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ENERGY LOSSES OF AN ELECTRON BUNCH MOVING ALONG THE AXIS OF A CIRCULAR WAVEGUIDE WITH PERIODICALLY PERTURBED WALL.

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Abstract

The field radiated by an electron bunch moving along the axis of a circular periodic waveguide is obtained from Maxwell's equations using a perturbation method. The guide shape is defined by its radius $a(z) = a_o \{1 + \varepsilon s(z)\}$ where a_o denotes the mean guide radius, $s(z)$ the guide geometry and ε a small perturbation parameter. Electromagnetic field is calculated up to second order in ε. The total energy loss suffered by the bunch and the potential acting on individual particles are related to the following parameters : charge, shape and width of the bunch, energy, guide geometry, by comparatively simple formulas. The influence of these parameters is calculated for a gaussian bunch and a rounded iris waveguide. This example clarifies under what conditions energy losses are virtually independent of energy and shows the importance of the bunch width.

Assumptions made in the problem

The bunch is moving with a constant velocity v. Its current density is given by $j_z = i(t-z/v)/\pi\rho^2$ where $i(t-z/v)$ is the instantaneous current and ρ the bunch radius. The waveguide has a period b. Its radius is described by the equation [1,2]

$$a(z) = a_o \{1 + \varepsilon s(z)\} \qquad (1)$$

where ε denotes the perturbation parameter. The periodic function s(z) which characterizes the wall geometry is taken to have zero mean value.

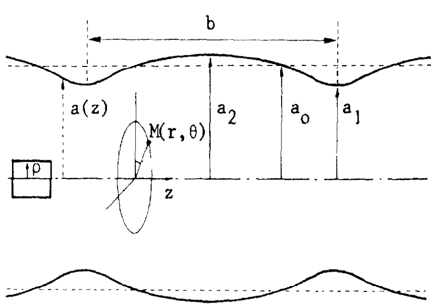

Fig. 1

The Fourier series expansion of s(z) is :

$$s(z) = \sum_{p=-\infty}^{+\infty} C_p \exp(j\, p \frac{2\pi z}{b}) \quad \text{with } C_o = 0 \qquad (2)$$

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Method of radiated field calculation

The field radiated by the bunch is calculated from Maxwell's equations, taking into account the boundary condition at $r = a(z)$. Because of the circular symmetry this field is entirely described by the asimuthal component H_θ of the magnetic field. In the other hand, it is convenient to introduce the variable $\tau = t - z/v$ and to use the variables (r,z,τ) instead of (r,z,t) because we are only interested by the field travelling with the bunch. The H_θ propagation equation can be written with $\gamma = 1/\sqrt{1-v^2/c^2}$:

$$\frac{\partial}{\partial r}\frac{1}{r}\left[\frac{\partial(rH_\theta)}{\partial r}\right] + \frac{1}{v^2\gamma^2}\frac{\partial^2 H_\theta}{\partial \tau^2} + \frac{\partial^2 H_\theta}{\partial z^2} - \frac{2}{v}\frac{\partial^2 H_\theta}{\partial\tau\partial z} = \frac{\partial j_z}{\partial r} \qquad (3)$$

and the boundary condition

$$\frac{\partial(rH_\theta)}{\partial r} = \frac{d\,a(z)}{dz}\left[-\frac{1}{v}\frac{\partial(rH_\theta)}{\partial \tau} + \frac{\partial(rH_\theta)}{\partial z}\right] \qquad (4)$$

The field $H_\theta(r,z,\tau)$ is expanded in Dini's series[1,2], Fourier's series and Fourier's integral as far as its r,z,τ dependance are concerned, respectively.

$$H = \int_{-\infty}^{+\infty} \sum_{p=-\infty}^{+\infty} \sum_{n=1}^{\infty} u_{np}(\nu) J_1(x_{on}\frac{r}{a_o}) e^{jp\frac{2\pi z}{b}} e^{j2\pi\nu\tau} d\tau \qquad (5)$$

J_o, J_1 denote the Bessel functions and x_{on} the n^{th} root of $J_o(x)$

$u_{np}(\nu)$ is then given by the equation :

$$\left[\nu^2 - \frac{2p}{b} v\gamma^2 \nu + v^2\gamma^2 \left(\frac{p^2}{b^2} + \frac{x_{on}^2}{4\pi^2 a_o^2}\right)\right] u_{np}(\nu) =$$

$$\frac{v^2\gamma^2}{2\pi^2 a_o^2\, b\, J_1(x_{on})} \int_{-b/2}^{+b/2}\int_{-\infty}^{+\infty} \left[\frac{\partial(rH_\theta)}{\partial r}\right]_{r=a_o} e^{-j2\pi\nu\tau} d\tau\; e^{-jp\frac{2\pi z}{b}} dz$$

$$-\delta_{po}\frac{v^2\gamma^2}{2\pi^2 a_o^2 \rho J_1(x_{on})} I(\nu)\; J_1(x_{on}\frac{\rho}{a_o}) \qquad (6)$$

$I(\nu)$ is the Fourier transform of $i(\tau)$ and $\delta_{po} = 1$ if $p=0$ and $\delta_{po} = 0$ if $p \neq 0$.

The field derivative at $r = a_o$ appears at the second member of this equation and may be calculated by a Taylor's series expansion of the function on the boundary condition

$$\frac{\partial(rH_\theta)}{\partial r} + \varepsilon a_o s \frac{\partial^2(rH_\theta)}{\partial r^2} + \frac{\varepsilon^2 a_o^2 s^2}{2}\frac{\partial^3(rH_\theta)}{\partial r^3} + \dots =$$

$$\varepsilon a_o \frac{ds}{dz}\left[\left(-\frac{1}{v}\frac{\partial(rH_\theta)}{\partial\tau} + \frac{\partial(rH_\theta)}{\partial z}\right) + \varepsilon a_o s\left(-\frac{1}{v}\frac{\partial^2(rH_\theta)}{\partial r\partial\tau} + \frac{\partial^2(rH_\theta)}{\partial r\partial z}\right) + \dots\right] \qquad (7)$$

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The field is then calculated by a perturbation method in which H_θ is taken as the sum of the magnetic field $H_\theta^{(o)}$ radiated by the bunch in a circular waveguide of constant radius a_o and corrective terms expanded* in series of ε.

$$H_\theta = H_\theta^{(o)} + \varepsilon H_\theta^{(1)} + \varepsilon^2 H_\theta^{(2)} + \dots \varepsilon^i H_\theta^{(i)} + \dots \quad (8)$$

This expansion together with the Taylor's series of boundary condition gives $H_\theta^{(1)}$ from the knowledge of $H_\theta^{(o)}$, then $H_\theta^{(2)}$ from the knowledge of $H_\theta^{(o)}$ and $H_\theta^{(1)}$ and so on.

Energy losses and longitudinal electric field

This method allows the calculation of the electromagnetic field by mode superposition of the perturbed waveguide. Keeping only synchronous modes we deduce easily energy losses and longitudinal electric field acting on a particle up to the second order in ε

Energy loss per period of the bunch :

$$W = \frac{-\varepsilon^2}{c\varepsilon_o} \sum_{p=1}^{\infty} p\, C_p\, C_{-p} \sum_{n=1}^{\infty} \left[\Phi(\nu_{np}) - \Phi(\nu'_{np}) \right] \quad (9)$$

where

$$\Phi(\nu) = \frac{2\nu\gamma}{\rho} \sqrt{1 - \frac{x_{on}^2\, b^2\, c^2}{4\pi^2 a_o^2\, p^2 \nu^2 \gamma^2}}\; |I(\nu)|^2 \frac{I_1\left(\frac{2\pi\nu\rho}{v\gamma}\right)\left(I_o \frac{2\pi\nu r}{v\gamma}\right)}{\left[I_o\left(\frac{2\pi\nu\, a_o}{v\gamma}\right)\right]^2} \quad (10)$$

and for a bunch of vanishing radius.

$$\Phi(\nu) = 2\pi \sqrt{1 - \frac{x_{on}^2\, b^2 c^2}{4\pi^2 a_o^2\, p^2 \nu^2 \gamma^2}}\; \frac{\nu |I(\nu)|^2}{\left[I_o\left(\frac{2\pi\nu\, a_o}{v\gamma}\right)\right]^2} \quad (11)$$

I_o, I_1 are modified Bessel functions, ε_o the free-space permittivity and N the greatest integer satisfying :

$$x_{oN} \leq 2\pi\, a_o\, v\gamma\, p/\, bc \quad (12)$$

and ν_{np} the roots of the $u_{np}(\nu)$ coefficient in the equation (6) :

$$\left.\begin{matrix}\nu_{np} \\ \nu'_{np}\end{matrix}\right\} = \frac{pv\,\gamma^2}{b} \left[1 \pm \frac{v}{c} \sqrt{1 - \frac{x_{on}^2\, b^2 c^2}{4\pi^2\, a_o^2\, p^2 v^2 \gamma^2}} \right] \quad (13)$$

If the particles velocity is strictly equal to the speed of light (infinite γ)

$$W = -\frac{\varepsilon^2 2\pi}{\varepsilon_o c} \sum_{p=1}^{\infty} p\, C_p\, C_{-p} \sum_{n=1}^{\infty} \nu_{np}\, |I(\nu_{np})|^2 \quad (14)$$

with

$$\nu_{np} = \frac{cp}{2b} + \frac{b}{p}\, \frac{x_{on}^2\, c}{8\pi^2 a_o^2} \quad (15)$$

*A refined perturbation method (Poincaré method) has been also used which gives virtually the same numerical results.

Longitudinal electric field $E_z(\tau)$ acting on the bunch particle for γ infinite

$$E_z(\tau) = -\frac{2}{\varepsilon_o c} \sum_{p=1}^{\infty} p\, C_p\, C_{-p} \sum_{n=1}^{N} \int \dot{i}(\tau - x) \sin 2\pi\nu_{np} x\, dx \quad (16)$$

$\dot{i}(\tau)$ is derivative of $i(\tau)$.

Examples

The bunch is supposed to have a zero radius and a gaussian longitudinal distribution. The total charge is 10^9 particles and the standard deviation $l_o = c\,\tau_o$ is of the order of millimeter. The shape of the waveguide is described by $s(z) = \cos^8 \pi z/b$, where the period b=3,5 cm and the smallest radius a_1 = 1,152 cm. The largest radius a_2 is normally chosen in a way consistent with the perturbation method that is to say $(a_2 - a_1)$ small compared with a_o. The inner radius a_1 and the period b are virtually those of the SLAC.

The set of curves of Fig.2 are applicable to a_2= 1,3cm. They show that the energy loss initially increases quickly with γ and eventually reaches a plateau. The smaller the bunch length, the more quickly is the energy loss leveling reached.

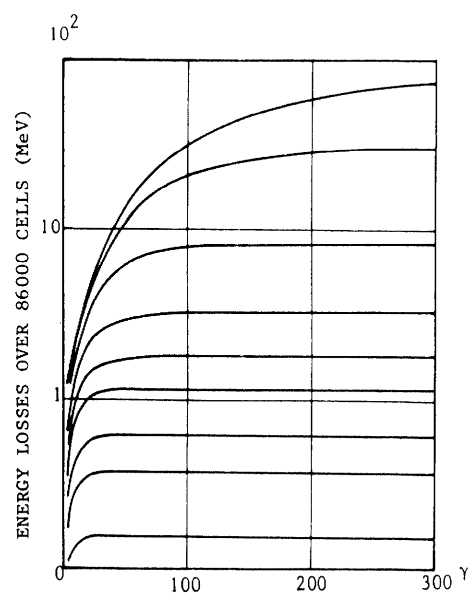

Fig.2 - Energy losses expressed in terms of γ and the bunch length l_o.

The set of curves of Fig.3 for γ infinite show both the influence of bunch length and of corrugation depth. The dotted curves represent the case where $\varepsilon\, s(z)$ is much too large for the application of the perturbation method. Nevertheless, the order of magnitude agrees with the values measured in references [5] or calculated in references [6,7] for the SLAC.

Fig.4 indicates the evolution of the acting field $E_z(\tau)$ when γ infinite for a_2 = 1,3cm and l_o= 0.75mm. The result is similar to the one calculated for a cylindrical cavity[8].

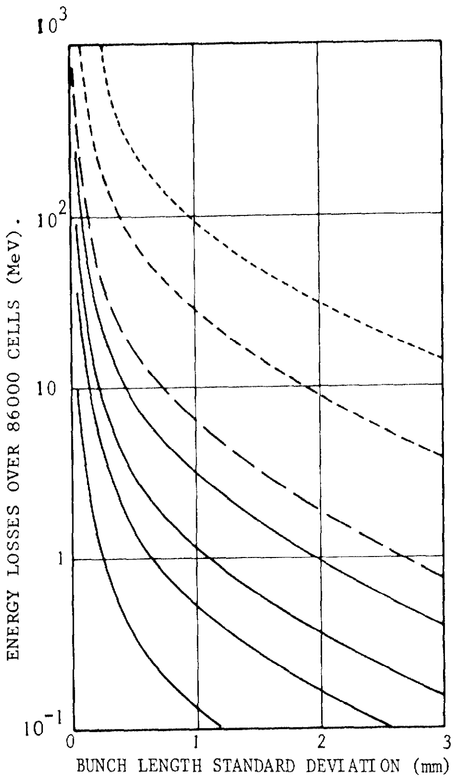

Fig.3 - Energy loss versus bunch length (l_o) and corrugation depth (a_2).

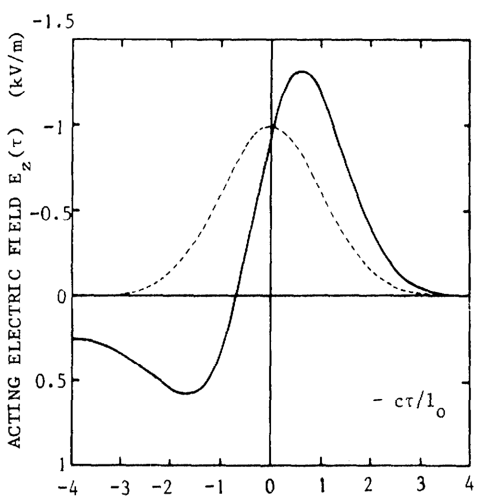

Fig.4 - Longitudinal electric field acting on a gaussian bunch.
The dotted line represents the bunch shape.

Non periodic structure

The results obtained for periodic structures may be extended to non periodic structure by taking the limit when the period b goes to the infinity (in this limiting process the shape of the perturbation is maintained). Under those conditions the shape of the structure Fig.5 is again described by the equation (1), s(z) goes to zero as z goes to infinity. If we call $S(\zeta)$ the Fourier transform of s(z) the total energy loss in the case where γ is infinite is :

$$W = \frac{-\varepsilon^2}{2\pi\varepsilon_o c} \int_o^\infty \zeta |S(\zeta)|^2 \sum_{n=1}^{\infty} \nu_n(\zeta) \left|I(\nu_n(\zeta))\right|^2 d\zeta$$

with

$$\nu_n(\zeta) = \frac{c}{2}\zeta + \frac{x_{on} c}{8\pi^2 a_o^2 \zeta} \frac{1}{\zeta}$$

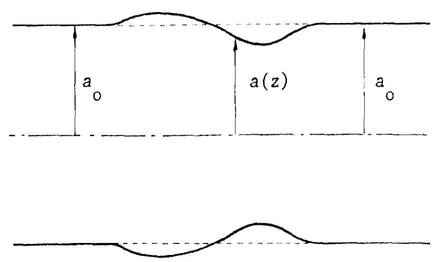

Fig.5

Conclusion

We have calculated the energy loss suffered by an arbitrary current distribution for the case where the waveguide is weakly perturbed in periodic fashion, up to second order in the perturbation parameter. In principle, the loss of energy originates from a coupling to all the TM waveguide modes that are synchronous with the electron beam, but in fact only a few modes are significant because of the fast decay of the radiated field spectral density. In this respect, the following points should be noted.

a) in the expression of the energy loss the bunch shape enters through the square of the current distribution spectral density.

b) the geometry of the guiding structure enters through the factor $p\, C_p\, C_{-p}$ and the value of ν_{np}

c) a finite value of γ contributes to the reduction of the number of relevant term in the series.

We have shown that the energy loss reaches a limit as γ goes to infinity. The shorter the bunch length, the higher is that limit.

Finally our results can also be applied to non periodic structures.

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