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ANALYSIS OF TRANSVERSE COHERENT INSTABILITY IN KEK BOOSTER

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Summary

Coherent transverse instability observed in KEK booster was analyzed with a free string model, taking into account of the chromaticity of the bunched beam. The build-up time of the instability is represented by an eigen value of a matrix equation. The results of the calculation explain quite well experimental observations. The instability is induced by the interaction of the beam with ferrite loaded kicker magnet.[1] The interaction is represented experimentally and theoretically with a mutual inductance. Using an equivalent circuit of the magnet, the induced field and the build-up time of the instability were calculated. So far many theoretical studies have been made but satisfactory agreement is not yet obtained partly because of the difficulty of a correct estimation of the source of the interaction.[2~6] The analysis of the kicker magnet is rather easy when the current terminator is removed. This leads to a thorough description of the instability and gives a good agreement with the observations.[7]

Equivalent circuit of kicker magnet

The kicker magnet for the fast beam extraction is composed of the lamination of ferrite cores and electric plates and the one-turn coil is terminated with a matched resistance. The instability becomes more rapid when the terminator is removed. The following description is made on the condition of the magnet without the terminator. The high frequency response of the magnet is represented with an LC distributed circuit. In regard to the primary current passing through the gap of the magnet, the induced current in the circuit is given by mutual inductance

$$M = (a - b\Delta x)\ , \qquad (1)$$

where $a = 1.01 \times 10^{-7}$, $b = 1.60 \times 10^{-12}$ in unit of MKS, Δx is the current position in the gap and M is per unit core. This is because that the magnetic flux produced by the primary current is enclosed by the one-turn coil through the capacitance.

Induced current by AC current

The relation between the primary current i_1 and induced current is represented with the equivalent circuit in Fig.1. For the n-th mesh, following relation holds

$$\Omega^2(I_{n-1} + I_{n+1} - 2I_n) - \delta\dot{I}_n - \ddot{I}_n = -\gamma\ddot{i}_1\ , \qquad (2)$$

where $\Omega = 1/\sqrt{LC}$, $\delta = R/L$, $\gamma = M/L$ and $i_1 = i_0 e^{j\omega t}$. With a smooth approximation $I_n = I(na) = I(z)$ and a Fourier expansion, (2) can be written as

$$\ddot{Y}_m(t) + \delta\dot{Y}_m(t) + \Omega_m^2 Y_m(t) = (-)f_m(t)\ , \qquad (3)$$

where

$$I(z,t) = \sum_{m=1}^{\infty} Y_m(t)\sin(\frac{m\pi z}{\ell_B})\ ,$$

$$f_m(t) = \begin{cases} \dfrac{4i_0\gamma\omega^2}{m} e^{j\omega t} & \text{for } m = 1,3,5\ \ldots \\ 0 & \text{otherwise}\ , \end{cases} \qquad (4)$$

$$\Omega_m = \Omega\frac{m\pi}{N}\ ,$$

and the boundary condition $I_0 = I_N = 0$ was used. The Ω_m is the resonance frequency of the magnet. The solution of (3) for odd m is

$$Y_m(t) = A_m e^{j(\omega t + \alpha_m)}\ , \qquad (5)$$

where

$$A_m = \frac{4i_0\gamma}{m\pi}\frac{\omega^2}{\sqrt{(\omega^2-\Omega_m)^2 + (\omega\delta)^2}}\ , \qquad (6)$$

$$\alpha_m = \arctan\left(\frac{\omega\delta}{\omega^2-\Omega_m^2}\right)\ .$$

The solution for even m damps with time. The A_m becomes large around the resonance and then $\alpha_m \simeq -\frac{\pi}{2}$ as shown in Fig.2.

Interaction between bunched beam and kicker magnet

The horizontal position of the ℓ-th particle which makes betatron and synchrotron oscillation is given by

$$\Delta x_\ell = Z_\ell e^{j(\omega_\beta t + \omega_\xi \tau_\ell)}\ , \qquad (7)$$

where ω_β is the betatron oscillation frequency, $\omega_\xi = \frac{\xi}{\eta}\omega_\beta$ and τ_ℓ is the time of arrival at the kicker magnet relative to the synchronous particle. The ξ is the chromaticity and $\eta = \alpha - \frac{1}{\gamma^2}$ where α is the momentum compaction factor and γ is the relativistic energy. Since (7) is independent of the momentum deviation, we regard that all the particles arriving at τ_ℓ have the same phase, and they are treated as a section of a free string.

Circulating current of the ℓ-th section is given by

$$i_\ell = i_0\rho_\ell D(t - nT - \tau_\ell)\ , \qquad (8)$$

where $\rho_\ell (\simeq \cos\frac{\pi}{\tau}\tau_\ell)$ is the charge distribution and $D(t - nT - \tau_\ell) = 1$ only when $t \simeq nT + \tau_\ell$, and otherwise $D = 0$. T is the revolution period and n is an integer. The magnetic flux induced by this current within the coil is

$$\phi_\ell = (a - b\Delta x_\ell)i_\ell\ . \qquad (9)$$

As for the induced current in the n-th mesh, the same relation with (2) holds except for the substitution of the right hand side with $-\ddot{\phi}_\ell/L$. Expanding $D(t\text{-}nT\text{-}\tau_\ell)$

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in Fourier series, we get a solution I(z,t) similar to (5).

The equation of the motion of the s-th section which is forced by the induced field is

$$\ddot{x}_s + \omega_\beta^2 x_s = \frac{ev\mu_0}{m_0\gamma h} \sum_\ell < I(z,t) > D(t-nT-\tau_s) \quad , (10)$$

where e is the electric charge, v the velocity of the beam, μ_0 the magnetic susceptibility of vacuum, $m_0\gamma$ the mass of proton and h the gap of the magnet. <I(z,t)> is the average in the magnet. Inserting I(z,t) and $D(t-nT-\tau_s)$ previously obtained or defined, we get after some approximations

$$\dot{Z}_s = j \sum_\ell \sum_m \sum_k H_{mk} Z_\ell \rho_\ell \Delta\tau_\ell e^{j(k\omega_0-\omega_\xi)(\tau_s-\tau_\ell)} \quad , (11)$$

where

$$H_{mk} = \frac{2}{\omega_\beta} \frac{e\mu_0 \ell_B}{m_0\gamma h} \frac{1}{T^2} \frac{bi_0}{L} \frac{1}{(m\pi)^2} \frac{\omega_k^2}{\sqrt{W}} e^{j\alpha_{mk}} \quad ,$$

$$W = (\omega_k^2 - \Omega_m^2) + (\omega_k \delta)^2 \quad ,$$

$$\alpha_{mk} = \text{arc tan } (\frac{\omega_k \delta}{\omega_k^2 - \omega_m^2}) \quad , \quad (12)$$

$$\omega_k = k\omega_0 + \omega_\beta \ .$$

The ℓ_B is the length of the magnet, and the relation $v\Delta\tau_s \doteq \ell_B$ was used.

Since Z_s is periodic with the period T, we expand it as

$$Z_s = \sum_{n=0}^{\infty} \{a_n \sin(n\omega_0\tau_s) + b_n \cos(n\omega_0\tau_s)\} \quad , (13)$$

where ω_0 is the revolution frequency. Substitution of this into (11) leads to

$$\dot{a}_\mu(t) = (-) \sum_\ell \sum_m \sum_k \sum_p H_{mk} \rho_\ell \Delta\tau_\ell G^-_{\mu k} \{a_p \sin p\omega_0\tau_\ell + b_p \cos p\omega_0\tau_\ell\} e^{-j(k-q)\omega_0\tau_\ell} \quad , (14)$$

where

$$G^\pm_{\mu k} = g((\mu-k+q)\pi) \pm g((\mu+k-q)\pi) \quad ,$$

$$g(y) = \sin(y)/y \quad , \quad (15)$$

and $q = \omega_\xi/\omega_0$. The summation over ℓ leads to

$$\sum_\ell = b_p F^+_{pk} - j a_p F^-_{pk} \quad , \quad (16)$$

where

$$F^\pm_{pk} = \frac{\pi\tau}{4} \{f((p-k+q)\pi r) \pm f((p+k-q)\pi r)\} \quad ,$$

$$f(x) = \frac{\cos x}{(\frac{\pi}{2})^2 - x^2} \quad , \quad (17)$$

and $r = \tau/T$. The τ is the bunch length. The $F^\pm_{pk}$ is a rapidly decreasing even function, and large when $p\pm(k-q) \approx 0$. Assuming $a_\mu = A_\mu e^{j\Delta\omega t}$ and $b_\mu = B_\mu e^{j\Delta\omega t}$ and substituting into (14) we get

$$\Delta\omega \cdot A_\mu = j \sum_m \sum_k \sum_p H_{mk} G^-_{\mu k} (B_p F^+_{pk} - jA_p F^-_{pk}) \quad . (18)$$

Similarly

$$\Delta\omega \cdot B_\mu = \sum_m \sum_k \sum_p H_{mk} G^+_{\mu k} (B_p F^+_{pk} - jA_p F^-_{pk}) \quad . (19)$$

The equation (18) and (19) form a matrix equation and $\Delta\omega$ is the eigen value and A_μ's and B_μ's are the elements of the eigen vector.

Numerical calculation and comparison with experiments

The imaginary part of the eigen value gives the build-up time of the instability. Let $\Delta\omega = \alpha - j\beta$, then the beam is unstable for $\beta > 0$. The maximum β of many eigen values is shown in Fig.3. It becomes largest around 17 msec after beam injection and the build-up time $\tau_\gamma = \frac{1}{\beta_{max}} = 2.2$ msec. These are in accord with the experiments as shown in Fig.4. In the eigen vector corresponding to the maximum β, the elements A_2, A_3, B_1, B_2 and B_3 are relatively large, so that we expect to see these modes in the booster beam.

The ΔR signal by a position monitor is proportional to the real part of $\rho_\ell \Delta x_\ell$,

$$\Delta R_\ell \propto \sum_\mu A_\mu \cos(\pi\sigma_\ell)\sin(2\pi\mu\sigma_\ell)\cos 2\pi(\nu n+q\sigma_\ell) + \sum_\mu B_\mu \cos(\pi\sigma_\ell)\cos(2\pi\mu\sigma_\ell)\cos 2\pi(\nu n+q\sigma_\ell) \quad , (20)$$

where $\sigma_\ell = \tau_\ell/\tau$, t = nT (n: integer). Figure 5 shows the calculated multi-traces of various single modes and Fig.6 the observed ones. Agreement is good.

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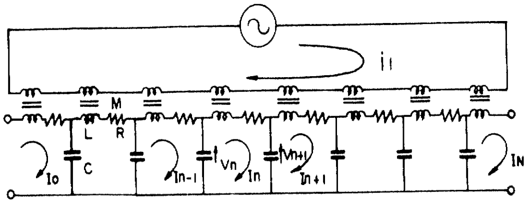

Fig.1 Equivalent circuit of kicker magnet.

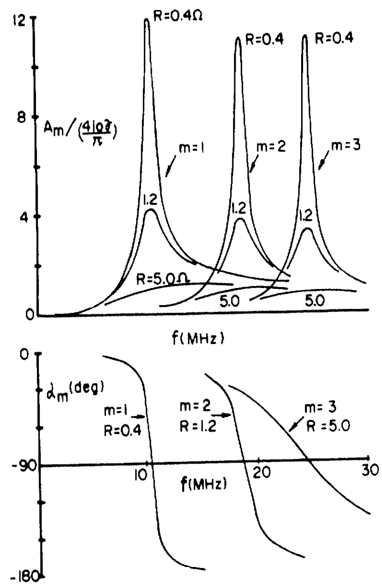

Fig.2 Amplitude of induced current A_m, and phase lag α_m.

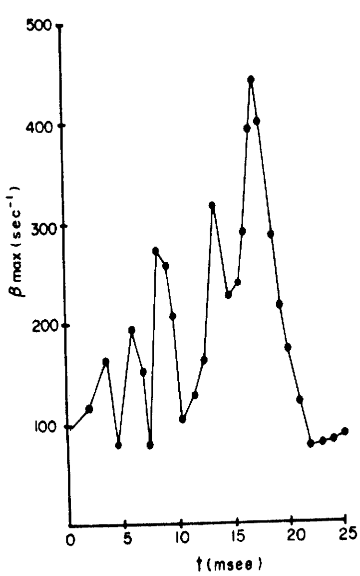

Fig.3 Inverse build up time β_{max}.

Fig.4 Horizontal ΔR signal during the acceleration.

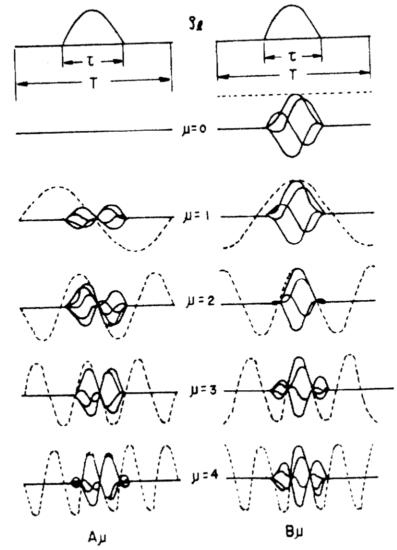

Fig.5 Calculated multi-traces of ΔR for various single modes.

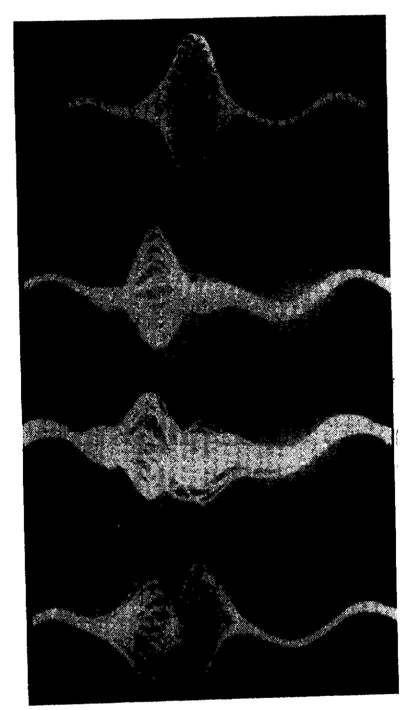

Fig.6 Observed multi-traces of ΔR signal (20 μs/div).