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THE VARIATION OF γ_t WITH $\Delta p/p$ IN THE CERN ISR

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Summary

The variation of the transition energy $\gamma_t m_o c^2$ across the momentum aperture in the ISR has been determined by measuring the non-linear change of the revolution frequency as a function of the radial displacement and the momentum deviation and by measuring the phase oscillation frequency variation across the aperture while operating close to transition energy. The ratio of the relative slopes of γ_t and γ was found to be $\frac{d\gamma_t/\gamma_{tc}}{d\gamma/\gamma_c} \approx -0.75$. This value could be changed to nearly zero by strongly exciting the sextupole magnets. All measurements are in good agreement with each other and with computations carried out with a modified version of the program AGS. The change of γ_t across the aperture causes an area change of the empty buckets while traversing the beam during phase displacement acceleration. This leads to a momentum blow-up.

Introduction

The transition energy $\gamma_t m_o c^2$ as well as the particle energy itself depends on the radial position inside the magnet aperture. Particles with different momentum deviations will therefore pass through transition energy at different times during an acceleration cycle. This "Johnsen effect"[1] has been studied mainly in connection with transition crossing. However, since some consequences of this effect are also important for storage rings, several measurements of the relevant parameters have been carried out at the ISR. Many authors have investigated transition energy effects. Here the theory given in references [1,2] and [3] is followed and adapted to apply to the measurement results.

Consideration is given to a storage ring with a fixed magnetic field. The variations of several longitudinal phase plane parameters are investigated as a function of the deviation of the beam momentum from the central momentum (a subscript 'c' refers to the central value of the parameter concerned).

A momentum deviation δ given by

$$p = p_c\left(1 + \frac{\Delta p}{p_c}\right) = p_c(1+\delta)$$

produces a change in the particle velocity βc

$$\beta = \frac{p/m_o c}{\sqrt{1+(p/m_o c)^2}} = \beta_c\left[1 + \frac{1}{\gamma_c^2}\delta - \frac{3}{2}\frac{\beta_c^2}{\gamma_c^2}\delta^2 \ldots\right]$$

and of its energy $\gamma m_o c^2$

$$\gamma = \gamma_c\left[1 + \beta_c^2\delta + \frac{1}{2}\frac{\beta_c^2}{\gamma_c^2}\delta^2 \ldots\right] .$$

The length L of the orbit is also changed[1]

$$L = L_o\left[1 + \alpha_1\delta + \alpha_1\alpha_2\delta^2 \ldots\right] \qquad (1)$$

where the coefficients α_1 and α_2 are completely determined by the beam optic. The change in length is related to the change in the average radial position.

Hence

$$\frac{\langle\Delta R\rangle}{R_c} = \alpha_1\delta + \alpha_1(\alpha_2 - w_2)\delta^2 \qquad (2)$$

where the 'wiggling factor' w_2

$$w_2 = \frac{1}{2\alpha_1 L_c}\int_0^{L_c}\left(\frac{d\alpha_{p1}}{ds}\right)^2 ds$$

gives the contribution to the orbit length due to the angle of the off-momentum orbit. This factor can be computed since it contains only the derivative of the first order dispersion α_{p1}; for the ISR $w_2 \approx 0.44$.

The transition energy $\gamma_t m_o c^2$ is defined as the energy at which the derivative of the revolution time $T = L/\beta c$ with respect to momentum p vanishes, i.e.

$$\frac{dT}{dp} = \frac{1}{c\beta}\left(\frac{dL}{dp} - \frac{L}{p\gamma_t^2}\right) = 0 \quad .$$

This gives γ_t as a function of δ

$$\gamma_t = \frac{1}{\sqrt{\alpha_1}}\left(1 - \left(\frac{1-\alpha_1}{2} + \alpha_2\right)\delta\right) = \gamma_{tc}\left(1 - \left(\frac{\beta_{tc}^2}{2} + \alpha_2\right)\delta\right)$$

where $\beta_{tc}^2 = 1 - \frac{1}{\gamma_{tc}^2}$. We consider here only the linear change of γ_t across the momentum aperture and neglect higher order terms in δ. The ratio of the relative slopes of γ_t and γ is

$$\frac{d\gamma_t/\gamma_{tc}}{d\gamma/\gamma_c} = -\frac{1}{\beta_c^2}\left(\frac{\beta_{tc}^2}{2} + \alpha_2\right) \approx -\left(\frac{1}{2} + \alpha_2\right)$$

The relative change of the revolution frequency ω is

$$\frac{\Delta\omega}{\omega_c} = \eta_c\delta - \left(\frac{\alpha_2}{\gamma_{tc}^2} + \frac{3}{2}\frac{\beta_c^2}{\gamma_c^2} + \frac{\eta_c}{\gamma_{tc}^2}\right)\delta^2 \qquad (3)$$

with $\eta_c = 1/\gamma_c^2 - 1/\gamma_{tc}^2$. The local relative derivative of ω with p is

$$\eta = \frac{p}{\omega}\frac{d\omega}{dp} = \frac{1}{\gamma^2} - \frac{1}{\gamma_t^2} = \eta_c - \left(\frac{2\alpha_2}{\gamma_{tc}^2} + \frac{\beta_{tc}^2}{\gamma_{tc}^2} + \frac{2\beta_c^2}{\gamma_c^2}\right)\delta$$

The small amplitude phase oscillation frequency Ω_s around the equilibrium momentum $p_c(1+\delta)$ changes with this momentum. We have

$$Q_s^2 = \frac{\Omega_s^2}{\omega^2} = \frac{h\, e\, \hat{V}_{RF}\cos\phi_s}{2\pi m_o c^2}\,\frac{\eta}{\beta^2\gamma}$$

with h = harmonic number, $\hat{V}_{RF}$ = RF voltage and ϕ_s = synchronous phase angle (defined so that $\eta \cdot \cos\phi_s \geq 0$). We obtain for the dependence of Q_s^2 on δ

$$Q_s^2 = Q_{sc}^2\left[1 - \frac{1}{\eta_c\gamma_{tc}^2}\left(2\alpha_2 + \frac{(2+\beta_c^2)\gamma_{tc}}{\gamma_c^2} - (2-\beta_c^2-\beta_{tc}^2)\right)\delta\right]$$

or

$$\frac{Q_s^2 - Q_{so}^2}{Q_{sc}^2} \approx -\frac{1}{\gamma_{tc}^2\eta_c}\left(2\alpha_2 + \frac{3\gamma_{tc}^2}{\gamma_c^2}\right)\delta =$$

$$= -\frac{1}{\eta_c}\left(2\alpha_2 + \frac{3\gamma_{tc}^2}{\gamma_c^2}\right)\frac{\langle\Delta R\rangle}{R_c} \qquad (4)$$

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The dependence of the phase oscillation frequency on momentum is sometimes called "longitudinal chromaticity"[4]. For large amplitudes the dependence of η on δ has to be taken into account in deriving the phase oscillation equation. This leads to a non-linear phase motion[3,4].

Some of the effects already discussed can be very important if a machine is operated close to its transition energy. This is illustrated in Fig. 1, where the ISR is assumed to operate in such a way that the central orbit is at transition energy, i.e. $\gamma_c = \gamma_{tc}$. Fig. 1a) shows the change of γ and γ_t with $\Delta p/p_c$ while Fig. 1b) shows the change of revolution frequency ω, η, and Ω_s. The revolution frequency has a maximum ω_c in the centre. For a frequency of the acceleration system being slightly less than $h\omega_c$, there are two buckets, one on the outside being above transition energy ($\cos\phi_s < 0$) and one on the inside being below transition energy ($\cos\phi_s > 0$). These buckets are considerably distorted by the non-linear effects and have a limited momentum acceptance[3]. Very short bunches could be obtained by operating a storage ring close to transition energy[5] where η is very small. For a given energy spread σ_E/E the bunch length σ_s is

$$\frac{\sigma_s}{R_c} = \frac{\eta}{Q_s \beta^2}\frac{\sigma_E}{E_c} = \sqrt{\frac{2\pi\, m_o c^2}{h\, e\, \hat{V}_{RF}\cos\phi_s}}\sqrt{\frac{\gamma\eta}{\beta^2}}\frac{\sigma_E}{E_c} \qquad (5)$$

However this only applies when the higher order terms and the energy spread are small[3].

The variation of the longitudinal phase plane parameters can be of practical importance for storage rings:

a) The area of the empty RF bucket changes while traversing a stack during phase displacement acceleration. This can lead to a momentum blow-up which is clearly observed in the ISR[6] and could be important for larger machines.

b) The non-linearity of the phase motion could change the radiation damping partition numbers for large amplitudes[7].

c) By operation of a storage ring close to transition energy short bunches can be obtained if the non-linear terms can be controlled. Hence short pulses of synchrotron radiation could be produced as well as coherent synchrotron radiation if the bunches are made very short.

d) It has been suggested that the dependence of Ω_s on p could lead to a longitudinal head-tail instability[4].

2. Measurements

Due to the long beam lifetime, the accurate measurement and control of the magnet parameters, and the good beam diagnostics, the ISR is a very suitable machine for measuring the change of γ_t across the momentum aperture. Many measurements were made close to transition energy. The beam had to be either accelerated from 3.5 GeV/c or decelerated from 11.8 or 15.4 GeV/c (injection momentum) using the technique described in reference [8] for bunched beam acceleration.

2.1 Variation of the bunch frequency with position

Using (2) and (3) the revolution frequency of the bunches can be directly related to the average radial displacement <ΔR>

$$\frac{\Delta\omega}{\omega_c} = \eta_c \gamma_{tc}^2 \frac{\langle\Delta R\rangle}{R_c} - \gamma_{tc}^4\left(\frac{\alpha_2}{\gamma_c^2} - w_2\eta_c + \frac{3}{2}\frac{\beta_c^2}{\gamma_c^2} + \frac{\eta_c}{\gamma_{tc}^2}\right)\left(\frac{\langle\Delta R\rangle}{R_c}\right)^2 \qquad (6)$$

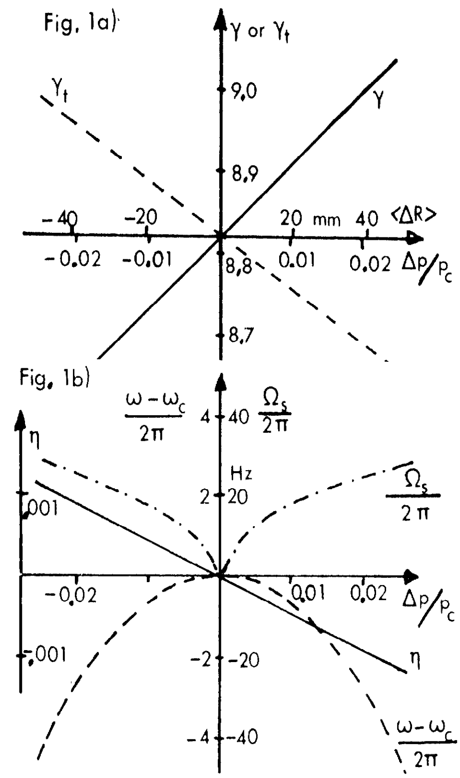

Fig. 1. Variation of γ_t and γ, phase oscillation frequency Ω_s ($\hat{V}_{RF}$ = 12 kV), difference in revolution frequency $\omega - \omega_c$ and η across the momentum aperture for the ISR operating at transition energy.

A bunched beam was displaced across the aperture and its revolution frequency measured as a function of average radial position. The bunch frequency was measured using a high precision frequency synthesiser (0.1 Hz accuracy) and the mean radial position was obtained from the normal closed orbit measuring system (Fig. 2).

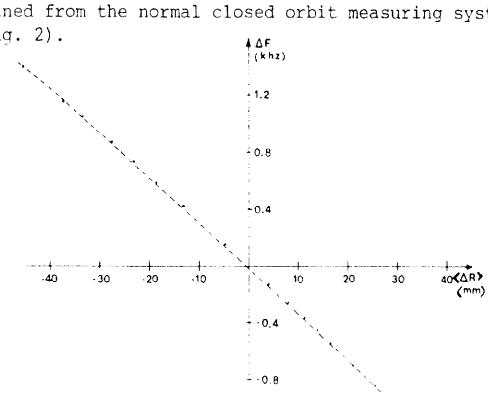

Fig. 2. Measurement of revolution frequency as a function of radial position.

2.2 Variation of the beam revolution frequency with momentum setting

In this case the field level of the ISR was changed to different central momentum settings (p_c) while a debunched beam of constant momentum p_b was circulating. The revolution frequency ω_b of the beam only changes due to the variation in orbit length (1)

$$\frac{\omega_b - \omega_{co}}{\omega_{co}} = -\frac{1}{\gamma_{tc}^2}\frac{p_b - p_c}{p_c} + \frac{1}{\gamma_{tc}^2}\left(\frac{1}{\gamma_{tc}^2} - \alpha_2\right)\left(\frac{p_b - p_c}{p_c}\right)^2 \qquad (7)$$

where ω_{co} is the revolution frequency for $p_b = p_c$. The revolution frequency of the debunched beam was measured by the longitudinal Schottky scan device. The working line was kept constant throughout the experiment.

The measured results plotted as $(p_c-p_b)/(f_b-f_c)$ against (f_b-f_c) are shown in Fig. 3.

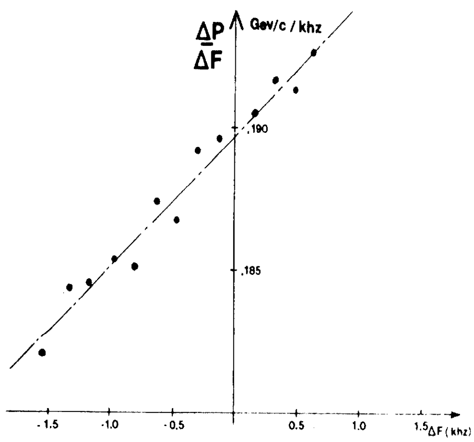

Fig. 3. Measurement of the relation between the revolution frequency of the debunched beam and the momentum setting.

2.3 Variation of Q_s^2 with radial position

The beam was decelerated to $\gamma \approx 9.1$ and the small amplitude phase oscillation frequency Ω_s measured as a function of radial position $\langle \Delta R \rangle$ (see Fig. 4). The change of γ_t across the aperture is obtained with (4) and Fig. 4.

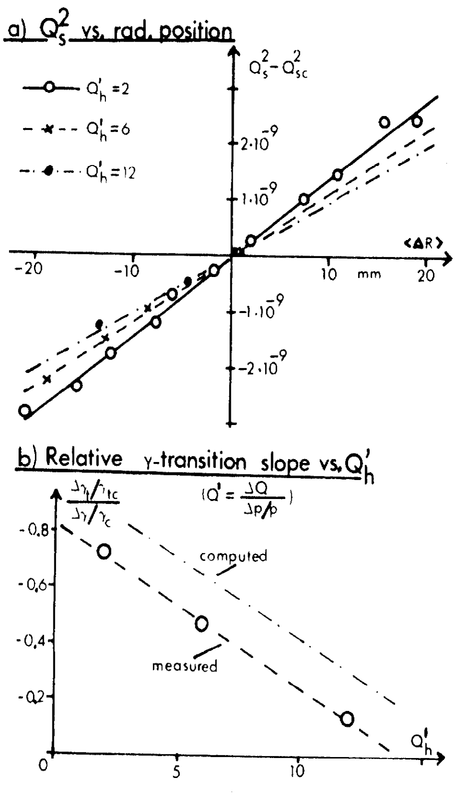

Fig. 4. Measurement of γ_t from Q_s.

The slope of γ_t can be changed with sextupole magnets[1,2]. The measurements were repeated for different sextupole settings, expressed here by their transverse effect in $Q' = \frac{\Delta Q}{\Delta p/p_c}$. The resulting effect on the slope of γ_t is shown in Fig. 4b) together with computed values. The slight discrepancy may be explained by a slight error in Q'_h; this had not been measured at the energy at which the measurements were made.

2.4 Results and comparison with computations

The results of the different measurement methods are listed in the table. They agree quite well with each other. The change of γ_t across the momentum aperture has also been computed using a modified version of AGS.

Measurement method	γ	Q'_h	Measured γ_t	Measured $\frac{d\gamma_t/\gamma_{tc}}{d\gamma/\gamma_c}$	Computed γ_t	Computed $\frac{d\gamma_t/\gamma_{tc}}{d\gamma/\gamma_c}$
ω vs. $\langle \Delta R \rangle$	12.6/9.2	∿2	8.82	- 0.73	8.82	
ω vs. p_b-p_c	12.6	∿2	8.87	- 0.79	"	- 0.90
Q_s^2 vs. $\langle \Delta R \rangle$	9.1	∿2	8.82	- 0.72	"	
"	"	∿6	"	- 0.47	"	- 0.70
"	"	∿12	"	- 0.15	"	-0.42

For an 8C-type working line ($Q \sim 8.6$, $Q' \sim 2$) we have in the ISR $\gamma_t = 8.82$, $\frac{d\gamma_t/\gamma_{tc}}{d\gamma/\gamma_c} = -0.75$, $\alpha_2 = 0.25$, wiggling factor $w_2 = 0.44$.

2.5 Observations of stored bunches close to transition energy

Bunched beams were accelerated or decelerated close to transition energy. The bunches, being kept stored very close to transition, were very short as expected from equation (5). A bunched beam with a fixed position ($\Delta R > 0$) was decelerated until transition energy was reached in the centre of the aperture ($\gamma_{tc} = \gamma_c$); Fig. 1. By keeping the magnetic field constant and jumping the RF phase the bunch was quickly decelerated across the aperture (and across transition energy) into the other bucket which is below transition. This operation, which is rather tricky and lossy, was performed for academic interest only.

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