THE EFFICIENCY OF ACCELERATION BY PHASE DISPLACEMENT IN THE PRESENCE OF RF NOISE

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Summary

During acceleration by phase displacement the particle synchrotron motion is non linear around the exterior of the RF bucket. Simultaneous numerical evaluation of this motion for a large number of 'particles' allows calculation of relevant 'beam' statistical parameters which can be used to define the relative efficiency of the acceleration technique. This procedure has been used in a computer program which calculates the reduction in efficiency of acceleration by phase displacement in the presence of RF noise and when the bucket parameters vary during the RF sweep. The results show that the variation of γ_t with $\Delta p/p$ produces a significant increase in the momentum blow-up per accelerating sweep. It is also shown that RF noise (frequency modulation) causes particles to penetrate the bucket separatrix and become decelerated within the bucket. This results in a significant reduction in the efficiency of acceleration and greatly contributes to the intensity losses. Extensive experimental results have confirmed the computed results.

1. Introduction

In the ISR beams of very high intensity (up to 35 A) are routinely accelerated by phase displacement from 26.6 to 31.4 GeV/c[1,2]. The efficiency of this acceleration process may be defined by three principal parameters :

(i) The increase of the rms momentum width of the beam (δp_{rms}) per sweep,

(ii) The momentum increase ($\overline{\Delta p}$) of the centre of gravity of the stack per sweep and

(iii) The intensity loss per sweep due to particles which become trapped inside the RF buckets (ΔI).

A computer simulation[3] of phase displacement on a beam with zero initial momentum spread showed

$$(\delta p_{rms})_{th} = \sin\phi_s \cdot \overline{\delta p_s} = \Gamma \, \overline{\delta p_s} \qquad (1)$$

where ϕ_s is the stable phase angle and $\overline{\delta p_s}$ is the mean momentum spread of a stationary bucket having otherwise the same RF parameters. It is also well known that the ideal momentum gain resulting from a phase displacement sweep is

$$\overline{\Delta p}_{th} = \overline{\Delta p}_b = \frac{A_b}{2\pi} \cdot \frac{p_c}{\beta\gamma} \qquad (2)$$

Recent measurements showed significant discrepancies with calculations based on equations (1) and (2). Preliminary experiments indicated that the momentum blow-up (δp_{rms}) and the intensity loss per sweep were strongly affected by the injection of band-limited white noise to the RF system.

2. Computation of Efficiency Parameters

The motion of a particle in longitudinal phase space is described by[5]

$$\ddot{\phi} + \frac{\Omega^2}{\cos\phi_s} \{\sin\phi - \sin\phi_s\} = 0 \qquad (3)$$

where ϕ is the phase of the RF waveform and $\frac{\Omega}{2\pi}$ is the

synchrotron frequency for oscillations of small amplitude.

Solution of the above equation is normally restricted to small amplitude oscillation inside the bucket and with ϕ_s and Ω being constant. However, for the case of phase displacement of stacks with a large relative momentum spread the synchrotron motion may involve both large and small amplitude oscillations as well as time-dependent coefficients (ϕ_s and Ω). For this case equation (3) is analytically insoluble but may be solved (as a function of time) numerically. The effect of frequency modulation (RF noise) may be computed by making ϕ_s time-dependent.

Frequency spectra measurements made in the ISR showed that the inherent modulation due to 'noise' was very narrow band at the mains frequency (50 Hz) and its harmonics. Hence a sinusoidal noise variation was used for the analysis

$$\phi_s = \phi_{so} + \hat{\phi} \sin \omega_n t \qquad (4)$$

where $\frac{\omega_n}{2\pi}$ is the frequency of the 'noise'.

Phase displacement acceleration is particularly advantageous for stacks with a large momentum spread. As a consequence of the large range (in $\Delta p/p$) covered by the RF bucket, the variations of bucket parameters (with $\Delta p/p$) must be taken into account. In particular measurements have shown[6] that the γ_t varies in the ISR as

$$\frac{\Delta\gamma_t}{\gamma_t} = -\frac{\Delta p}{p} \qquad (5)$$

Such a variation causes all RF parameters to vary during the scan of the beam, e.g. the bucket area A_b, the synchrotron frequency for small amplitudes Ω; the $\Gamma(\sin\phi_s)$ also changes due to its dependence on df/dt.

It is also important to know when a particle crosses the bucket separatrix either into or out of the bucket. This can be done by comparing the position of the particle in phase space with the position of the separatrix.

$$\dot{\phi}_{sep}(\phi) = \left\{ \frac{2\Omega^2}{\cos\phi_s}(\cos\phi - \cos\phi_{1e} + \phi\sin\phi_s - \phi_{1e}\sin\phi_s) \right\}^{\frac{1}{2}} \qquad (6)$$

where ϕ_{1e} is the unstable fixed point = $(3\pi - \phi_s)$. The number of particles which remain inside the bucket as it is decelerated across the lower momentum aperture limit gives the intensity loss caused by the sweep.

The computations are initialized by representing the coasting beam by a large number of points in the $\phi, \dot{\phi}$ plane. The points are distributed randomly between 0 and 2π in ϕ and distributed in $\dot{\phi}$ according to the initial density distribution of the beam. For the ISR a uniform distribution is a reasonable approximation. After initialisation, the position of each particle ($\phi_i, \dot{\phi}_i$) is calculated as a function of time by solution of equation (3) by a 4th order Runge Kutta numerical method. At regular time intervals the statistical parameters related to efficiency are calculated by converting the $\dot{\phi}_i$ to the more measurable quantity Δp_i

$$\Delta p_i = -\frac{p_b}{2\pi f n} \{\dot{\phi}_i - \dot{\phi}_s\} \qquad (7)$$

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where P_b is the momentum at the centre of the RF bucket.

At each time interval, the mean and rms of the momentum distribution are calculated as well as the number of particles inside and outside the bucket. The momentum blow-up is calculated from the usual quadratic addition

$$\Delta p^2_{rms}(t) = \Delta p^2_{rms}(0) + \delta p^2_{rms}(t) \qquad (8)$$

The momentum gain is simply

$$\overline{\Delta p}(t) = \overline{p}(t) - \overline{p}(0) \qquad (9)$$

By defining the time (or $\Delta p/p$) at which the end of the scan occurs, the statistical parameters for a complete sweep are evaluated.

The experimental measurements of the stack momentum width and centre of gravity were performed by computer analysis of the longitudinal Schottky scan[4]. The measurement of the number of particles which are inside the RF bucket at the end of the scan was performed by suitable triggering of the dc beam intensity monitor just before and just after the RF bucket had reached the inner aperture limit. Each experimental point is the result of many phase displacement scans. The RF 'noise' was input as shown in Fig. 1.
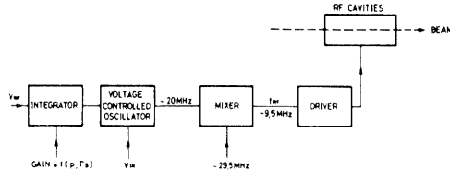

Fig. 1   Injection of 'noise' to the RF system

3.   Discussion of Results

In Fig. 2 the computed momentum blow-up is plotted as a function of Γ; also shown is the theoretical momentum blow-up expressed by equation (1). It can be seen that for the case of no noise and no γ_t variation the computed results agree very well with the results from existing theory. In addition the computed momentum gain per sweep ($\overline{\Delta p}$) corresponded to the theoretical value with a high degree of accuracy. These results confirm the accuracy of the computational technique. Also shown in Fig. 2 is the computed variation of δp_{rms} when the γ_t varies across the machine momentum aperture. It can be seen that for lower values of Γ (the operational value is 0.1) a significant increase in the momentum blow-up is produced. Blow-up rates corresponding to the computed values have been measured during
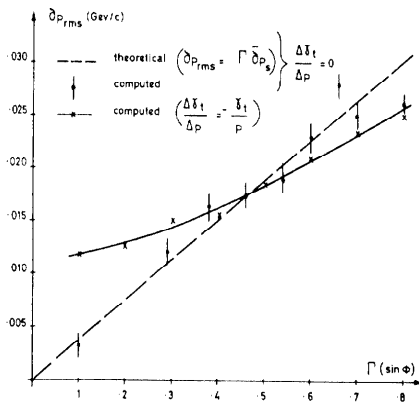

Fig. 2   Momentum blow-up as function of Γ

normal acceleration in the ISR. At higher Γ values the computed blow-up rates converge towards the theoretical value even when γ_t varies. A Γ value of 0.25 was chosen for all tests with RF noise. Higher values of Γ are not of any real importance.

The variation of the efficiency parameters with the amplitude of the RF noise is shown in Fig. 3.
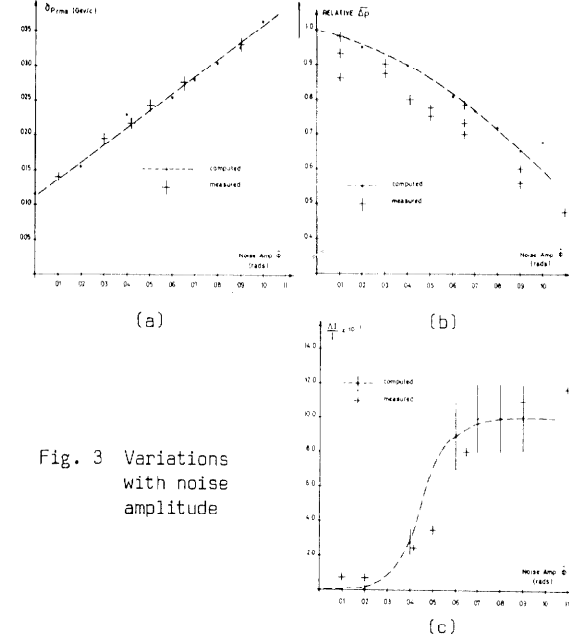

Fig. 3   Variations with noise amplitude

The computed results are in good agreement with those measured. The momentum blow-up increases linearly with the noise amplitude. Plot (b) shows the relative momentum increment ($\overline{\Delta p/\Delta p}_{th}$) decreasing with noise amplitude. Both of these effects may be explained qualitatively by the fact that the RF noise causes particles to enter and spend some time inside the RF bucket. During the time spent inside the bucket these particles are decelerated rather than accelerated. This behaviour will obviously increase the momentum width artificially and bias the centre of gravity of the stack towards lower momentum. Plot (c) shows the increase of intensity losses per sweep with the noise amplitude. The agreement between computed and measured results is surprisingly good in view of the poorer statistics. More interesting is the behaviour of the efficiency parameters with the frequency of the 'noise' (Fig. 4). Both measured and computed results show that the reduction in efficiency is maximum at noise frequencies just below the synchrotron frequency for small amplitudes (around $0.8\ f_{so}$). Based on these measurements, a large effort was concentrated on the reduction of the inherent noise at 50 Hz by building a completely new low-noise electronic control for phase displacement. At the same time the RF parameters for phase displacement were chosen so as to place the dangerous frequencies ($0.8\ f_{so}$) as far as possible from the prevailing (50 MHz) mains frequency. These measures resulted in a significant improvement in the efficiency of acceleration[2].

It is also interesting to observe the motion of individual particles which penetrate the separatrix. An example of a particle which remained inside the RF bucket for a significant number of phase oscillations is shown in Fig. 5 and Fig. 7.

Fig. 5(a) shows the trajectory of a particle as it approaches and becomes trapped inside the bucket. This particle spirals inwards until it approaches the centre of the bucket, then its motion changes and the particle spirals outwards again (Fig. 5(b). The same motion is shown in Fig. 7 as a function of time. The amplitude

f the synchrotron oscillations decreases as the fre-
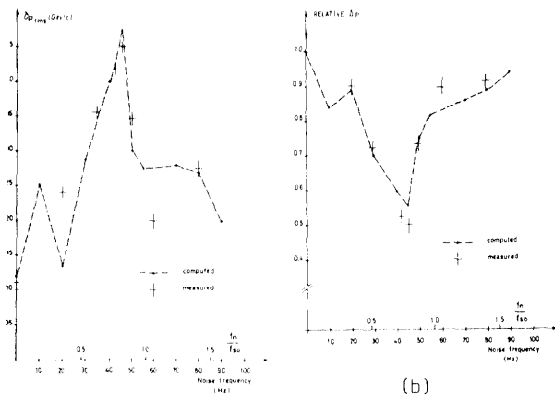uency of the synchrotron oscillations increases.

(a)

(b)

Fig. 4 Variation
with noise
frequency

(c)

Fig. 7 Phase plane
motion with
time

(a)

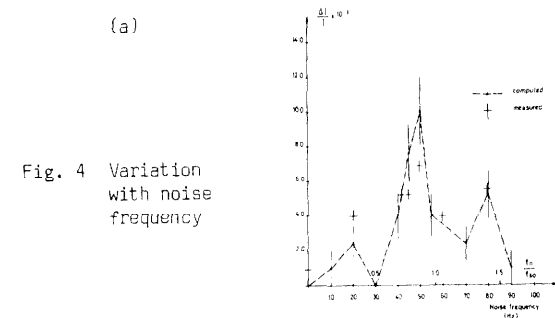(b)

(a)   (b)

Fig. 5 Phase plane trajectories

Fig. 6 shows a plot of the computed transient
behaviour of the momentum blow-up as a function of the
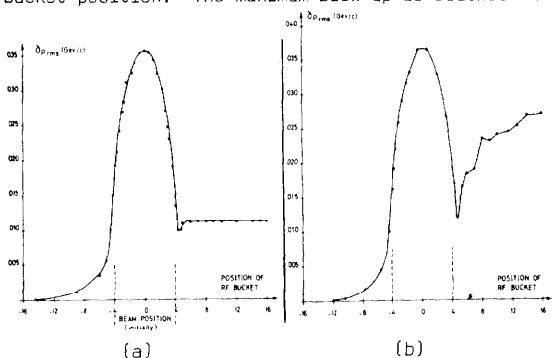bucket position. The maximum blow-up is reached when

(a)   (b)

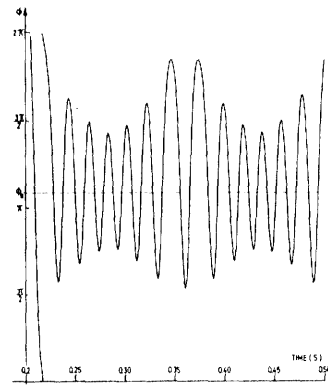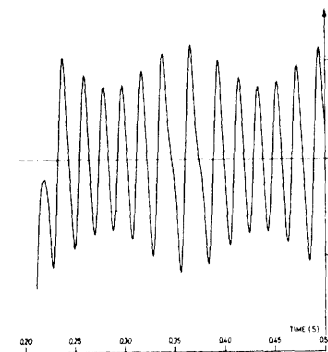Fig. 6 Transient behaviour of momentum width

the bucket is in the exact middle of the stack. In the
absence of RF noise (Fig. 6(a)) the momentum width
stabilises after the bucket leaves the stack. However,
in the presence of RF noise (Fig. 6(b) particles con-
tinue to drop out of the bucket for some time after
exit from the stack, causing an increased momentum
width.

4. Conclusions

The efficiency with which acceleration by phase
displacement is performed can be greatly influenced by
FM of the RF ('noise) and by variations in γ_t with
$\Delta p/p$. RF 'noise' at frequencies around 0.8 times the
synchrotron frequency for small oscillations is parti-
cularly harmful. These effects can be evaluated with
good accuracy by performing numerical solutions to the
equations of synchrotron motion. RF 'noise' causes
some particles to enter the RF bucket, then spiral
towards its centre and back towards the exterior of
the bucket. This type of motion can continue for
several periods and produces a reduction in the effi-
ciency of the acceleration.

5. References

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