

STABLE PARTICLE MOTION IN A LINEAR ACCELERATOR WITH SOLENOID FOCUSING*

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Abstract

We derived the equation governing stable particle motion in a linear ion accelerator containing discrete rf and either discrete or continuous solenoid focusing. We found for discrete solenoid focusing that

$$\cos \mu = (1 + d\Delta) \cos \theta/2 + \left(\frac{\lambda\Delta}{\theta} - \frac{d\theta}{2\lambda} - \frac{\theta\Delta d^2}{4\lambda} \right) \sin \theta/2$$

$$\Delta = 1/f \text{ and } \lambda + 2d = \beta\lambda,$$

where μ , θ , f , λ , and d are the phase advance per cell, precession angle in the solenoid, focal length of the rf lens, length of the solenoid in one cell, and the drift distance between the center of the rf gap and the effective edge of the solenoid. The relation for a continuous solenoid is found by setting d equal to zero. The boundaries of the stability region for θ vs Δ with fixed λ and d are obtained when $\cos \mu = \pm 1$.

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Introduction

The stability relations are derived for a linear ion accelerator with solenoidal focusing for the discrete case with solenoids in the drift tubes and for the continuous case with the accelerator contained in one continuous solenoid. We begin by giving the first-order transport matrix for the solenoid.¹ Next we calculate the transport matrix for one cell and derive the stability relation for the discrete solenoid. Finally, we examine the case of the continuous solenoid and show that it is a special case of discrete focusing. The stability curves for both the continuous and discrete solenoids are shown in Figs. 1 and 2, respectively.

First-Order Solenoid Transport Matrix¹

We first consider the interior region of the solenoid where the magnetic field is assumed constant and directed along the solenoid axis (z axis). A particle with nonzero transverse velocity v_T will spiral in

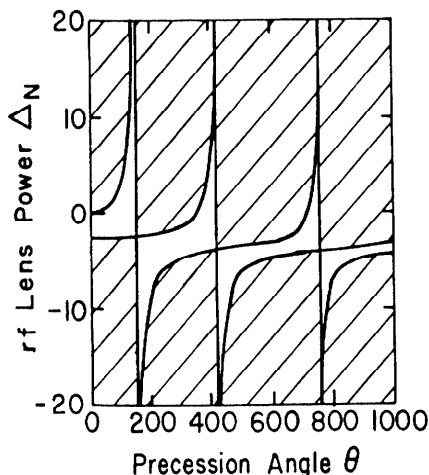
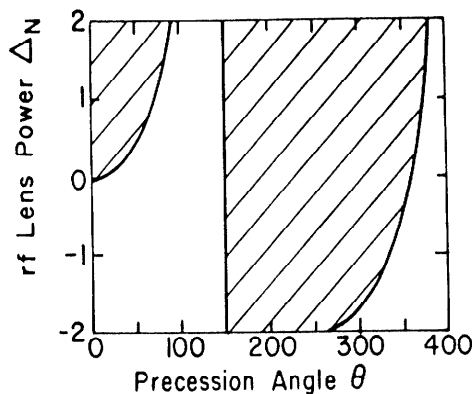
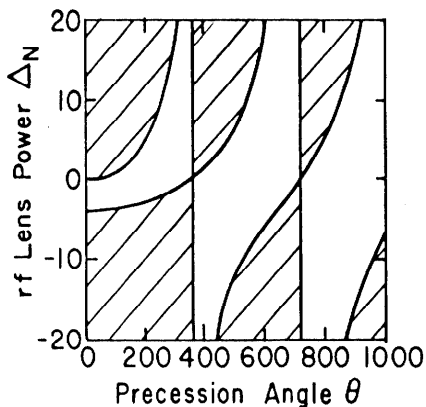
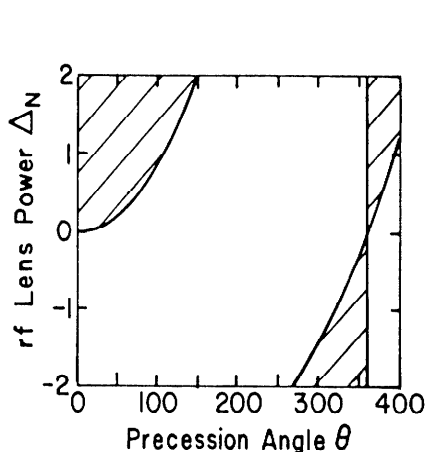


Fig. 1. Stability diagram for the continuous solenoid. Nonshaded areas are stable regions. Delta (Δ_N) is $\beta\lambda$ divided by the focal length of the rf lens and θ is the precession angle in the solenoid.

Fig. 2. Stability diagram for the discrete solenoid. Nonshaded areas are stable regions. Delta (Δ_N) is $\beta\lambda$ divided by the focal length of the rf lens and θ is the precession angle in the solenoid of length $1/2 \beta\lambda$.

the magnetic field and will project a circle in the xy plane. Let

B_0 = magnetic field strength directed along z,
 v_T = transverse velocity of the particle,
 v_z = longitudinal velocity,
 ℓ = length of the solenoid,

and θ = the precession angle
 = $eB_0\ell/(mv_z)$. (1)

where e and m are the electric charge and mass of the particle. The transport matrix for the interior of the solenoid \underline{S} is

$$\underline{S} = \begin{bmatrix} 1 & (\ell \sin \theta)/\theta & 0 & \ell(1 - \cos \theta)/\theta \\ 0 & \cos \theta & 0 & \sin \theta \\ 0 & \ell(\cos \theta - 1)/\theta & 1 & (\ell \sin \theta)/\theta \\ 0 & -\sin \theta & 0 & \cos \theta \end{bmatrix} \quad (2)$$

where \underline{S} operates on the vector \underline{V}

$$\underline{V} = \begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}$$

and

$$x' = dx/dz \\ y' = dy/dz$$

The fringe field transform \underline{F} is

$$\underline{F} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \theta/2\ell & 0 \\ 0 & 0 & 1 & 0 \\ -\theta/2\ell & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

The total transport matrix \underline{M}_s for the solenoid is

$$\underline{M}_s = \underline{F}(-\theta)\underline{S}\underline{F}(\theta) = \begin{bmatrix} C^2 & SC/\alpha & SC & S^2/\alpha \\ -SC\alpha & C^2 & -S^2\alpha & SC \\ -SC & -S^2/\alpha & C^2 & SC/\alpha \\ S^2\alpha & -SC & -SC\alpha & C^2 \end{bmatrix} \quad (4)$$

with $S = \sin(\theta/2)$, $C = \cos(\theta/2)$, and $\alpha = \theta/2\ell$.

The matrix \underline{M}_s may be written as

$$\underline{M}_s = \underline{M}\underline{R},$$

where

$$\underline{R} = \begin{bmatrix} C & 0 & S & 0 \\ 0 & C & 0 & S \\ -S & 0 & C & 0 \\ 0 & -S & 0 & C \end{bmatrix}$$

is a rotation matrix and

$$\underline{M} = \begin{bmatrix} C & S/\alpha & 0 & 0 \\ -S\alpha & C & 0 & 0 \\ 0 & 0 & C & S/\alpha \\ 0 & 0 & -S\alpha & C \end{bmatrix}$$

is block diagonal. Thus we decouple x and y and write for x,

$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \begin{bmatrix} C & S/\alpha \\ -S\alpha & C \end{bmatrix} \begin{bmatrix} x_1 \\ x_1' \end{bmatrix} \quad (5)$$

Transport with Discrete Solenoid

One cell in a linac consists of one-half of an rf defocusing lens followed by a drift, a solenoid, a drift, and one-half of the rf defocusing lens. The transport matrix \underline{M}_c for the cell is

$$\underline{M}_c = \underline{f} \underline{D} \underline{M} \underline{D} \underline{f}, \quad (6)$$

where \underline{f} is the rf lens, \underline{D} is a drift, and \underline{M} is the solenoid in the rotated frame

$$\underline{f} = \begin{bmatrix} 1 & 0 \\ \Delta/2 & 1 \end{bmatrix} \quad (7)$$

$$\underline{D} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \quad (8)$$

$$\underline{M} = \begin{bmatrix} C & S/\alpha \\ -S\alpha & C \end{bmatrix} \quad (9)$$

and

$$\underline{M}_c = \begin{bmatrix} (1+d\Delta)C + \left(\frac{\ell\Delta}{\theta} - \frac{d\theta}{2\ell} - \frac{\theta\Delta d^2}{4\ell}\right)S & 2Cd + 2\ell S/\theta - S\theta d^2/2\ell \\ C\Delta\left(1 + \frac{d\Delta}{2}\right) + S\left(\frac{\ell\Delta^2}{2\theta} - \frac{\theta}{2\ell}\left(1 + \frac{d\Delta}{2}\right)^2\right) & (1+d\Delta)C + \left(\frac{\ell\Delta}{\theta} - \frac{d\theta}{2\ell} - \frac{\theta\Delta d^2}{4\ell}\right)S \end{bmatrix} \quad (10)$$

($\Delta = 1/\text{focal length of the rf lens}$). Because one period consists of an integral number of cells, n, one has a relation between θ , Δ , ℓ , d and $\cos \mu$, where²

$$\cos \mu = 1/2 \text{Tr}(\underline{M}_c)$$

$$= (1 + d\Delta) \cos \theta/2 + \left(\frac{\ell\Delta}{\theta} - \frac{d\theta}{2\ell} - \frac{\theta\Delta d^2}{4\ell}\right) \sin \theta/2 \quad (11a)$$

Continuous Solenoid

The main calculational difference between the continuous case and the discrete solenoid case is that a rotation matrix does not exist, which when applied to \underline{S} , Eq. (2), decouples x from y in a fixed reference frame. We could approach this problem in the Larmor frame using canonical variables to obtain a decoupling in x and y for the solenoid but we would then have Coriolis forces in the gap. We sidestep this problem by making the following observation. The continuous solenoid case consists of the following transport stream.

S f S f S f

While the discrete case is

S F D f D F S F D f D F S ...

See Eqs. (2), (3), (7), and (8). The difference between the two is that f in the continuous case is $F D f D F$ in the discrete case. We, therefore, study $F D f D F$ and take the limit $d \rightarrow 0$ to obtain

$$\lim_{d \rightarrow 0} \begin{bmatrix} F & D & f & D & F \\ D & f & D & F & S \end{bmatrix} = \underline{f} .$$

Thus, the stability relation Eq. (11a) for the discrete case may be used for the continuous case by setting $d = 0$.

Results

The stability diagrams for continuous and discrete solenoids are given in Figs. 1 and 2. Δ_N is $\beta\lambda$ divided by the focal length of the rf lens and positive Δ_N corresponds to defocusing rf. The precession angle θ is proportional to the solenoid's magnetic field strength, length, and the particle's mass, charge, and velocity Eq. (1).

The nonstable region is shaded. For the discrete case, both the solenoid length ℓ and drift distance $2d$ were equal to $1/2 \beta\lambda$, following Smith and Gluckstern.³ Note that 1 cell = 1 period.

Each region is bounded by a line that is independent of the rf defocusing strength Δ . The reason for this may be seen by factoring the equation for $\cos \mu$ (Eq. 11a) into two pieces, one of which is independent and one is linearly proportional to Δ

$$\cos \mu = (C - dS \theta/2\ell) + (Cd + S\ell/\theta - d^2S\theta/4\ell)\Delta . \quad (11b)$$

A region boundary ($\cos \mu = \pm 1$) is independent of Δ if

$$Cd + S\ell/\theta - d^2S\theta/4\ell = 0 , \quad (12)$$

and

$$C - dS\theta/2\ell = \cos \mu = \pm 1 . \quad (13)$$

The consistency of these two equations is shown by solving for θ in Eq. (13) and substituting the result in Eq. (12). This results in the identity

$$C^2 + S^2 = \cos^2 \theta/2 + \sin^2 \theta/2 = 1.$$

Acknowledgments

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