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A SECOND-ORDER MAGNETIC OPTICAL ACHROMAT*

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Abstract

A design procedure is given for the elimination of all of the second-order transverse geometric and chromatic aberrations in a particular class of static-magnetic transport systems for charged-particle beams.[1]

Introduction

There are numerous applications for magnetic optical systems that transport beams of charged particles from one location to another such that the transverse phase-space configuration of the beam at the final position is a faithful reproduction of the beam at the point of origin. The precision to which this may be achieved depends upon the magnitude of the phase-space volume to be transmitted and upon the optical distortions (aberrations) introduced by the intervening transport system. It is the purpose of this paper to describe a relatively simple method of devising a class of beam transport systems which approach this ideal objective by eliminating all of the second-order geometric and chromatic aberrations at the end point of the system. We restrict the discussion to systems where the transverse phase-space volume is conserved and where space-charge effects may be neglected.

Basic Design Concepts

The following of a charged particle trajectory through a series of magnetic lenses may be expressed by matrix multiplication.[2,3] At any specified position in the system the arbitrary trajectory is represented by a vector X, whose components are the positions, angles, and momentum deviations of the arbitrary ray relative to some specified central trajectory. In this report we use the notation of the TRANSPORT program,[3] where the components of the vector X are $X_1 = x$, $X_2 = x'$, $X_3 = y$, $X_4 = y'$, $X_5 = \ell$, and $X_6 = dp/p$. As we are concerned with only the transverse coordinates and the momentum of the particles, the longitudinal component X_5 will be ignored for the remainder of the report.

The linear properties of each magnetic lens or a sequence of lenses are represented by a square matrix R, which describes the action of the magnet(s) on the particle coordinates as follows:

$$X_1 = RX_0 \qquad (1)$$

where X_0 is the initial coordinate vector and X_1 is the final coordinate vector of the particle under consideration. This linear matrix formalism is conveniently extended to include second-order terms (aberrations) by the addition of a matrix T_{ijk} as follows:

$$X_{i,1} = \sum R_{ij} X_{j,0} + \sum T_{ijk} X_{j,0} X_{k,0} \qquad (2)$$

The geometric terms as those for which i,j or k are equal to 1,2,3 or 4; and the chromatic terms as those for which j or k equal 6.

We define a second-order achromat as any system for which all R_{ij} and all T_{ijk} vanish for i = 1,2,3 or 4 and j or k equals 6, i.e., any system for which all of the first- and second-order transverse chromatic terms vanish. The particular solution we present here is further restricted to the special case where the transform matrix, from the beginning to the end of the system, is the identity matrix for both the x and y transverse planes.

Elimination of the Second-Order Geometric Aberrations

Consider a static magnetic-optical beam transport system composed of a series of N identical unit cells where each unit cell contains dipole and quadrupole magnetic field components. It is possible to choose the dipole and quadrupole components for each cell such that the linear transfer matrix R, representing the first-order transverse optics of the total system, is equal to the identity matrix. This corresponds to a 2π phase shift between the beginning and the end of the transport system. It then follows from the theory of second-order beam-transport optics,[2] that the resulting system has vanishing second-order transverse geometric aberrations provided that the number of unit cells N, comprising the total system, does not equal one or three. Furthermore, it can be shown that if N = 4 or more, the addition of eight sextupole components to the system, four for the x-plane and four for the y-plane, is sufficient to eliminate all of the second-order chromatic aberrations and at the same time still have vanishing second-order geometric aberrations.

The proof that all second-order geometric aberrations will vanish under these circumstances is seen by writing the integrals used to calculate these terms in a form involving the phase shift ψ and the multipole strengths $K_n(\psi)$. n = 0 is the dipole term, n = 1 is the quadrupole term, and n = 2 is the sextupole term. For a system of N repetitive unit cells making up a total phase shift of $\psi = 2\pi$, the second-order geometric terms in T_{ijk} are generated by integrals of the form:

$$\int_0^{2\pi} K_n(\psi) \cos^{\ell}\psi \sin^{m}\psi \, d\psi \qquad (3)$$

where

$$K_n(\psi) = \left(\frac{1}{n!}\right)\left(\frac{1}{B\rho}\right) \left.\frac{\partial^n B_y}{\partial x^n}\right|_{x=y=0}$$

and $(\ell+m) = 3$ for the dipole and sextupole contributions (see Ref. 4 for a derivation of K_n). The quadrupole components do not contribute to the second-order geometric terms but the dipole and sextupole components do. Transforming this integral to the complex plane, it assumes the form

$$\int_0^{2\pi} K_n(\psi)\left[e^{i\psi} + e^{-i\psi}\right]^{\ell} \cdot \left[e^{i\psi} - e^{-i\psi}\right]^{m} d\psi \qquad (4)$$

Expanding and ignoring the numerical coefficients, the final result may be expressed as a sum of terms containing two basic integral forms: i.e.,

$$\int_0^{2\pi} K_n(\psi)\, e^{\pm i\psi} \, d\psi \qquad (5)$$

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and

$$\int_0^{2\pi} K_n(\psi)\, e^{\pm 3i\psi}\, d\psi \qquad (6)$$

Evaluating these integrals for a repetitive unit cell structure, the dipole or sextupole components may each be viewed as "vector additions in the complex plane," where $K_n(\psi)$ is the amplitude of the vector and ψ is its phase for Eq. (5) and 3ψ is the phase of the vector for Eq. (6). Both integrals vanish when N, the number of unit cells comprising a 2π betatron phase shift, does not equal one or three. $N = 1$ is excluded because there is no possibility for a vector cancellation, and $N = 3$ is excluded because all of the vector components in Eq. (6) add constructively. Both integrals vanish for any other integer value of N.

Elimination of the Second-Order Chromatic Aberrations

Sextupoles may be used to eliminate second-order chromatic aberrations if dipole components are present in the lattice to provide momentum dispersion and hence coupling to the off-momentum trajectories. At least four sextupole components are needed in each transverse plane to permit coupling to all of the trajectories and at the same time allow the second-order geometric aberrations to vanish.

One such solution is a unity transform system composed of four or more identical unit cells. Two sextupole components are introduced into each unit cell, one for the x-plane and one for the y-plane. The x-plane sextupoles are positioned where the x-plane monoenergetic beam envelope is large compared to the y-plane beam envelope. Similarly the y-plane sextupole components are positioned at a location where the y-plane beam envelope is large compared to the x-plane envelope. This maximizes the relative coupling coefficients to the chromatic terms in each transverse plane and thereby minimizes the strength of the sextupole components required for the correction process. These sextupole components may be thought of as providing additional "quadrupole-like" gradient focusing elements for the off-momentum trajectories. The strengths of the two sextupole components are adjusted to make the chromatic terms vanish in both the x and y planes. This consists of solving two simultaneous linear equations. The remarkable result is that all of the second-order chromatic terms vanish simultaneously with the introduction of only the two variables, the x-plane and y-plane sextupole strengths that are introduced into each unit cell.

Other sextupole patterns are also permissible, all of which have the common characteristic that at least four appropriately positioned sextupole components are needed in each transverse plane in order to couple to all of the off-momentum trajectories and to have vanishing second-order geometric aberrations at the end point. The eight sextupoles are required to achieve a unity transform matrix valid to second-order in the optics.

High Order Optical Aberrations

Aberrations of higher than second order should also be considered when formulating a particular solution for an achromat. They arise from two primary sources: (a) those which are inherent in the basic design of the first-order lattice, and (b) those which arise from the introduction of the sextupole correcting elements. In the discussions above it has been assumed that the total length of the achromat corresponds to a 2π phase shift. But the results quoted are also valid for systems whose length is a multiple of a 2π phase shift. Under these circumstances the sextupole correcting elements may be distributed over a longer distance, measured in units of phase shift. A particularly interesting example is when the number of first-order unit cells N making up each 2π phase shift section is four or more and is an even integer. The sextupole components may then be introduced in pairs, the elements of each pair being identical and separated by a phase shift of π in both transverse planes. The transformation matrix between them is then equal to minus the identity matrix. If under these circumstances the two sextupoles are of equal strength and of the same polarity, then for all monoenergetic trajectories, corresponding to the momentum of the central trajectory, the effect of the first sextupole on the trajectories at the end of the system is uniquely cancelled by the second sextupole. This cancellation is valid to all orders in the monoenergetic geometric optics to the extent that the phase shift over the length of the sextupole is negligible. Using this principle, it is then possible to formulate beam transport systems which have no second- or higher-order geometric aberrations introduced by the sextupole correcting elements.

Some Examples of Second-Order Achromats

Example 1

One typical example of an achromat is a separated function FODO array of alternating strong-focusing quadrupoles (Q) with interspersed dipoles (B), sextupoles (S), and drift spaces. An acceptable unit cell is the following symmetric array of magnetic elements:

Q(x) S(x) B(x) S(y) Q(y)Q(y) S(y) B(x) S(x) Q(x)

where

Q(x) is a quadrupole focusing in the x-plane and defocusing in the y-plane.

Q(y) is a quadrupole focusing in the y-plane and defocusing in the x-plane.

S(x) is a sextupole with strong coupling to the x-plane and weak coupling to the y-plane.

S(y) is a sextupole with strong coupling to the y-plane and weak coupling to the x-plane.

B(x) is a dipole whose magnetic midplane lies in the x-plane.

The optical equivalent of the above FODO array is shown in Fig. 1.

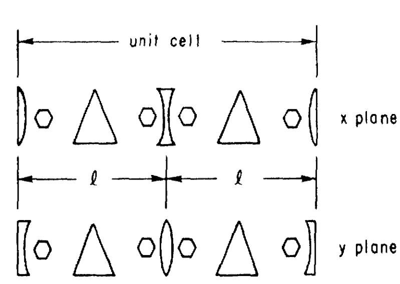

Fig. 1. A typical separated function unit cell for a second-order achromat. The lenses represent quadrupoles, the triangles dipoles, and the hexagons sextupoles.

An assembly of four or more such unit cells adjusted to have a total phase shift of 2π constitutes a second-order achromat when the sextupole components are adjusted to make the second-order chromatic aberrations vanish.

As an alternative, the sextupoles may be introduced into the unit cell in an asymmetric manner as follows:

$$Q(x)\ S(x)\ B(x)\ Q(y)\ S(y)\ B(x)$$

Example 2

A unit cell may also be generated by using a combined function magnet as shown in Fig. 2.

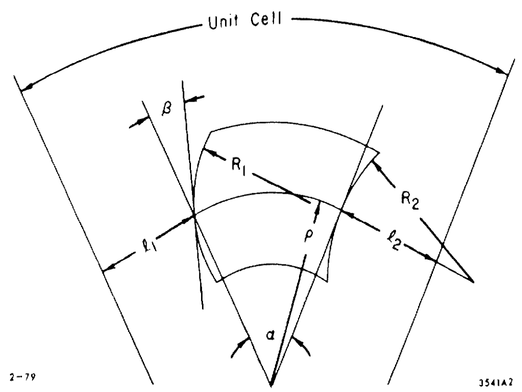

Fig. 2. An example of a combined function unit cell for a second-order achromat.

The strength of the dipole component is equal to the bending angle α. The dipole provides dispersion and first-order focusing in the radial plane. A quadrupole component, focusing in the non-bend plane and defocusing in the bend plane, is introduced via the rotated input face of the magnet; and two sextupole components are introduced via the curved surfaces, R_1 and R_2, on the entrance and exit faces of the magnet. The unit cell then consists of the combined function magnet and a drift space preceeding and following it. The total achromat is composed of at least four such unit cells adjusted to a total phase shift of 2π.

Example 3

An example of an extended 6π phase-shift achromat, having non-interlaced sextupole pairs, is illustrated in Fig. 3.

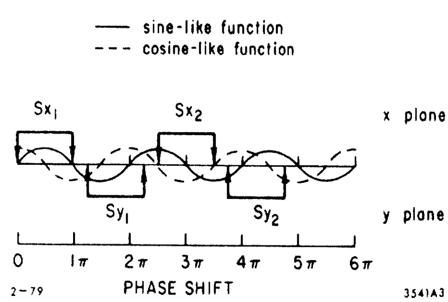

Fig. 3. A typical lattice arrangement for an extended, 6π phase shift, second-order achromat using non-interlaced sextupole pairs.

The phase shift in each transverse plane is chosen to be the same. The correcting sextupoles are introduced in pairs with the individual members of each pair being identical and separated by a phase shift of π. The respective pairs, labeled Sx_1, Sx_2, Sy_1, and Sy_2 are not interlaced and therefore do not introduce second- or higher-order geometric distortions. The distance of separation is chosen such that the strengths of Sx_1 and Sx_2 are the same as are Sy_1 and Sy_2.

The 6π phase shift achromat is most applicable to those systems where it is desirable to avoid higher order geometric aberrations caused by the interlacing of the sextupoles. An example of this is a chromatic correction system for large storage rings, discussed elsewhere in this conference.[6] Another example is in the design of secondary charged particle beams where residual tails in the transverse spatial distribution at the end point is important.

Summary

Several examples of second-order achromats have been studied using the computer programs TRANSPORT[3] and TURTLE[5]. Other studies have been made using the achromat principle to make chromaticity corrections for large storage rings.[6] In addition, secondary beams have been designed based on the achromat principle which have significant improvement in the transmitted phase-space volume.[7] From the study of these few examples it is evident that there are many potential applications for the achromat concept.

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