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HIGH ENERGY ELECTRON COOLING TO IMPROVE THE LUMINOSITY AND LIFETIME IN COLLIDING BEAM MACHINES*

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Abstract

Electron cooling can be applied to improve the performance of high energy colliding beams of hadrons and e⁻p storage rings. Normal beam excitations such as multiple scattering, resonance growth, or beam-beam interaction can be controlled, leading to longer beam lifetimes, and in some situations to higher luminosities and larger tolerable tune shifts. The electrons, in a small storage ring, are cooled by radiation and heated by the hadron beam. An equilibrium is reached in which the hadron beam is cooled. The electron beam requires strong cooling by "wigglers". We have designed a simple cooling experiment for the Fermilab synchrotron.

Relativistic proton or antiproton beams in circular accelerators or storage rings can be damped by interactions with electrons ($\gamma_p=\gamma_c$) circulating in a storage ring.[1,2] The proton or antiproton beam is damped and the electron beam is heated in the interaction. In order to reduce the blowup of the electron beam, synchrotron radiation in bending magnets or wiggler magnets and subsequent acceleration in an RF system is required, thus the electron beam is cooled. This is schematically shown in Fig. 1.

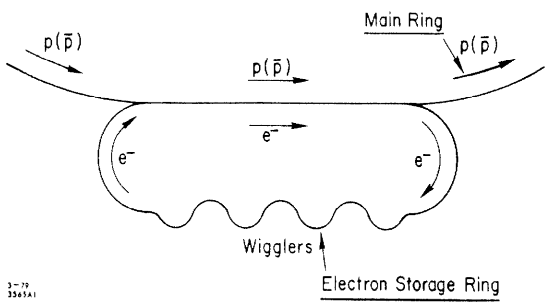

Fig. 1. High energy electron cooling plan.

There are three useful features of relativistic electron cooling for high energy colliding beam devices. (1) The damping increases the luminosity lifetime of the colliding beams and should make the beams more stable to higher order resonances, etc. (2) The damping decreases the beam size and under certain circumstances should lead to an increased luminosity for a given number of particles in each beam. (3) The damping may increase the tolerable beam-beam tuneshift limit ($\Delta\nu$).

For proton-antiproton storage rings where $N_{\bar{p}}$, the number of antiprotons, is limited the addition of electron cooling can result in a dramatic improvement in the machine performance. To illustrate this point we consider the collision of equal number of protons (N_p) and antiprotons ($N_{\bar{p}}$) in a storage ring with 1 TeV (the energy doubler at Fermilab). The available luminosity is given by

$$\mathscr{L} = 5.6\times 10^{22}\,\frac{(\Delta\nu)}{\beta^*}\,N_{\bar{p}}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$$

where β^* is the beta at the interaction point. Relativistic electron cooling results in a damping of the size of the beam so that for a given number of antiprotons the maximum tolerable tune shift limit is reached. The luminosity therefore depends only on $N_{\bar{p}}$, $\Delta\nu$ and β^*. Both $N_{\bar{p}}$ and $\Delta\nu$ may be further increased thus resulting in an increase in $\mathscr{L}$. $N_{\bar{p}}$ is increased because the $\bar{p}$ refilling time of a $\bar{p}p$ storage ring is increased if the luminosity lifetime is increased. We believe that $\Delta\nu$ may also be increased in analogy to the larger $\Delta\nu$ that can be tolerated for an e^+e^- machine (0.06) which has a damping mechanism (synchrotron radiation damping) compared to the maximum $\Delta\nu$ of 0.005 usually assumed for proton-proton storage rings. Figure 2 shows the luminosity that can be achieved as a function of these parameters for $\bar{p}p$ machines.[2]

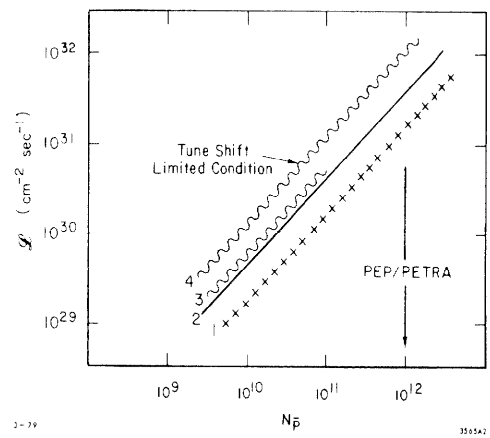

Fig. 2. Luminosity for 1000 GeV proton-antiproton storage ring as a function of $N_{\bar{p}}$, $\Delta\nu$ and β^*. (1) $\beta^* = 2.5$, $\Delta\nu = 0.005$; (2) $\beta^* = 2.5$, $\Delta\nu = 0.01$; (3) $\beta^* = 1$, $\Delta\nu = 0.005$; (4) $\beta^* = 1$, $\Delta\nu = 0.01$.

A similar gain in luminosity might be realized for e⁻p storage rings by damping the proton beam emittance and increasing the tolerable tune shift limits. The proton beam should be damped until its dimensions are comparable to the electron beam in the storage ring.

The idea of using relativistic electron cooling was originally considered and discarded by Budker and his colleagues because of the constraints it implied an electron storage ring.[3] Recently this subject was reconsidered at meetings held at Lawrence Berkeley Laboratory and the University of Wisconsin.[1,2] For the early work see the proceedings of these meetings. We present here two additional developments. (1) Refinements in the thermodynamical calculations of the relaxation of the hadron beam and the electron beam. (2) The conceptual design of a simple storage ring that can be used to carry out an experiment on relativistic electron cooling. In particular such an experiment would be used to extend our knowledge of the performance of high current electron storage rings in the few hundred MeV energy range.

We briefly outline the thermodynamic equilibrium of this system. In the absence of interactions between the two beams, we can write the following equations for the rms beam emittance ($\varepsilon = \sigma^2/\beta$)

$$\frac{d\varepsilon_p}{dt} = D_p \qquad (1)$$

$$\frac{d\varepsilon_e}{dt} = -\frac{2}{\tau}\varepsilon_e + D_e \quad . \qquad (2)$$

We assume both beams are round, namely, that they have the same horizontal and vertical emittance.

In the absence of diffusion-like processes and of damping effects, the emittances are normally considered

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invariants. The diffusion coefficient D_p on the right-hand side of (1) is primarily given by gas scattering and similar effects. This diffusion is not compensated by damping and will cause a linear increase of the beam emittance with time. In Eq. (2), τ is the synchrotron radiation-damping time and D_e the quantum-fluctuation diffusion coefficient. The electron beam would have an equilibrium emittance which is given by

$$\bar{\varepsilon}_e = \frac{1}{2}\tau D_e \quad . \tag{3}$$

This equilibrium value is reached in the e-folding time $\tau/2$.

Observe that τ and D_e depend strongly not only on the beam energy but also on the electron-beam storage ring lattice.

We now modify Eqs. (1) and (2) to include the beam-beam interaction, which is supposed to lead to "cooling" of the proton beam at the cost of some "heating" of the electron beam. Because of the large energy and since the electron beam is already focused by the lattice quadrupoles and RF cavities, we ignore space-charge effects on the trajectory of the electrons. It is easily verified that at larger energies

$$\theta_\| << \nu\theta_\perp \quad ,$$

where $\theta_\|$ and $\theta_\perp$ are respectively the longitudinal and transverse relative momentum spreads. This is true for both beams. The coupled equations for the beam emittances are

$$\frac{1}{\beta_p^*}\frac{d\varepsilon_p}{dt} = -\frac{2}{\tau_p}\left(\frac{\varepsilon_p}{\beta_p^*} - \frac{m_e}{m_p}\frac{\varepsilon_e}{\beta_e^*}\right) \tag{4}$$

$$\frac{1}{\beta_e^*}\frac{d\varepsilon_e}{dt} = -\frac{2}{\tau_e}\left(\frac{\varepsilon_e}{\beta_e^*} - \frac{m_p}{m_e}\frac{\varepsilon_p}{\beta_p^*}\right) \tag{5}$$

where β_p^* and β_e^* are the lattice beta-values at the interaction region and m_p and m_e are the masses at rest. The relaxation times are given by

$$\frac{1}{\tau_p} = \frac{6\pi e^4 L}{m_p m_e c^4}\frac{\eta_e}{\beta^4\gamma^5}\frac{I_{e/e}}{(a_e^2+a_p^2)(\theta_e^2+\theta_p^2)^{3/2}} \tag{6}$$

and

$$\frac{1}{\tau_e} = \frac{6\pi e^4 L}{m_p m_e c^4}\frac{\eta_e}{\beta^4\gamma^5}\frac{I_{p/e}}{(a_e^2+a_p^2)(\theta_e^2+\theta_p^2)^{3/2}} \tag{7}$$

Here β and γ are the usual relativistic factors which are common to both beams, η_p is the ratio ℓ/c_p of the interaction region length to the proton ring circumference, η_c is defined in a similar way and accordingly, I_e and I_p are the beam currents within a bunch, L is the Coulomb logarithm (~15) which we assume to be the same for both beams, finally a_e and a_p are the two beams transverse radii.

Ignoring intrabeam scattering we can solve these equations to obtain the equilibrium conditions. By the following we shall assume that the motion of the bunches in the two rings is synchronized in such a way that $\eta_p = \eta_e$; this occurs when one bunch interacts always with the same one in the other beam. Also the bunches in the two beams are assumed to have about the same length.

From the definition of emittance we have

$$a^2 = \varepsilon\beta^* \quad \text{and} \quad \theta^2 = \varepsilon/\beta^* \tag{8}$$

which we can use in the right-hand side of (6) and (7). Then (4) and (5) can be replaced by

$$\frac{d\varepsilon_p}{dt} = -2K_p \frac{\frac{\varepsilon_p}{\beta_p^*} - \frac{m_e}{m_p}\frac{\varepsilon_e}{\beta_e^*}}{(\beta_e^*\varepsilon_e + \beta_p^*\varepsilon_p)\left(\frac{\varepsilon_e}{\beta_e^*} + \frac{\varepsilon_p}{\beta_p^*}\right)^{3/2}} \tag{9}$$

$$\frac{d\varepsilon_e}{dt} = -2K_e \frac{\frac{\varepsilon_e}{\beta_p^*} - \frac{m_e}{m_p}\frac{\varepsilon_p}{\beta_p^*}}{(\beta_e^*\varepsilon_e + \beta_p^*\varepsilon_p)\left(\frac{\varepsilon_e}{\beta_e^*} + \frac{\varepsilon_p}{\beta_p^*}\right)^{3/2}} \tag{10}$$

with

$$K_p = \frac{6\pi e^4 L}{m_p m_e c^4}\frac{\eta_p I_{e/e}}{4\ 5}\beta_p^* \tag{11}$$

$$K_e = \frac{6\pi e^4 L}{m_p m_e c^4}\frac{\eta_e I_{p/e}}{4\ 5}\beta_e^* \tag{12}$$

In absence of intrabeam scattering an equilibrium emittance of the proton beam would be reached

$$\varepsilon_p = \frac{\varepsilon_0\varepsilon_e}{\varepsilon_e - \bar{\varepsilon}_e}$$

where $\bar{\varepsilon}_e$ is given by Eq. (3) and

$$\varepsilon_0 = \frac{1}{2}\left(\tau\frac{K_e}{K_p}\frac{m_e}{m_e}\right)D_p = \frac{1}{2}\tau_0 D_p \tag{13}$$

where τ_0 represents the proton or antiproton beam "cooling" time. Where

$$\tau_0 = \tau\left(\frac{m_p}{m_e}\right)\frac{\eta_e}{\eta_p}\frac{\beta_e^*}{\beta_p^*}\frac{I_p}{I_e} \tag{14}$$

Crudely this formula indicates that the proton damping time will be longer than the synchrotron damping time since $m_p > m_e$ and in a realistic situation $\eta_p << \eta_e$. However it is conceivable that $I_e >> I_p$ for low energy electron storage rings and thus τ_0 may be reduced. In practice τ_0 will depend on the lattice of the storage rings, the effects of intrabeam scattering the amount of synchrotron radiation in the electron storage ring and the maximum electron current. Typically for $I_e)_{peak}$ ~ 5 amps, E_e ~ 125 MeV and a proton storage ring such as the SPS and Fermilab (but with a very good vacuum in each case) the damping times will be of order 100-1000 seconds. This time is short enough to be useful in realistic situations since $\tau_0 << \tau$ lifetime due to beam blowup.

Intrabeam scattering in the electron storage ring causes an increase in this cooling time. The effect of the intrabeam scattering depends very much on the dispersion of the lattice. In the following we shall consider only the average contribution, namely we shall spread smoothly the dispersion around the machine. Also we shall assume the same amount of dispersion in both planes. We consider the case of energies of both beams is well above the transition energies of the respective machines.

If the dispersion is considerably high and because $\theta_\| << \gamma\theta_\perp$ the diffusion due to intrabeam scattering in the transverse plane can be decoupled from the longitudinal emittance of the beam.[4] If all the other effects are ignored one would have

$$\frac{d\varepsilon}{dt} = \frac{Q}{\varepsilon^{3/2}} \tag{15}$$

and

$$Q = \frac{6\pi e^4 L\beta^{1/2} I/e}{m^2 c^4 \beta^4 \gamma^3 \gamma_T^2} \tag{16}$$

where $\bar{\beta}$ is the average value of the beta-function around the lattice and γ_T is the ratio of the transition energy to the rest energy.

The coupled equations for the emittances now become

$$\frac{d\varepsilon_p}{dt} = D_p - 2K_p \frac{\dfrac{\varepsilon_p}{\beta^*_p} - \dfrac{m_e}{m_p}\dfrac{\varepsilon_e}{\beta^*_e}}{(\beta^*_e\varepsilon_e + \beta^*_p\varepsilon_p)\left(\dfrac{\varepsilon_e}{\beta^*_e} + \dfrac{\varepsilon_p}{\beta^*_p}\right)^{3/2}} + \frac{Q_p}{\varepsilon_p^{3/2}} \quad (17)$$

and

$$\frac{d\varepsilon_e}{dt} = D_e - \frac{2}{\tau}\varepsilon_e - 2K_e \frac{\dfrac{\varepsilon_e}{\beta^*_e} - \dfrac{m_p}{m_e}\dfrac{\varepsilon_p}{\beta^*_p}}{(\beta^*_e\varepsilon_e + \beta^*_p\varepsilon_p)\left(\dfrac{\varepsilon_e}{\beta^*_e} + \dfrac{\varepsilon_p}{\beta^*_p}\right)^{3/2}} + \frac{Q_e}{\varepsilon_p^{3/2}} \quad (18)$$

The equilibrium solution is obtain by setting $d\varepsilon_p/dt = d\varepsilon_e/dt = 0$.

Intrabeam scattering effects can be reduced by one of two techniques:[4] (1) reduce the dispersion in the machine to very low values and (2) arrange the electron beam emittances such that

$$\sigma_{x'} = \sigma_{z'} = \sigma_y$$

i.e., an isotropic distribution of velocities in the rest frame of the beam particles. Our calculation including intrabeam scattering indicate that the cooling time τ_0 probably does not increase appreciably in a practical situation.

We now turn to a preliminary design of a small storage ring that could be used to study relativistic electron cooling at Fermilab or CERN.

The ring, Fig. 3, could be situation in the present tunnel at Fermilab at one of the 15 m medium straight sections. The proton and electron beams would coincide in one of the straight sides of the electron ring, where the electrons would cool the protons. The electrons would then radiate their transverse energy away in the four 90° bending magnets and in wiggler magnets in the straight side opposite the cooling region.

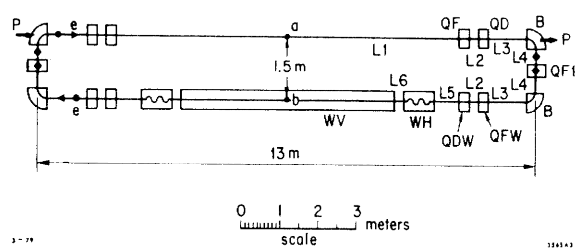

Fig. 3. Lattice of electron storage ring or cooling protons.

In designing the lattice we tried to satisfy a number of requirements, not all of which are easily compatible. We list them briefly, not necessarily in the order of importance.

1. The proton and electron beams should have roughly equal transverse dimensions and angles so that the cooling proceed efficiently. That implies comparable emittances, about 10^{-8} mrad, and beta-values, in the range 50-100 m.

2. So that longitudinal collisions between electrons and protons do not excite transverse electron oscillations, the dispersion function should be zero or small in the cooling region.

3. Electron damping times should be as short as possible. This leads to strong fields in the dipoles and to the addition of wiggler magnets.

4. The ring should be stable over the expected electron energy range.

The emittance requirement also leads to zero or low dispersion in the straight sections, because large dispersion in the bending magnets or wigglers would give large emittance due to quantum excitation.

We first worked on a lattice without wiggler magnets.[5] This ring has reflection symmetry about both axes, and two superperiods, while the ring with wigglers has only left-right reflection symmetry and one superperiod (see Fig. 3). Zero dispersion in the straights uniquely determines the strength of the QF1 quadrupoles. Next, one may specify the beta-function values at point A and adjust QF and QD to produce a waits at the center of QF1. One does not always find solutions, nor do those found always seem feasible with regard to the betatron tunes, emittance, or chromaticity.

The lattice elements of the ring design selected are shown in Fig. 3 and Table I, and the operating and orbit parameters in Tables II and III, respectively. Apart from the long damping times, the ring might cool if vertical emittance were introduced by use of solenoids or skew quadrupoles.

Table I. Lattice Elements

Type Name	Length (m)	Wiggler Off	Wiggler On
Drifts			
L1	4.45		
L2	0.20		
L3	1.00		
L4	0.35		
L5	0.65		
L6	0.20		
Bends		Magnetic Radius ρ(m)	
B	0.3927	0.25	0.25
Quads		Gradient $K = G/B\rho\,(m^{-2})$	
QF	0.30		3.4896
QD	0.30		-4.0865
QF1	0.30		10.2440
QFW	0.30		6.1816
QDW	0.30		-3.6581
Wigglers		Magnetic Radius ρ(m)	
WH (horizontal)	0.80	--	0.25
WV (vertical)	2.80	--	0.25
Sextupoles		Strength K' (m^{-2})	
SL4			-161.66
SQF1			74.55

Table II. Operating Parameters

Energy	E	125	MeV
Magnetic Field in Main Bends	B_o	1.6678	Tesla
Rigidity	$B\rho$	0.41695	Tesla-m
Edge Angle of 90° Bands		0	Degrees
Wigglers:			
Length of One Bending Period	LWP	0.20	m
Number Periods in WH	N_n	4	
Number Periods in WV	N_v	14	
Length of Each Wiggler Pole	LWP	0.10	m
Wiggler Magnetic Field	B_w	1.6678	Tesla
Machine Circumference	$2\pi R$	28.5708	m
Average Radius	R	4.54718	m
Revolution Time	Trev	0.0953	µs
Energy Radiated/Turn (Wigglers On)	U_o	0.482	keV

Table III. Orbit Parameters

		Wigglers Off	Wigglers On	
Betatron Tunes	ν_x	2.36	6.28	
	ν_y	0.28	2.26	
Beta-Functions in Cooling Region				
Center	β_x	40	50	m
	β_y	40	120	m
Dispersion	η	0	0	m
Beta-Functions at Center of Vertical Wiggler	β_x	40	.15	m
	β_y	40	.5	m
	η	0	0	m
Transition Energy	γ_t	7.07	7.77	
Damping Times	τ_x	.444	.053	s
	τ_y	.283	.049	s
	τ_E	.120	.024	s
Emittances $\varepsilon = \sigma^2/\beta$	ε_x	1.43×10^{-8}	$.29 \times 10^{-8}$	m-rad
	ε_y	0	$.68 \times 10^{-8}$	m-rad
$\Delta\ell\ \Delta\beta\gamma$	ε_ℓ	.0078	.0019	m
Energy Spread	σ_p/p	.00020	.00021	

To reduce damping times we investigated use of wiggler magnets in the ring. We found that if the orbit bends through large angles, that it is nearly impossible to make the complete ring stable. Therefore, we propose short period wigglers, as indicated in Fig. 4. Different combinations of horizontal and vertical wigglers were tried, introduction of further quadrupoles among them and use of gradients. We concluded that no further quadrupoles are needed or gradients in the wigglers. The combination shown in Figs. 3,4, and Tables I, II, and III appeared to maximize stability vs. momentum change and gave good emittances. However, use of only a single vertical wiggler magnet also looks attractive. For this case we obtained emittances $\varepsilon_x = .18 \times 10^{-8}$ mrad, $\varepsilon_y = 1.89 \times 10^{-8}$ mrad, so skew quadrupoles or solenoids would be needed. In order to increase the bunch length a small cavity could be included to add a third harmonic to the bunches.

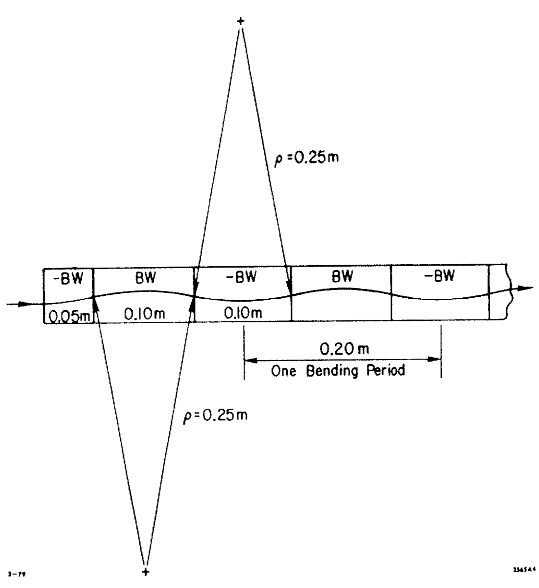

Fig. 4. Schematic of wiggler magnet.

The ring has a rather large chromaticity, but fortunately the momentum spread of the electrons is small. We obtain the best results by placing chromaticity correcting sextupoles in the L4 drifts, and as windings in the QF1 quadrupoles. These are indicated as diamonds in Fig. 3. With these adjusted to bring the slope of the tunes vs. momentum to zero, one obtains a parabotic dependence, which gives stability in $\Delta p/p$ from about $-8\sigma_p \le \Delta p/p \le 10\sigma_p$.

All of the lattice design, emittance, and chromaticity calculations were done using the SYNCH computer program.[6]

Reasonable cooling times for the antiproton beam require a peak electron current of several amperes, an emittance of about 1×10^{-8} mm mrad and a momentum spread of about 1×10^{-3} of the electron beam. The resulting high current density causes instability in the electron machine by electron-electron scattering in the bunches. The effect mainly occurs in the horizontal plane, where the dispersion is not zero, which is in the small sides of our race track. It leads mainly to a growing horizontal emittance, but also the vertical emittance and the momentum spread are influenced. If the growth rate by synchrotron radiation, one gets an equilibrium state in the machine. Because of the different damping and growth rates in all three planes the equilibrium state has not automatically the desired emittances and momentum spread.

To study the problem we used a program of Hübner, Möhl and Sacherer, which is based on a paper of Piwinski.[4] To find the equilibrium state we neglected quantum excitation as we are far from the limit where radiation excitation becomes important.

With a peak current in the bunch of 5A we find $\varepsilon_H = 3 \times 10^{-7}$, $\varepsilon_V = 3 \times 10^{-9}$, $\Delta p/p = 2.7 \times 10^{-3}$. The horizontal emittance and the momentum spread are much too big. By an increase of the vertical emittance the problem can be solved. This can be done either by a much stronger horizontal damping by more wigglers or a coupling between the horizontal and the vertical plane. The coupling can be made by skewed quadrupoles or a solenoid.

To counteract the blowup of the proton or antiproton beam by rest gas scattering and beam-beam interaction an average current of about 1 Amp has to be stored in the electron cooling ring. As the beam becomes more stable with increasing energy one directly injects at 125 MeV. This can be done with one of the three injectors: a linac, race track microtron, or a combination of linac and race track microtron.

In conclusion the addition of a damping mechanism to high energy proton or antiproton beam is useful to increase stability and luminosity in these machines. Damping times of 100-1000 seconds seem feasible with our present knowledge of the required parameters of the electron storage ring. We have designed a small storage ring that can be used to carry out an experiment of an electron cooling in order to demonstrate the principle and explore the further possibilities of this technique.

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