IEEE Transactions on Nuclear Science, Vol. NS-26, No. 3, June 1979

THE REVISED SKEW QUADRUPOLE SYSTEM FOR COUPLING COMPENSATION IN THE CERN INTERSECTING STORAGE RINGS (ISR)

P.J. Bryant

Abstract

With the existing and proposed installations of solenoid detector magnets and low-β schemes, the original coupling compensation scheme in the ISR was no longer sufficiently versatile to provide adequate compensation under all conditions. A brief description of the new scheme, which was installed early in 1978, is given. This scheme can excite coupling vectors at any phase so that it is able to compensate solenoids and large localized quadrupole errors in low-β schemes as well as random magnet tilts. The design of the scheme is also strongly influenced by the fact that there are no regions with zero horizontal dispersion in the ISR and the skew quadrupoles have to be specially arranged so as to avoid exciting vertical dispersion. The experience gained with this scheme and the methods used for measurement and correction of the machine are illustrated by practical examples.

1. Introduction

In the ISR, it is necessary to be able to compensate the linear, second-order, zero-harmonic, coupling resonance $Q_x - Q_z = 0$. The excitation of this resonance can be expressed in the form of a complex coupling coefficient, C,[1] given by

$$C = \frac{1}{2\pi R}\int_0^{2\pi}\sqrt{\beta_x\beta_z}\left[K + \frac{MR}{2}\left(\frac{\alpha_x}{\beta_x}-\frac{\alpha_z}{\beta_z}\right) - \frac{iMR}{2}\left(\frac{1}{\beta_x}+\frac{1}{\beta_z}\right)\right] \exp\left\{i\left[(\mu_x-\mu_z)-(Q_x-Q_z)\theta\right]\right\}d\theta \;, \quad (1)$$

where:

$K(\theta) = \frac{1}{2}\frac{R^2}{B\rho}\left(\frac{\partial B_x}{\partial x}-\frac{\partial B_z}{\partial z}\right)$, skew field term,

$M(\theta) = \frac{R}{B\rho}B_\theta$, axial field term,

x, z = transverse coordinates,

θ = axial distance over average machine radius, R,

μ_x, μ_z = betatron phases,

β_x, β_z = betatron amplitude functions,

$B\rho$ = magnetic rigidity.

In the above formulation of C, the origin of θ, μ_x and μ_z define the observer's position. A change of origin affects only the phase of C, i.e. $|C|$ is constant for all observers. Although mathematically the choice of the origin is immaterial, there is a reason for choosing an origin for which the phase term $[(\mu_x - \mu_z) - (Q_x - Q_z)\theta]$ averages to zero around the machine, i.e. a symmetry point. Under these conditions, randomly distributed skew gradients, e.g. from magnet tilts, will in most cases give a purely real value for C, which is intuitively correct.

Theoretically, it is equally possible to base a compensation scheme on either skew quadrupoles or solenoids. Although each case must be judged separately, a coupling compensation scheme for a whole machine is better designed with skew quadrupoles. The substitution of practical values into eq. (1) shows that for a given contribution to C, quadrupoles are far less space and power consuming. Of course, skew quadrupoles can excite vertical momentum dispersion but this effect can be suppressed or turned to advantage by designing the scheme to correct median plane tilts as well.

ISR Division, CERN
CH-1211 Geneva-23

2. Original ISR Coupling Compensation Scheme

The original skew quadrupole scheme in the ISR (Fig. 1.a)) comprised two series chains. The magnet positions had been determined by a computer search for layouts which excited very little vertical momentum dispersion but at the same time could decouple the horizontal and vertical motions in the presence of randomly tilted magnets. Not surprisingly, both these chains gave almost purely real coupling vectors, since random tilts around the whole machine lead to an averaging over the phase term in eq. (1). At first, this scheme was adequate, but large localized errors, such as a tilted quadrupole in a low-β insertion or an axial field detector magnet, are not easily corrected by such a scheme unless by chance the phase corresponds to the correcting quadrupole chains. It became clear that a more universal scheme, which could compensate coupling vectors, C, of any phase, would be required.

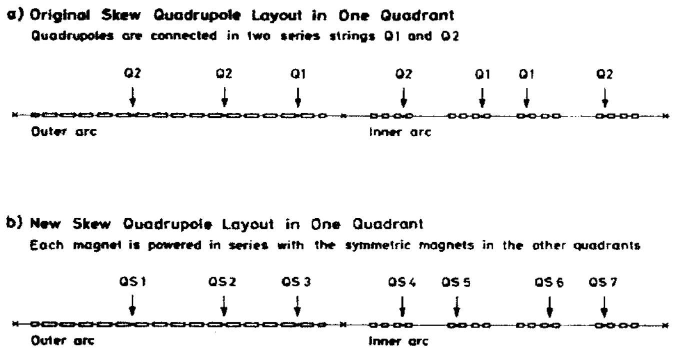

Fig. 1 Original and Revised Skew Quadrupole Schemes

3. Calculation of Coupling Compensation Schemes

A practical skew quadrupole scheme needs to be able to separate the two functions of coupling control and vertical dispersion or median plane tilt control. For convenience, the coefficient, C, from eq. (1) can be divided into real and imaginary parts and the integral replaced by a summation over N quadrupoles.

$$\begin{matrix}\text{Real}\\ \text{Imag.}\end{matrix}\; C = \frac{1}{2\pi}\sum_{i=1}^{N}\left(\sqrt{\beta_x\beta_z}\,\ell\, k\right)_i \begin{matrix}\cos\\ \sin\end{matrix}\left[(\mu_x-\mu_z)-(Q_x-Q_z)\theta\right]_i \;, \quad (2)$$

where:

the i^{th} quadrupole has length ℓ_i and

strength $k_i = \frac{1}{B\rho}\left(\frac{dB_x}{dx}\right)_i$.

The excitation of the vertical dispersion can be found by modifying the well known closed orbit equation derived by Courant and Snyder[2] to give

$$\alpha_{pz}(s) = \frac{\sqrt{\beta_z(s)}}{2\sin(\pi Q_z)}\sum_{i=1}^{N}\left(\sqrt{\beta_z}\,\alpha_{px}\,\ell\, k\right)_i \cos\left(Q_z\pi - |\mu_z(s)-\mu_{z,i}|\right) \;, \quad (3)$$

where:

α_{px}, α_{pz} = horizontal and vertical momentum compaction functions,

s = position of the observer.

4. Direct Method

Clearly, the excitation of the vertical dispersion in eq. (3) can be wholly suppressed by situating the skew quadrupoles where $\alpha_{px} = 0$. By choosing positions with as nearly as possible $\pi/2$ difference in the phase term of eq. (2), C can be fully controlled. This strategy is very common, but unfortunately α_{px} is never zero in the ISR and it is necessary to search for special quadrupole configurations.

5. Harmonic Method

This method[3] allows control of both coupling and median plane tilt. In brief, it treats α_{pz} as a closed orbit distortion expressed as the sum of its harmonics in a form analogous to that used for closed orbits in Ref. 2.

$$\alpha_{pz}(\phi) = \beta_z^{\frac{1}{2}}(\phi) \sum_{m=0}^{\infty} \left(\frac{Q_z^2}{Q_z^2 - m^2} \right) \left[a_m \cos(m\phi) + b_m \sin(m\phi) \right], \tag{4}$$

where:

ϕ = normalized phase = $\int_0^s ds/Q_z\,\beta_z$,

a_m, b_m = harmonic amplitudes of the modified skew gradient distribution ($\beta_z^{3/2}\,\alpha_{px}\,k$) in ϕ.

α_{pz} can be expressed directly as a harmonic series

$$\alpha_{pz} = \beta_z^{\frac{1}{2}}(\phi) \sum_{m=0}^{\infty} \left[A_m \cos(m\phi) + B_m \sin(m\phi) \right] . \tag{5}$$

Equations (4) and (5) relate the harmonic amplitudes in α_{pz} to those of the skew gradient distribution, giving:

$$\begin{matrix} A_m \\ B_m \end{matrix} = \frac{1}{\pi Q_z} \left(\frac{Q_z^2}{Q_z^2 - m^2} \right) \sum_{i=1}^{N} \left(\beta_z^{\frac{1}{2}}\,\alpha_{px}\,\ell\,k \right)_i \begin{matrix} \cos \\ \sin \end{matrix} (m\phi_i) . \tag{6}$$

Thus, for N skew quadrupoles, N different equations can be formulated, inverted and the magnet distribution found to excite a particular set of α_{pz} harmonics A_m and B_m. If eq. (2) is included in the matrix, then (N-2) harmonics of α_{pz} and the real and imaginary coupling coefficients can be controlled by the N quadrupoles. To get good results, N must be large enough that the harmonics outside the matrix are of a sufficiently high order to be attenuated by the $Q_z^2/(Q_z^2 - m^2)$ term. In practice, it has been found better to omit the zero order harmonic, which has a form similar to the coupling coefficient, from the matrix.

Such a scheme works well in the ISR, if the quadrupole distribution is symmetric and uniform and providing there are sufficient lenses that harmonics of orders up to twice the Q-value can be included in the matrix. Unfortunately, there is virtually no space left in the ISR outer arcs and it was not possible to install the necessary additional quadrupoles. The method, however, has been tested with good results for closed orbit correction with 36 correctors[4].

6. Insertion Method

For the ISR, it was decided to relinquish control of the median plane tilt and to concentrate on the coupling. This method relies on making local bumps in α_{pz} in the inner and outer arcs of the machine with net real or imaginary coupling. Linear combinations of these bumps then give any desired coupling vector. As the α_{pz} is localized in the bumps in the arcs, there will be no median plane tilt in the intersection regions and hence no loss of luminosity.

Using a standard minimization program, the computer was used to search for suitable bumps. The initial choice of the quadrupole positions is, of course, important. Ideally, the quadrupoles should

a) fall into two groups which are separated by as nearly as possible $\pi/2$ in the phase term $[(\mu_x - \mu_z) - (Q_x - Q_z)\theta]$,

b) be at maxima in $\sqrt{\beta_x \beta_z}$,

c) but at minima in $\sqrt{\beta_z}$.

For the ISR, this was best achieved by shifting two of the inner arc quadrupoles as shown in Fig. 1.b). With this layout of 7 series chains of 4 quadrupoles, a satisfactory scheme was calculated. Table 1 gives the calculated maximum α_{pz} inside the bumps and the residual values of α_{pz} and tilt at the intersections for $|C| = 0.05$. Since it is somewhat harder for the scheme to excite vectors along the imaginary axis, these maximum values generally occur at 90° in phase.

TABLE 1

Maximum $|\alpha_{pz}|$ and tilt values for $|C| = 0.05$ for standard ISR working conditions

Parameter \ Working condition		8C or FP line	ELSA line	steel low-β scheme
Max. $\lvert\alpha_{pz}\rvert$	[m]	0.071	0.070	0.061
Max. $\lvert\alpha_{pz}\rvert$ at intersection	[m]	0.018	0.018	0.006
Max. tilt at intersection	[mrad]	7.8	8.2	2.8

7. Measurement of C

The testing and operation of the skew quadrupole scheme depends almost entirely on the coupling meter at present installed in the ISR[5]. Since this meter can only measure $|C|$, the phase of coupling vector can only be found indirectly by making various combinations with known vectors, i.e. calculated vectors.

It should be mentioned that since $|C|$ is independent of the origin, i.e. of the observer's position, the pick-up used by the coupling meter has no bearing on the choice of origin discussed earlier. The origin is, in fact, only determined once a "known vector", e.g. the coupling due to the first circuit QS1 is ascribed a phase.

8. Checking and Applying the Revised Coupling Scheme

In order to check the revised compensation scheme, the amplitude and phase of the coupling vectors associated with each circuit was measured. Since only $|C|$ can be measured, as explained above, this could only be done indirectly. Firstly, the amplitude of the coupling in the basic machine $|B|$ was measured. A series of seven measurements was then made of resultants $|B + QS_i|$, where QS_i is the calculated vector excited by the i^{th} quadrupole chain. By varying the phase of B, a best fit can then be found for the measured data. This fit is shown in Fig. 2 and the measured and calculated data are recorded in Table 2. In reality, this is a method for measuring the phase of the machine coupling vector B, but the quality of the fit obtained is a test of the validity of the theoretical values calculated for quadrupole chains. This quality expressed as

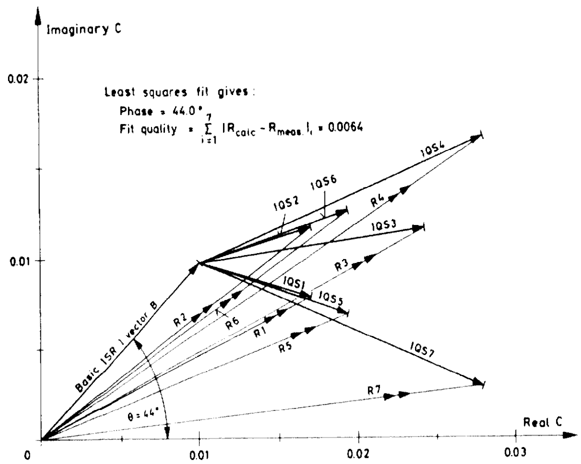

Fig. 2 Test of the Individual Skew Quadrupole Circuits (Working condition ELSA, Q_x, $Q_z \simeq 8.9$)

TABLE 2

Test of Individual Circuits in Ring 1

Assumed theoretical data			Best fit is for $\theta = 44^{o}$		
Circuit	$\|C\|$	Phase	Resultant	Measured	Calcul.
1QS1	0.0072	-15.47^{o}	$\|R1\|$	0.020	0.0187
1QS2	0.0072	15.44^{o}	$\|R2\|$	0.020	0.0206
1QS3	0.0143	7.38^{o}	$\|R3\|$	0.028	0.0269
1QS4	0.0192	21.13^{o}	$\|R4\|$	0.030	0.0325
1QS5	0.0097	-17.09^{o}	$\|R5\|$	0.021	0.0205
1QS6	0.0097	17.07^{o}	$\|R6\|$	0.023	0.0231
1QS7	0.0192	-21.16^{o}	$\|R7\|$	0.028	0.0281
			$\|B\|$*	0.014	

*$|B|$ is machine coupling.

$$\sum_{i=1}^{7} \left|R_{calc.} - R_{meas.}\right|_i = 0.0064$$

shows the scheme to be correct within the measuring accuracy of ±0.001.

Once the individual circuits had been checked, the excitation of real and imaginary coupling vectors were tested in an analogous way (Fig. 3 and Table 3). The vector B needed to correct the machine is shown as a dashed line.

It is only by chance that the bare machine vector in Ring 1 - Fig. 2 has the same phase as in Ring 2 - Fig. 3. Without low-β and solenoid, Ring 2 had a bare machine vector at 11.5^{o} with $|C| = 0.007$. This gives the effect of the low-β scheme and the solenoid as a vector of amplitude $|C| = 0.0049$ at a phase of 92^{o}.

Acknowledgements

The author would like to thank the many members of the ISR Division who have helped and encouraged him in this work.

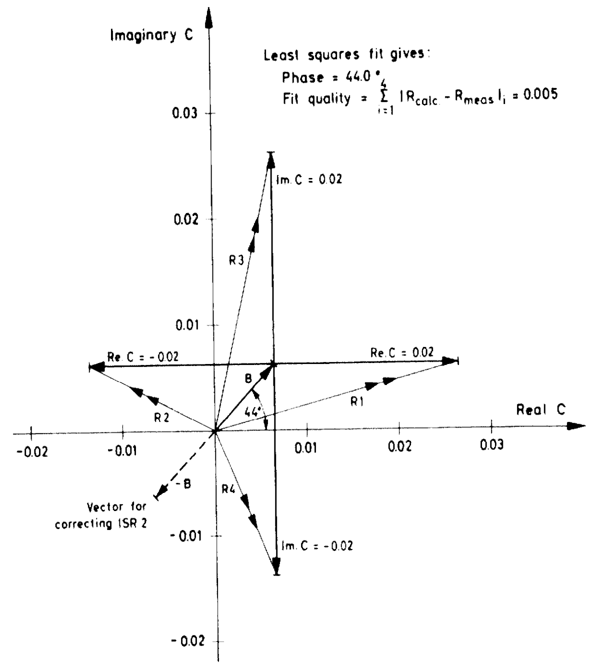

Fig. 3 Test of Real and Imaginary Coupling Vectors and Measurement of Machine Excitation (Working condition steel low-β and superconducting solenoid, Q_x, $Q_z \simeq 8.9$)

TABLE 3

Test of Excitation of Real and Imaginary Vectors and a Measurement of the Ring 2 Coupling

Vectors added by quadrupole scheme		Best fit is for $\theta = 44^{o}$		
C	Phase	Resultant	Measured	Calcul.
Re.C = 0.02	0^{o}	$\|R1\|$	0.025	0.0273
Re.C = -0.02	0^{o}	$\|R2\|$	0.013	0.0149
Im.C = 0.02	90^{o}	$\|R3\|$	0.027	0.0271
Im.C = -0.02	90^{o}	$\|R4\|$	0.015	0.0152
		$\|B\|$	0.009	

References

1. G. Guignard, The general theory of all sum and difference resonances in a three-dimensional magnetic field in a synchrotron, Div. Report CERN ISR-MA/75-23 (1975).
2. E.D. Courant and H.S. Snyder, Theory of the alternating gradient synchrotron, Annals of Physics, Vol. 3, pp. 1-48 (1958).
3. P.J. Bryant, On the use of skew quadrupoles in the ISR, Div. Report CERN ISR-MA/75-26 (1975).
4. B. Autin and P.J. Bryant, Closed orbit manipulation and correction in the CERN-ISR, Proc. VIIIth Int. Conf. on High Energy Accelerators, CERN (1971).
5. P.J. Bryant, P. Galbraith, J.P. Gourber, G. Guignard, K. Takikawa, Measurement of the excitation of the coupling resonance $Q_h - Q_v = 0$, Proc. 1977 Particle Accelerator Conference, Chicago (1977).