minimized by reducing the strength of harmonic 54 - see below - or by changing the machine tune.

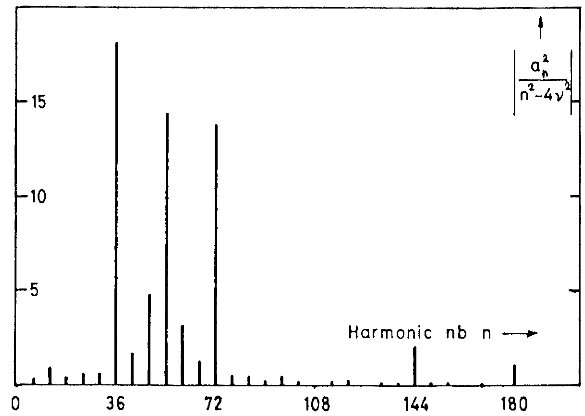

Fig. 2 Spacial harmonics of the sextupole distribution.

The same exercise has been performed for the vertical tune : according to the measurements, this effect is much less pronounced in the vertical plane, because the 54th harmonic of the D sextupoles distribution is much smaller than in the horizontal case, and the corresponding sum term in (2) is small and negative. The $\bar{b}$ term is also small, as the D sextupoles have little influence on the dispersion function .

Simulation with the AGS program

The AGS computer program,[4], allows the calculation of the betatron functions and tunes for off-momentum particles. When using the SPS lattice, with the two sextupole families adjusted so as to compensate the natural chromaticity, one obtains the curve A of Fig. 3, which is comparable with the 200 GeV measurements. This calculated curvature can be split into three parts :

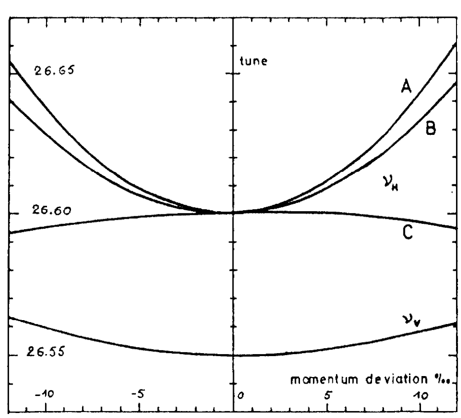

Fig. 3 AGS simulation of SPS chromaticity.

- Thanks to their regular distribution, the F sextupoles can be divided into two independent interlaced families, which allows to cancel harmonic 54, while keeping a proper compensation of the machine chromaticity. In this way, AGS gives the curve B of Fig. 3 which exhibits somewhat less curvature than A.

- The effect of the other harmonics can be estimated by calculating for a given $\Delta p/p$ the changes in the lattice functions β_H and α_p , induced by the presence of the sextupoles. Curve C of Fig. 3 is obtained by substracting this contribution from curve B.

- The residual small curvature of C is intrinsic to the bare machine : AGS shows that even without sextupoles, the machine tunes do not vary linearly with $\Delta p/p$.

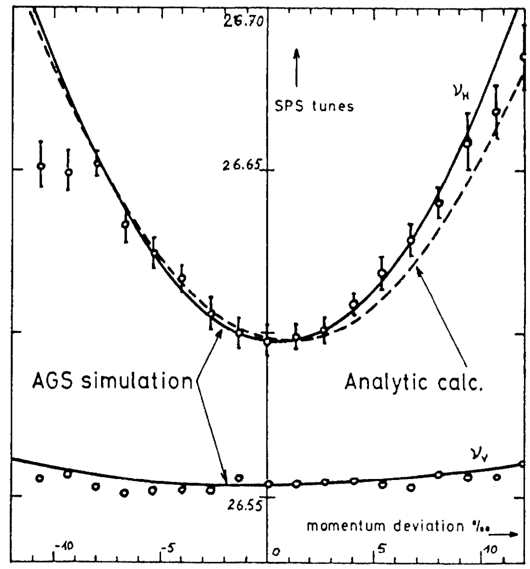

Fig. 4 Non-linear chromaticity at 10 GeV.

In order to see if AGS simulation fit the experimental data, one has to take into account the variation of the uncorrected chromaticities with energy as given by (1). This was done by putting in the AGS input data suitable sextupolar components in the field of the main bending magnets and then adjusting the machine sextupoles for compensation of the effective chromaticities at each energy. Excellent agreement was obtained this way and Fig. 4 shows an example for 10 GeV. Note that for $\Delta p/p > 9$ ‰, one is close to $3\nu_0 = 80$ and measurements are doubtful, as part of the beam is lost. The tune variation deduced from the analytic calculations is also in very good agreement with the experiments, but departs slightly from AGS simulation for large positive $\Delta p/p$, because of the limitation of equation (2) to the second order.

Momentum dependence of the transition timing

This effect, known as the "Johnsen effect", [7], is supposed to be at the origin of some losses just after transition when beams with large momentum spread are accelerated in the SPS, [8]. Let us write the variation ΔL of the orbit length L up to the second order in the momentum deviation $\delta = \Delta p/p$:

$$\frac{\Delta L}{L} = \alpha_1 \delta (1+ \alpha_2 \delta) \quad , \quad \text{with } \alpha_1 = \frac{1}{\gamma^2_{tr_0}} \qquad (7)$$

where γ_{tr} is the transition energy of the reference particle ($\delta = 0$). It can be shown, [9], that the spread in transition timing arising from the momentum spread in the beam is given by :

$$\Delta t = -(1.5 + \alpha_2)\delta \cdot \frac{\gamma_{tr_0}}{\dot{\gamma}} \qquad (8)$$

So particles pass transition at the same time only if $\alpha_2 = -1.5$.

Evaluation of α_2

A particle with a momentum error δ has an orbit deviation $x = \alpha_p\delta = \alpha_{p_0}\delta + \alpha_{p_1}\delta^2 \ldots$. The perturbed dispersion function α_p can be calculated up to second order in δ, [1], which results in :

$$\alpha_{p_0} = \nu_0^2 \sqrt{\beta_0} \sum_n \left(\frac{\beta_0^{3/2}}{\rho}\right)_n \cdot \frac{e^{in\phi}}{\nu^2 - n^2} \qquad (9)$$

$$\alpha_{p_1} = -\alpha_{p_0} + \nu_0^2 \sqrt{\beta_0} \sum_n \frac{F_n^1 \, e^{in\phi}}{\nu^2 - n^2} \qquad (10)$$

where F^1 is defined by (6). The change in orbit length, up to second order in δ, is now obtained by calculating the elementary trajectory length and integrating it over the circumference.

$$\frac{\Delta L}{L_0} = \frac{\delta}{L_0}\int_{C_0} \frac{\alpha_{p_0}}{\rho}\, ds + \frac{\delta^2}{L}\int_{C_0}\left(\frac{\alpha_{p_1}}{\rho} + \frac{\alpha'^2_{p_0}}{2}\right) ds \qquad (11)$$

In fact, (11) contains other quadratic terms, coming from orbit changes in the magnetic lenses, but these terms can completely be neglected for the SPS. After integration with the help of (9) and (10), (11) can be identified with (7), leading to :

$$\alpha_1 = \frac{1}{\gamma_{tr_0}} = \frac{\nu_0}{2\pi R}\int_0^{2\pi} \frac{\beta_0 \alpha_{p_0}}{\rho}\, d\phi \qquad (12)$$

$$\alpha_2 = -1 + \frac{\nu_0}{R\alpha_1}\sum_n \left(\frac{\alpha_{p_0}}{\sqrt{\beta_0}}\right)_{-n} F_n^1 + \frac{1}{2\alpha_1}\int_{C_0} \alpha'^2_{p_0}\, ds \qquad (13)$$

The last term in (13) can be evaluated using the AGS output for $\alpha'p(s)$ and amounts to .826 for the SPS. A first evaluation of α_2 is obtained by keeping only the zero harmonic component of the sum term in (13), which with some approximation boils down to $-2\xi_0 + \Delta\xi$, ξ_0 being the natural chromaticity of the machine and $\Delta\xi$ the chromatic correction given by the sextupoles.

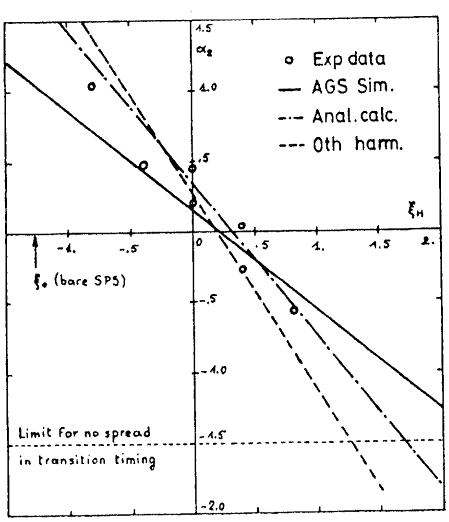

Fig. 5 Variation of α_2 with horizontal chromaticity.

Comparison with measurements

α_2 can be deduced from the observed linear variation of the transition timing versus the beam mean radial position, for a given machine chromaticity, and the results are plotted in Fig. 5 as a function of the effective SPS chromaticity. The fact that the zero harmonic approximation of (13) fits better the experimental data than the complete evaluation of (13) with a full Fourier analysis of the relevant functions, is not very significant owing to the accuracy of the measurements.

Fig. 5 also shows the α_2 variation deduced from AGS outputs. The discrepancy with (13) comes from the approximation used in AGS to calculate the transition energy for an off-momentum particle, [4]. Anyway the suppression of the spread in transition timing would require an effective chromaticity of the order of $\xi_H = 1.6$ which cannot be tolerated around transition.

Conclusion

Second order effects in momentum deviation of the beam have been observed in the SPS. Both computer simulations with the AGS program and analytic calculations using Fourier expansions of the lattice functions agree well with the experimental data. Furthermore this latter method allows to see how these effects could be minimized, should it prove to be necessary for the future SPS projects.

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