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OPTIMIZATION OF LUMINOSITY FOR e-p COLLISIONS

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1. Introduction

At present CERN is studying a large electron-positron storage ring, LEP[1,2] which, if it were sited at CERN, would also permit e-p collisions with protons from the SPS. The high centre-of-mass energy available makes this an attractive option despite the fact that only one interaction region could be provided. In this paper two different operating modes are discussed and an algorithm given for maximizing the luminosity in each case. The luminosity so calculated will be an important factor in evaluating the physics potential of such a facility.

2. Assumptions

2.1 Protons

It will be assumed that protons are brought out from the SPS into an external bypass[3] to collide with electrons (or positrons) in LEP. This minimizes the perturbation to the SPS, which remains essentially unaltered. The characteristics of the SPS for e-p collisions have been optimized in the course of the CHEEP study[4] and most of these values will be adopted. However, only 50 proton bunches will be required[5] instead of 60 so the bunch charge may be increased to give the same circulating current. Preliminary experiments in the SPS indicate that this is not an unreasonable extrapolation[6]. The horizontal beta function β_{ph} will be changed slightly as a result of the optimization.

2.2 Electrons

It will be assumed that collisions occur in one of the LEP interaction regions with essentially no change in the electron ring geometry to minimize the extra synchrotron radiation produced near the interaction region. LEP should therefore be sited sufficiently close to the SPS to make this layout possible with a reasonable bypass and should be oriented in such a way that a LEP insertion is at the point of nearest approach to the SPS. The parameters of the electron ring are from LEP Version 8[2]. The insertion requirements are less stringent than for e^+e^- and it will be assumed that both beta functions β_{ev}, β_{eh} can be optimized (this is less restrictive than assuming the same beta ratio[7] as LEP and results in a higher luminosity). Wigglers will be used to maintain the electron emittance constant at all energies[1].

2.3 General

Two operating modes will be considered. In parasitic mode e-p collisions occur at the same time as e^+e^- collisions in the other seven insertions which are assumed to have precedence and hence determine the circulating current and number of bunches. In the e-p insertion the electrons and positrons would be separated by electrostatic plates preventing e^+e^- collisions and the protons would only interact with one beam. In dedicated mode only one beam circulates in LEP, which can be optimized for e-p collisions. This yields a higher luminosity in the e-p insertion but the other seven insertions remain idle. In both modes the SPS would be totally dedicated to e-p. In order to obtain an upper limit to the luminosity a zero crossing angle is assumed though it is by no means obvious that this is technically feasible. The numerical values assumed are given in Table 1.

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Table 1. Numerical Values Adopted

	Protons	Electrons	
Energy	$130 \le E_p \le 270$	$20 \le E_e \le 80$	GeV
Particles per bunch	$N_{po} = 4.0\times10^{11}$	$N_{eo} = 1.48\times10^{12}\times E_e/80$**	
No. of bunches parasitic mode	$k_p = 10$	$k_e = 4$	
No. of bunches dedicated mode	$K_p = 50$	$K_e = 220$	
Vertical emittance*	$\varepsilon_{pv} = 10\times10^{-6}/\beta\gamma$	$\varepsilon_{ev} = 4.28\times10^{-9}$	πradm
Horizontal emittance*	$\varepsilon_{ph} = 20\times10^{-6}/\beta\gamma$	$\varepsilon_{eh} = 6.85\times10^{-8}$	πradm
Vertical beta function	$\beta_{pv} = 0.6$	β_{ev} optimized	m
Horizontal beta function	β_{ph} optimized	β_{eh} optimized	m
Maximum tune shift	$\Delta Q_p = 0.005$ or 0.01	$\Delta Q_e = 0.06$	

* for protons $\varepsilon = (2\sigma)^2/\beta$; for electrons $\varepsilon = \sigma^2/\beta$
** in parasitic mode.

3. The Equations

3.1 Beam-beam tune shifts

There are four separate constraints on the beam-beam tune shifts

$$\Delta Q_{ph} = \frac{r_p N_e \beta_{ph}}{2\pi\gamma_p \sigma_{eh}(\sigma_{eh} + \sigma_{ev})} \le 0.005\ (0.01) \tag{1}$$

$$\Delta Q_{pv} = \frac{r_p N_e \beta_{pv}}{2\pi\gamma_p \sigma_{ev}(\sigma_{eh} + \sigma_{ev})} \le 0.005\ (0.01) \tag{2}$$

$$\Delta Q_{eh} = \frac{r_e N_p \beta_{eh}}{2\pi\gamma_e \sigma_{ph}(\sigma_{ph} + \sigma_{pv})} \le 0.06 \tag{3}$$

$$\Delta Q_{ev} = \frac{r_e N_p \beta_{ev}}{2\pi\gamma_e \sigma_{pv}(\sigma_{ph} + \sigma_{pv})} \le 0.06 \tag{4}$$

The first step in the optimization is to choose the ratio of the beta values to equalize the tune shifts in the two planes for each beam. Thus from (1) and (2)

$$\frac{\beta_{ph}}{\beta_{pv}} = \frac{\sigma_{eh}}{\sigma_{ev}} = \left(\frac{\beta_{eh}}{\beta_{ev}}\frac{\varepsilon_{eh}}{\varepsilon_{ev}}\right)^{1/2} \tag{5}$$

and from (3) and (4)

$$\frac{\beta_{eh}}{\beta_{ev}} = \frac{\sigma_{ph}}{\sigma_{pv}} = \left(\frac{\beta_{ph}}{\beta_{pv}}\frac{\varepsilon_{ph}}{\varepsilon_{pv}}\right)^{1/2} \tag{6}$$

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and combining (5) and (6) defines the required beta ratios

$$\beta_{er} = \frac{\beta_{eh}}{\beta_{ev}} = \left(\frac{\varepsilon_{eh}}{\varepsilon_{ev}}\frac{\varepsilon_{ph}^2}{\varepsilon_{pv}^2}\right)^{1/3} \quad (7)$$

$$\beta_{pr} = \frac{\beta_{eh}}{\beta_{ev}} = \left(\frac{\varepsilon_{eh}^2}{\varepsilon_{ev}^2}\frac{\varepsilon_{ph}}{\varepsilon_{pv}}\right)^{1/3} \quad (8)$$

These two simple equations are the basis for the insertion design. Note that the optimum ratio of beta values for each beam is independent of energy. However, the ratio of electron to proton beta values should be varied as a function of the energies of the two beams. Putting in numerical values from Table 1 gives $\beta_{er} = 4$, $\beta_{pr} = 8$. The proton beta ratio is rather similar to that adopted for CHEEP[4] (10.8) and does not seem unreasonable. On the other hand the electron beta ratio is very different from that used in LEP[1,2] (16) and cannot be achieved with very small beta values. A minimum value of $\beta_{ev} = \beta_{pv}$ will be taken somewhat arbitrarily to show the influence of such a limit on the luminosity. A detailed insertion design would be needed to give a better estimate.

3.2 Number of particles per bunch

Imposing conditions (7) and (8) ensures that the tune shifts in the two planes are equal for each beam. The number of particles per bunch is then either determined by the tune shift or by the maximum bunch population given in Table 1, whichever is the smaller. Thus for the electrons

$$N_e = \frac{2\pi\gamma_p\varepsilon_{ev}}{r_p} \cdot \frac{\beta_{pr}+1}{\Delta Q_p} \cdot \frac{\beta_{ev}}{\beta_{pv}} \leq N_{eo} \quad (9)$$

while for the protons

$$N_p = \frac{\pi\gamma_e\varepsilon_{pv}}{2r_e} \; \frac{\beta_{er}+1}{\Delta Q_e} \; \frac{\beta_{pv}}{\beta_{ev}} \leq N_{po} \quad (10)$$

3.3 Luminosity

The luminosity for head-on collisions is given by

$$\mathcal{L} = \frac{f_e k_e N_e N_p}{2\pi(\sigma_{ph}^2 + \sigma_{eh}^2)^{1/2}(\sigma_{pv}^2 + \sigma_{ev}^2)^{1/2}} \quad (11)$$

where it is assumed that each time an electron bunch passes the interaction region it finds a proton bunch. However, a proton bunch will not necessarily collide with an electron bunch every time. Substitution of equations (7) and (8) gives the optimum luminosity.

$$\mathcal{L} = \frac{2 f_e k_e N_e N_p}{\pi\varepsilon_{pv}\beta_{pv}\beta_{er}\left\{1 + \frac{4\varepsilon_{ev}}{\varepsilon_{pv}}\left(\frac{\beta_{pr}}{\beta_{er}}\right)^2\frac{\beta_{ev}}{\beta_{pv}}\right\}^{1/2}\left\{1 + \frac{4\varepsilon_{ev}}{\varepsilon_{pv}}\frac{\beta_{ev}}{\beta_{pv}}\right\}^{1/2}} \quad (12)$$

In Fig. 1 the luminosity per bunch crossing at the top energy is plotted as a function of β_{ev}/β_{pv}, the only free variable in (12). At small values of β_{ev}, N_e is limited by (9) while N_p is at the maximum value N_{po}. At intermediate values of β_{ev} both N_e and N_p are limited, by (9) and (10) respectively. At high values of β_{ev}, N_p is limited by (10) while N_e is at the maximum value N_{eo}.

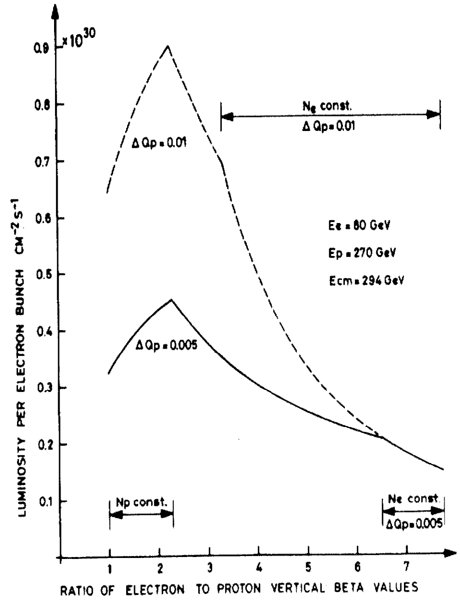

Fig. 1. Maximum luminosity per electron bunch

4. Performance

4.1 Parasitic mode

The maximum luminosity in parasitic mode is obtained at the second kink in the curves in Fig. 1. This corresponds to putting the maximum value of N_e into (9) and using the maximum permissible value of ΔQ_p (0.005 or 0.01). The value of β_{ev} which maximizes the luminosity is proportional to E_e/E_p and it will be assumed that the electron ring insertion allows the beta values to be tuned over the required range. This gives an upper limit to the luminosity in parasitic mode which is plotted in Fig. 2. The peak luminosity occurs at the highest centre-of-mass energy (294 GeV) and is $8.1\times10^{29}\text{cm}^{-2}\text{s}^{-1}$ for $\Delta Q_p = 0.005$ and $2.8\times10^{30}\text{cm}^{-2}\text{s}^{-1}$ for $\Delta Q_p = 0.01$. In this regime with N_e fixed, an increase in the permissible ΔQ_p allows both a reduction in the electron beam dimensions and an increase in N_p which accounts for the factor of nearly 3½ between the luminosities in the two cases. In Fig. 2 the effect of imposing a minimum value of β_{ev} is also shown.

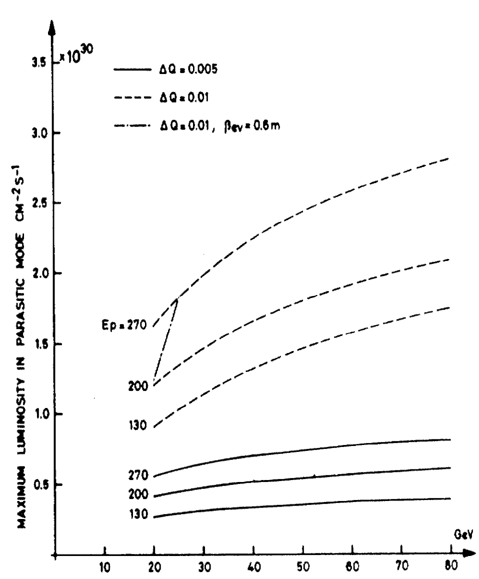

Fig. 2. Maximum luminosity in parasitic mode.

4.2 Dedicated mode

The maximum luminosity per bunch crossing is obtained at the first kink in the curves in Fig. 1. This corresponds to putting the maximum value of N_p into (10). The electron bunch charge is then smaller than that required for e^+e^- amd there is only one beam. There will be sufficient RF power to increase the number of bunches to the maximum at low electron energy and this is the optimum working condition. At high electron energies either the number of bunches or the charge per bunch must be reduced but it is always best to work with the highest number of bunches possible, contrary to some previous assumptions[7,8].

For fixed optics, the luminosity is proportional to N_p (determined by space charge forces in the proton ring) and $f_e\ k_e\ N_e$, which is just the circulating electron current divided by the electronic charge. (This assumes correct synchronization which can only be obtained with certain discrete values of k_b). At high electron energies the maximum circulating current is limited by the RF power available but is higher if the charge is distributed over many bunches. This is because higher mode losses are proportional to the peak charge, not the total charge[9]. In LEP Version 8 at 80 GeV the maximum current in four bunches in one beam is 13.4 mA but in 220 bunches it is 48.0 mA - a considerable increase. There is a second effect which also favours many bunches. The number of electrons per bunch is smaller permitting the electron beam dimensions to be reduced without exceeding the proton tune shift. The 'specific luminosity' per electron is thereby increased giving a further improvement in the luminosity. The maximum luminosity in dedicated mode at top energy is plotted in Fig. 3 as a function of the number of bunches (the circles indicate allowed values) and this shows unequivocally that the best operating condition is to use the greatest number of bunches.

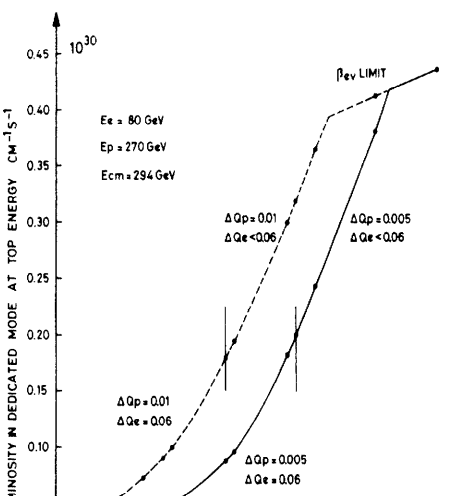

Fig. 3. Luminosity at top energy as a function of the number of electron bunches.

The maximum luminosity in dedicated mode as a function of energy is calculated as follows. The maximum number of protons per bunch N_p is always used. The program BEAMPARAM is used to evaluate the maximum number of electrons per bunch in 220 bunches as a function of energy. At high electron energies, the number of electrons is RF limited and β_{ev}/β_{pv} is calculated from (9). At low electron energies the number of electrons is RF tune-shift limited and β_{ev}/β_{pv} is calculated from (10). At very high and very low electron energies the optimum β_{ev} is lower than the permissible value and limiting β_{ev} causes the luminosity to decrease more rapidly than the optimum. This is plotted in Fig. 4. The maximum luminosity of the facility in dedicated mode is $1.7\times10^{32}cm^{-2}s^{-1}$ at 244 GeV c-m energy if $\Delta Q_p = 0.01$ and $0.93\times10^{32}cm^{-2}s^{-1}$ at 265 GeV c-m energy is $\Delta Q_r = 0.005$.

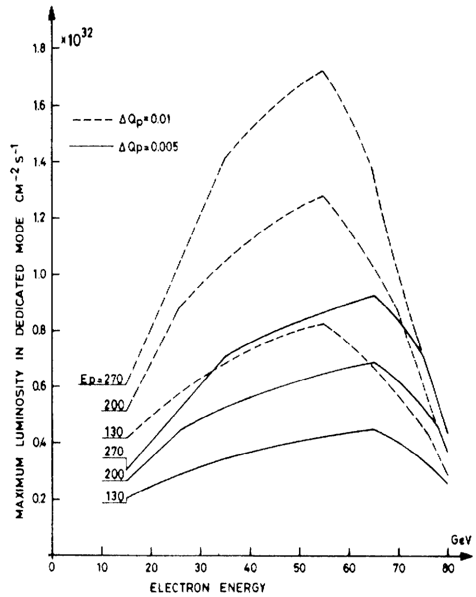

Fig. 4. Maximum luminosity in dedicated mode.

Conclusions

An algorithm for optimizing the luminosity in e-p collisions has been given. The ratio of the beta values for each beam is given by equations (7) and (8). β_{pv} is taken as the smallest possible value and β_{ev} is chosen to maximize the luminosity in the two operating modes - the smallest value of β_{ev} is not necessarily the best, particularly in parasitic mode. In dedicated mode it is demonstrated that the maximum number of bunches should be used at all energies.

The e-p facility which could be obtained by colliding electrons from LEP Version 8 with protons from the CERN SPS would cover the centre-of-mass energy range of 100-300 GeV with a maximum luminosity of at least $10^{32}cm^{-2}s^{-1}$ in dedicated mode but a factor of 60 less in parasitic mode.

References

1. The LEP Study Group, Design Study of a 15 to 100 GeV e^+e^- Colliding Beam Machine (LEP), CERN/ISR-LEP/78-17, 1978.
2. W. Schnell, Design Study of a Large Electron-positron Colliding Beam Machine (LEP), This conference (C2).
3. Y. Baconnier and W. Scandale, A Bypass for the SPS, SPS/AOP/DV/Note/78-23, 1978.
4. CHEEP Study Group, Physics with e-p Colliding Beams, CERN 78-2, 1978.
5. A. Hutton, Choice of the LEP Circumference for e-p Compatibility, LEP note 108, 1978 (unpublished).
6. J. Gareyte et al., Acceleration and Storage of a Dense Single Bunch in the CERN SPS, This conference (F9).
7. A. Hutton, e-p with LEP-70 and the SPS, CERN/ISR-TH/78-7, 1978.
8. E. Dasskowski et al., An e-p Facility at PETRA, DESY 78-2, 1978.
9. A. Hutton, Insertion Design for e^+e^- Storage Rings, This conference (F21).