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## A PROPOSED ORBIT AND VERTICAL DISPERSION CORRECTION SYSTEM FOR PEP*

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## Introduction

The scheme for minimization of the rms orbit errors in ISR ${ }^{1}$ and SPEAR ${ }^{2}$ will be used to correct the closed orbit errors in PEP. The effectiveness of this scheme has been studied for some alignment and field errors in the PEP magnets and position monitors. It has been found for orbit correction system in PEP consisting of 48 correctors and 96 monitors, both horizontal and vertical orbits can be kept below 0.5 mm rms values even allowing for a position monitor alignment error of 0.5 mm rms . This method of correction has also been found to be usable to reduce the rms value of the vertical dispersion without appreciably affecting the corrected orbits. The result of this study will be presented in this paper.

## The Correction Scheme

Let vectors $\vec{y}$ and $\vec{\eta}_{y}$ be the vertical orbit and dispersion at the position monitors. Let $\vec{\theta}$ be the dipole kicks at the correctors. The values of the orbit and dispersion vectors after correction are given by the sum of the measured values and the values introduced by the correctors as:

$$
\begin{equation*}
\vec{y}=\vec{y}_{m}+\vec{y}_{c} \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
\vec{\eta}_{y}=\vec{n}_{\mathrm{m}}+\vec{n}_{\mathrm{c}} \tag{2}
\end{equation*}
$$

If thin lens approximation is used for each magnet

$$
\begin{equation*}
\vec{y}_{c}=\overrightarrow{M \theta} \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
\vec{n}_{c}=N \vec{\theta} \tag{4}
\end{equation*}
$$

where the matrix elements are

$$
\begin{equation*}
M_{i j}=\frac{\sqrt{\beta y i} y_{y j}}{2 \sin \left(\pi v_{y}\right)} \cos \left(\pi v_{y}-\left.\right|_{y i}-\phi_{y j} \mid\right) \tag{5}
\end{equation*}
$$

and

$$
N_{i j}=\frac{\sqrt{\beta_{y i} \beta_{y j}}}{2 \sin \left(\pi v_{y}\right)} \cdot\left\{\begin{array}{l}
+\cos \left(\pi \nu_{y}-\left|\phi_{y i}-\phi_{y j}\right|\right), j \text { th dipole }  \tag{6}\\
+\sum_{q} \frac{\beta_{y q}\left(\frac{g l}{B \rho}\right)_{q}}{2 \sin \left(\pi \nu_{y}\right)} \cos \left(\pi v_{y}-\left|\phi_{y i}-\phi_{y q}\right|\right) \\
\cdot \cos \left(\pi \nu_{y}-\left|\phi_{y i}-\phi_{y j}\right|\right), q \text { quadrupoles } \\
-\sum_{s} \frac{\pi_{x s} \beta_{y s}\left(\frac{2 S l}{2 \rho}\right)_{s}}{2 \sin \left(\pi v_{y}\right)} \cos \left(\pi v_{y}-\left|\phi_{y i}-\phi_{y s}\right|\right) \\
\cdot \cos \left(\pi \nu_{y}-\left|\phi_{y i}-\phi_{y j}\right|\right), \text { s sextupoles. }
\end{array}\right.
$$

[^0]In these expressions the strengths of the quadrupole and sextupole magnets are given by $g=\partial B_{y} / \partial x$ and $2 S=\partial B_{y}^{2} / \partial x^{2} ; \ell=$ magnet length; $B \rho=$ particle rigid1ty; $\beta_{y}, \phi_{y}$ and $v_{y}$ are the betatron function, betatron phase and tune, respectively. The value of $\eta_{c}$ results from the dipole corrector and the change in the orbit at the quadrupole and sextupole magnets caused by the correction.

It can be shown ${ }^{2}$ easily that the corrector strengths corresponding to the minimum value of $y_{\text {rms }}$ is given by

$$
\begin{equation*}
\vec{\theta}=-\left(\tilde{M M}^{-1} \stackrel{\widetilde{M y}}{\mathrm{~m}}\right. \tag{7}
\end{equation*}
$$

with $\tilde{M}$ denoting the transpose of $M$, and a similar result for minimum $x_{r m s}$ by interchanging $y$ with $x$. For minimum $\eta_{y}$ rms the solution is

$$
\begin{equation*}
\vec{\theta}=-(\tilde{\mathrm{N} N})^{-1}{\widetilde{\mathrm{~N}} \overrightarrow{\mathrm{n}}_{\mathrm{m}}} \tag{8}
\end{equation*}
$$

Given a measured orbit vector we find the $n$ most effective correctors up to 48 as follows: First we calculate $y_{\text {rms }}$ using Eqs. (1), (3) and (7) for each of the 48 correctors and determine the corrector $C_{1}$ corresponding to the smallest $y_{r m s}$ values. We then calculate $y_{\text {rms }}$ for all possible pair of correctors $\left(C_{1}, C\right)$ and determine $C_{2}$ the second corrector which when paired with $C_{1}$ produces the smallest yrms value. This procedure is repeated to find the third corrector which when coupled with $C_{1}$ and $C_{2}$ makes $y_{\text {rms }}$ smallest. After n times, the best n correctors will be found. The strength of the corrector vector $\vec{d}$ for each $n$ is given by Eq. (7).

## The Corrected Orbit

The measured orbit and dispersion $\vec{y}_{m}$ and $\vec{\eta}_{m}$, were simulated by a computer code ${ }^{3}$ taking into account the surveying errors. The errors are assumed to be gaussian with the sigma values given below:

- vertical misalignment of quadrupoles 0.2 mm
- horizontal misalignment of quadrupoles 0.2 mm
- relative field error in the dipoles $0.01 \%$
- misalignment of secondary monuments 0.3 mm
- angular misalignment of major monuments $20 \mu \mathrm{rad}$.

The orbit and dispersion values produced by error values randomly selected with the above sigmas were calculated and displayed at the midpoint of every dipole, quadrupole, sextupole, position monitor and corrector around the ring (a total of 1044 points). A typical PEP configuration with $\nu_{x}=21.23, \nu_{y}=18.67$, $\beta_{\mathrm{y}}^{*}=0.11 \mathrm{~m}, \beta_{\mathrm{x}}^{*}=2.88 \mathrm{~m}$ and $n_{\mathrm{x}}^{*}=-.468 \mathrm{~m}$ was used for this study, where a * denotes the interaction point value.

In order to obtain a statistically significant result, we studied the orbit correction for 20 machines. For each machine, the measured orbit was simulated with a different set of random alignment and field errors. In all cases 48 correctors and 96 position monitors were used. Their layout is given in Fig. 6 for $1 / 12$ of the lattice (see the appendix).

Table 1 gives the average of the rms values of the corrector strength, the orbit and dispersion before and after correction over the 20 cases considered.

Table 1

| Monitor <br> Position Error mm | Horfzontal <br> Corrector <br> Strength <br> $\mu \mathrm{rad}$ |  | ```Horizontal Orbit Values mm``` |  | Vertical <br> Corrector Strength urad |  | Vertical Orbit Values mn |  | Vertical Dispersion Values mn |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| rms | rms | max | rms | max | rms | max | rms | max | rms | max |
| - | 0 | 0 | 6.5 | 26 | 0 | 0 | 15 | 89 | 525 | 2640 |
| 0 | 63 | 148 | 0.4 | 1.8 | 49 | 123 | 0.4 | 1.4 | 105 | 635 |
| 0.5 | 76 | 181 | 0.6 | 2.2 | 53 | 133 | 0.5 | 1.7 | 136 | 806 |
| 1.0 | 107 | 248 | 0.9 | 3.4 | 64 | 158 | 0.7 | 2.4 | 199 | 1148 |

In addition, Table 1 also gives the rms value of the maximum corrector strength, the orbit and dispersion before and after orbit correction over the 20 cases. The values after correction are shown for three position monitor alignment errors with sigmas equal to $0.0,0.5$ and 1.0 mm .

In practice, it may be unnecessary to use all of the 48 correctors everytime we correct the orbit. The effectiveness of the correction as a function of the number of corrector for a typical case is shown in Fig. 1. In this case, 15 correctors should be sufficient.


Fig. 1. Vertical closed orbit and dispersion versus number of correctors (rms of one "machine").

## The Corrected Dispersion

From the operating experience of SPEAR it may be necessary to reduce the vertical dispersion in PEP after the orbit is corrected. The effectiveness of the dispersion correction using the orbit correctors with the same minimization scheme as the orbit correction has been studied. Table 2 gives the results for a typical case. For this case it can be seen that both $\eta_{y}$ max and $\eta_{y}$ rms can be reduced by a factor of 5 with only 8 correctors. The effects upon the corrected orbit is relatively small since the strength of the correctors is less than $10 \%$ of those required for orbit correction. This method has been tried experimentally at SPEAR and obtained the predicted results.

## The Harmonic Representation

Some of the results described in the previous sections could be explained in terms of the harmonics of the orbit and dispersion. For example, Fig. 2 shows the harmonics of the vertical orbit before orbit correction for the case described in Fig. 1. It can be seen that only those harmonics near 19 are dominant.

Table 2

| $\begin{aligned} & \text { Number } \\ & \text { of } \\ & \text { Correctors } \end{aligned}$ | Vertical <br> Corrector Strengths urad |  | Vertical <br> Orbit <br> Values <br> mm |  | Vertical Dispersion Values mm |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | rms | $\max$ | rms | max | rms | max |
| 0 | 0 | 0 | 0.4 | 1.1 | 64 | 304 |
| 1 | 15 | 15 | 0.6 | 2.1 | 33 | 114 |
| 2 | 9 | 11 | 0.6 | 2.1 | 32 | 106 |
| 3 | 7 | 11 | 0.5 | 2.0 | 31 | 108 |
| 4 | 10 | 12 | 0.5 | 1.3 | 23 | 62 |
| 5 | 9 | 14 | 0.5 | 1.3 | 21 | 60 |
| 6 | 8 | 13 | 0.5 | 1.4 | 17 | 58 |
| 7 | 8 | 13 | 0.5 | 1.5 | 16 | 63 |
| 8 | 9 | 14 | 0.5 | 1.5 | 15 | 61 |



Fig. 2. Harmonics of the normalized vertical closed orbit (before correction).

After orbit correction, all of the dominant harmonics were reduced as shown in Fig. 3. This residual orbit


Fig. 3. Harmonics of the normalized vertical closed orbit (after correction).
produces a dispersion having only a few predominant harmonics, as shown in Fig. 5, so that only a small number of correctors will be needed for dispersion correction. Note that the dispersion before orbit correction has dominant harmonics not just centered around 19 but also near $19 \pm 6$ as shown in Fig. 4; 6 is the machine superperiodicity.


Fig. 4. Harmonics of the normalized vertical dispersion (before correction).


Fig. 5. Harmonics of the normalized vertical dispersion (after correction).

## Appendix

The effects of the orbit corrections have been examined for several systems with different position monitor and corrector locations. It has been found that the chosen system, which has correctors situated
close to position monitors, gives the smallest residual orbit with the minimum corrector strength. A schematic layout of this system is given in Fig. 6 for $1 / 12$ of the lattice.


$$
\begin{array}{ll}
\text { I.P. }=\text { Interaction Point } & Q=\text { Quadrupole Magnet } \\
\text { S.P. }=\text { Symmetry Point } & H=\text { Horizontally Correcting Dipole } \\
\text { B.P. }=\text { Beam Position Monitor } & V=\text { Vertically Correcting Dipole } \\
\text { I- } &
\end{array}
$$

Fig. 6. A layout of a half-superperiod showing the relative position of the position monitors, dipole correctors, and the ring quadrupole magnets (48 correctors).

## References

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