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### 1. Summary

Recent experiments on stochastic cooling have resulted in cooling rates several orders of magnitude higher than obtained previously in the ISR. Two cooling systems reduce betatron oscillations. A third system reduces momentum spread, using the so-called filter method. The favourable signal-to-noise ratio of this method has led to cooling times (e-folding of peak density) of 15 s with  $7 \cdot 10^7$  protons in ICE. Betatron cooling times are longer due to the lower signal-to-noise ratio. Simultaneous cooling in all three planes has yielded lifetimes of about 100 h, a value consistent with losses caused by single scattering on the residual gas. The existing stochastic cooling theory has been confirmed.

### 2. Introduction

All experiments were done with a view to application of stochastic cooling in the CERN Antiproton Accumulator<sup>2</sup> (AA). The main purpose was to check the existing theory<sup>3</sup> of momentum cooling. This theory is now believed to be complete. It includes the coherent correction towards a nominal momentum which a particle exerts on itself via the feedback over many million revolutions, the diffusion caused by signals from other particles and amplifier noise, and, finally, the dynamic reduction of Schottky signals due to the feedback action.

In addition to the verification of the theory, a novel method for fast  $\Delta p/p$  cooling was tried, i.e. the filter method<sup>1</sup>, where the coherent correction is a function of the deviation of a particle's revolution frequency from a nominal value.

The theory is believed to be sufficiently general to apply also to the method in which the correction is a function of the particle's radial position, R. Palmer's method<sup>4</sup>. Since the betatron cooling theory<sup>4</sup> has been derived along the same lines as the  $\Delta p/p$  cooling, a confirmation of the  $\Delta p/p$  theory is an indication that the betatron theory is also correct.

### 3. Principle of Momentum Cooling with the Filter Method

The signal from a sum pick-up of a single coasting particle with revolution frequency  $f_0$  is a series of delta functions. The voltage spectrum consists of lines separated by  $f_0$  with the height:

$$2ef_0R_p \quad (1)$$

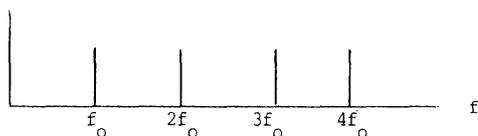


Fig. 1.  $R_p$  pick-up impedance.

The signal passes through a filter which changes polarity at each harmonic  $Hf_{0n}$  of a nominal revolution frequency that corresponds to a nominal momentum.

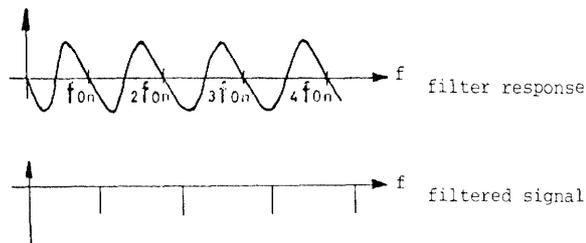


Fig. 2.

The filtered signal is fed via amplifiers to a series of wide-band accelerating/decelerating gaps. In a machine above transition a particle with  $f_0 > f_{0n}$  will accelerate itself through the signal it sets up in the feedback system until  $f_0 = f_{0n}$  (assuming proper polarity and that the delay in the loop equals the time of flight between pick-up and gap). Similarly, a particle with  $f_0 < f_{0n}$  will be decelerated towards the nominal momentum.

The method is particularly suited for fast cooling at low intensities since the pick-ups are connected directly to the preamplifier and the signal filtered only afterwards, in such a way that the signal-to-noise ratio remains high and independent of radial position. With the alternate method, based on the radial difference pick-up<sup>4</sup>, the signal-to-noise ratio is zero for a particle on the nominal orbit. More sum pick-ups can be accommodated in a given length. In ICE, signals from 16 ferrite loaded gaps were added in hybrid transformers prior to low-noise amplification and filtering.

### 4. Review of Momentum Cooling Theory

#### 4.1 The Fokker Planck equation

The equation has been derived in references 3 and 5. Both derivations are incomplete, as far as the diffusion term is concerned. A first experiment in the ISR<sup>7</sup>, however, confirmed the correctness of the diffusion term.

$$\frac{\partial \psi}{\partial t} = f_0 \frac{\partial}{\partial E} [\Delta E_c \psi] + \frac{1}{2} f_0 \frac{\partial}{\partial E} [\Delta E_{ic}^2 \frac{\partial \psi}{\partial E}] \quad (2)$$

where  $\psi = \frac{dN}{dE}$  particle density in energy (3)

$E =$  energy deviation from a nominal energy,  $E_n$  in eV (4)

#### 4.2 The coherent cooling term

$\Delta E_c$  is the single turn total energy change a particle with energy deviation  $E$  exerts on itself via the feedback system. It is the sum of the changes over all harmonics in the feedback system. The peak single particle voltage after observation, amplification, filtering and summation over harmonics  $H_1$  to  $H_2$  is:

$$\Delta E_c = \sum_{H=H_1}^{H_2} 2e f_0 \sqrt{R_p R_k} \text{Real} [g(H, E)] \quad (5)$$

where  $g(H, E)$  is the complex gain of the amplifier and filter for the  $H^{\text{th}}$  harmonic and  $E$ .

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The corresponding frequency is calculated as:

$$f = H \left( f_{\text{on}} + \frac{df_o}{dE} E \right) \quad (6)$$

#### 4.3 Heating due to particle and amplifier noise

$\Delta E_{\text{ic}}$  is the mean square incoherent energy change per turn for a particle with energy  $E$ .  $\Delta E_{\text{ic}}$  represents random energy changes, and is the result of the sum of the squared spectral noise densities over the harmonics of the revolution frequency belonging to  $E$ , as given by (6).

$$\Delta E_{\text{ic}}^2 = \underbrace{\sum_{H=H_1}^{H_2} 2e^2 R_p R_k^3 \psi \frac{dE}{df_o} \left| \frac{g^2(H,E)}{H} \right|}_{\text{particle noise}} + \underbrace{2kTR_k f_o \left| g^2(H,E) \right|}_{\text{amplifier noise}} \quad (7)$$

where  $R_k$  = gap resistance,  
 $2kT = 8.2 \cdot 10^{-21}$  W/Hz, assuming a 3 dB noise figure for the pre-amplifier.

#### 4.4 Dynamic change of Schottky signals due to the feedback correction

It is shown in references 3 and 6 that if we apply a perturbing voltage  $u_k(\omega_p)$  to the kicker, the open loop signal at the pick-up will be given by the complex integral

$$u_{\text{pu}}(\omega_p) = -u_k(\omega_p) \left( j f_o^2 e \sqrt{R_p R_k} \int_0^{+\infty} \frac{d\psi}{\omega_p - \omega(E)} dE \right) \quad (8)$$

where the term in the brackets is called the beam transfer function  $G(\omega)$ . The denominator of the integrand vanishes (singularity) at  $\omega(E) = \omega_p$  inside the integration interval. The difficulty is overcome in reference 6 by integrating up to  $\omega_p - \epsilon$  and continuing from  $\omega_p + \epsilon$ . This yields an imaginary term, corresponding to Cauchy's principal value. Furthermore, a real voltage

$$u_k(\omega_p) f_o^2 e \sqrt{R_p R_k} \frac{d\psi}{d\omega}$$

results from the singularity at  $\omega_p$ .

Reference 3 demonstrates that if  $g(\omega) \cdot G(\omega)$  is the complex open loop gain of the feedback system the beam transfer function  $G(\omega)$  changes the pick-up signals by the complex factor:

$$T(\omega) = \frac{1}{1 - G(\omega)g(\omega)} \quad (9)$$

and that the coherent cooling factor (5) should be replaced by

$$\Delta E_c = \sum_{H=H_1}^{H_2} 2ef_o \sqrt{R_p R_k} \text{Real} \left[ \frac{g(H,E)}{1 - G(H,E)g(H,E)} \right] \quad (10)$$

Similarly, the diffusion terms are modified with the square of  $|T(\omega)|$ :

$$E_{\text{ic}}^2 = \sum_{H=H_1}^{H_2} \left( \left| \frac{g(H,E)}{1 - G(H,E)g(H,E)} \right|^2 \right) \times \left( 2e^2 R_p R_k^3 \psi \frac{dE}{df_o} \frac{1}{H} + 2kTR_o R_k \right) \quad (11)$$

For cooling with a radial position dependent pick-up (Palmer method) the pick-up sensitivity with respect to radial position (i.e. with  $E$ ) replaces the filter characteristics in  $g(H,E)$  and should be included inside the integral in (8). The amplifier noise term in (11) is only to be multiplied with the electronic gain, since the pre-amplifier comes after the pick-up.

## 5. The ICE Machine

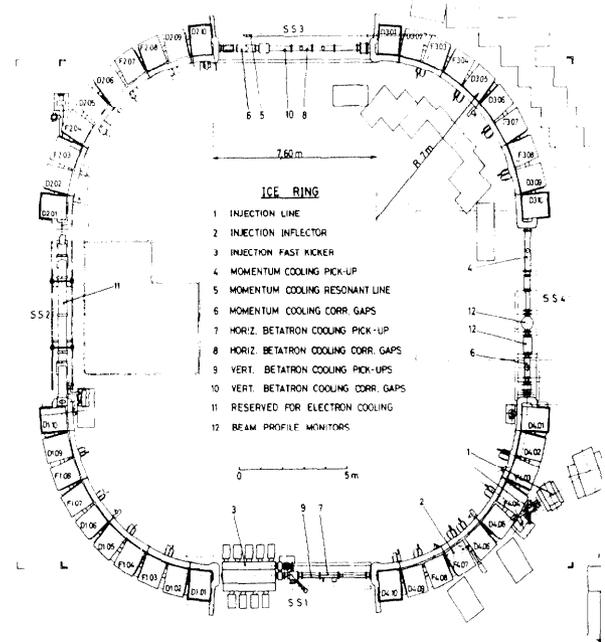


Fig. 3. General floor layout.

ICE is a strong focusing machine with four magnetic sectors and four straight sections, where protons from the CPS or secondary particles from a conversion target are injected and cooled. The injection is done with a pulsed inflector and a fast full-aperture kicker.

The magnets were built from the iron yoke and the coils of the dismantled g-2 ring (storage ring used to measure the magnetic moment of the muon). The pole pieces were modified to include the required gradient. Each quadrant consists of six defocusing and four focusing magnet blocks. All defocusing units are fitted with poleface windings for tuning and chromaticity correction ( $Q_H = 1.35$ ,  $Q_V = 1.55$ ). A completely new vacuum system was built, partly using existing spare parts from the ISR and the SPS. The working pressure is around  $10^{-9}$  torr ( $N_2$  equivalent). For stochastic cooling ICE was run at 1.73 and 2.1 GeV/c.

## 6. Stochastic Cooling Hardware

Each of the 16 sum pick-ups used for momentum cooling consists of a 6 cm long drift tube short-circuited to earth at one end, surrounded by a ferrite frame in the middle and connected to a 50Ω output lead at the other end. Thus the image current associated with the passage of a particle is coupled out. The ferrite choke prevents this current from being shorted to earth. The signals from the 16 pick-ups are

combined in a hybrid network before being fed into the preamplifier.

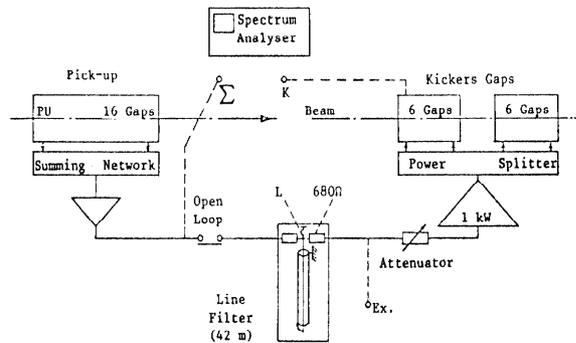


Fig. 4. General layout of  $\Delta p/p$  cooling system.

The filter consists of a  $66\Omega$ , 42 m long coaxial line, short-circuited at one end and having an outer diameter of 120 mm. The line provides, in parallel with the inductance, periodic notches and peaks.

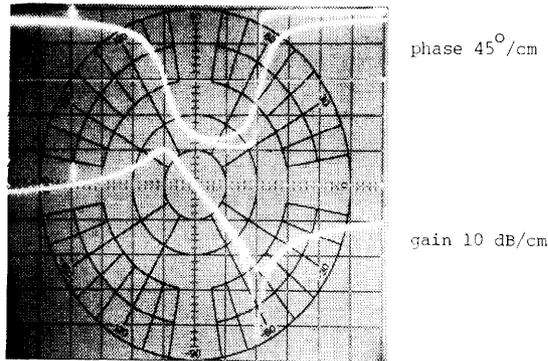


Fig. 5. Filter characteristics around the 21st notch with and without line loss compensation.

Incomplete signal suppression in the notches is the consequence of resistive line losses. The residual signal is annulled by injecting into the filter output a small fraction of the input in antiphase. The amplitude of the correction should increase as  $f^{3/2}$  since the skin depth decreases with  $f^{-1/2}$ . The correction signal is obtained as the difference signal at the end of two lines, one line having high losses, the other low losses. Both lines are fed with the filter input signal.

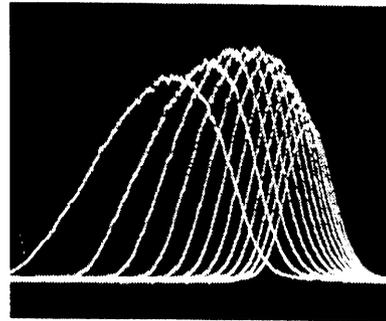
After filtering the signal is amplified by a 1 kW distributed amplifier and powers 12 accelerating gaps, similar to the pick-up gaps. These correctors are in two groups of six, separated by half a betatron wavelength to cancel the betatron heating caused by momentum corrections. The over-all characteristics are given in Fig. 9.

The betatron cooling systems consist of 30 cm long loop coupler pick-ups and kickers and work from 125 to 375 MHz, similar to the low-frequency betatron coolers in the ISR<sup>8</sup>.

## 7. Experimental Results

First evidence of momentum cooling was obtained in December 1977.

During the January 1978 shutdown six of the 12  $\Delta p/p$  correctors were reversed to obtain cancellation of horizontal kicks caused by electric field components.



First evidence of  $\Delta p/p$  cooling in ICE.

Fig. 6. Longitudinal Schottky scans at the 18th harmonic, taken at 1 min intervals. The scans are the result of spectrum analysis of longitudinal statistical beam structure. The vertical axis is  $\sqrt{dN/df}$ , the horizontal is frequency.

After the start-up, higher cooling factors and rates were quickly obtained. Two major improvements to the filter line followed:

- reduction of reflections resulting in smaller frequency errors of the resonant notches,
- passive compensation of filter line losses in the notches to increase the signal suppression.

The e-folding time for the peak density was further lowered to 15 s.

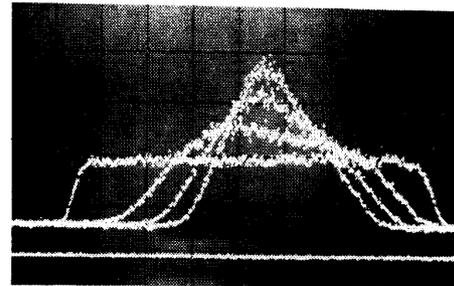


Fig. 7.  $8 \cdot 10^7 p$ , 10 s between scans, factor 4.75 in peak density in 30 s, initial e-folding time is 15 s, average value 20 s. Cent. frequency 60 MHz, 20 kHz/cm, total initial width  $\Delta p/p = 3.5 \cdot 10^{-3}$ . The vertical axis is proportional to  $\sqrt{\psi}$ .

Such high cooling rates were obtained with high electronic gains, also causing strong diffusion as can be seen from (7), the cooling being proportional to the electronic gain, the diffusion to its square.

Thus for low gains high density increase factors could be reached at the expense of the cooling rate.

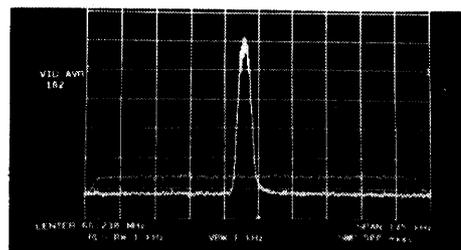


Fig. 8. 60 min, factor 28 in peak density,  $2 \cdot 10^7$  protons.

7.1 Computer simulation of cooling process at different intensities

Since the feedback system is the result of cascading many components such as pick-ups, signal combiners, amplifiers, filters, phase equalizers, wide-band gaps (and also the beam through its transfer function  $G(\omega)$ ) the overall gain characteristic  $g(\omega) \times T(\omega)$  becomes quite complicated. Thus  $g(\omega) \times T(\omega)$  is most conveniently expressed numerically and the Fokker Planck equation can be integrated by computer for a given initial distribution.

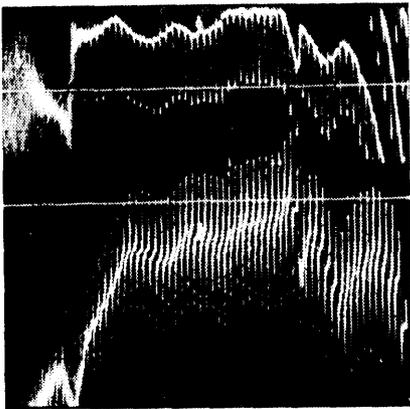
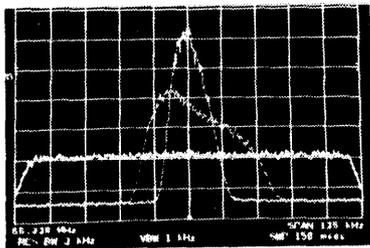
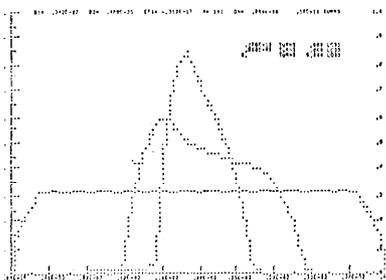


Fig. 9.  $g(\omega)$  between 0 and 240 MHz.

To check the theory with the experiment,  $g(\omega)$  of the system was recorded with a network analyser in 15 frequency points inside each of 45 harmonic ( $15 \leq H \leq 59$ ) bands. The 15 frequencies were chosen as harmonics of 15 equidistant revolution frequencies, representing 15 equidistant particle energies between +25 MeV and -25 MeV. The data were used in a computer program to calculate the evolution of initially flat distributions after four and eight minutes at different intensities  $N$ . The equivalent experiments were done with ICE.



(a)



(b)

Fig. 10. (a) Observed density evolution after four and eight minutes.  $7 \cdot 10^7$  protons.  
(b) Computed density evolution.

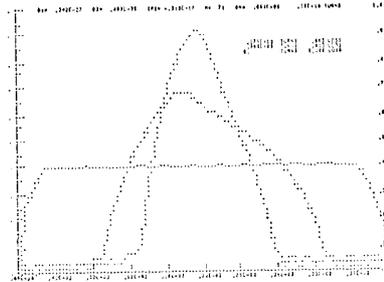
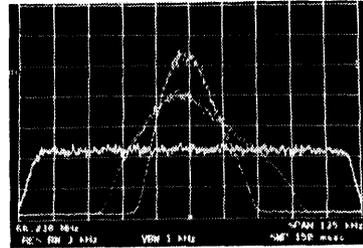


Fig. 11. As Fig. 10, but  $5 \cdot 10^8$  protons.

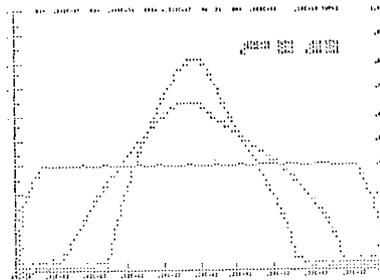
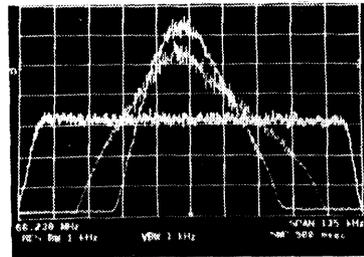
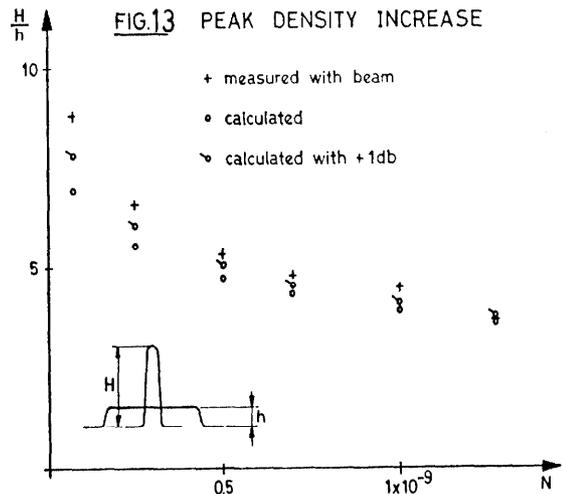


Fig. 12. As Fig. 10, but  $1.27 \cdot 10^9$  protons.



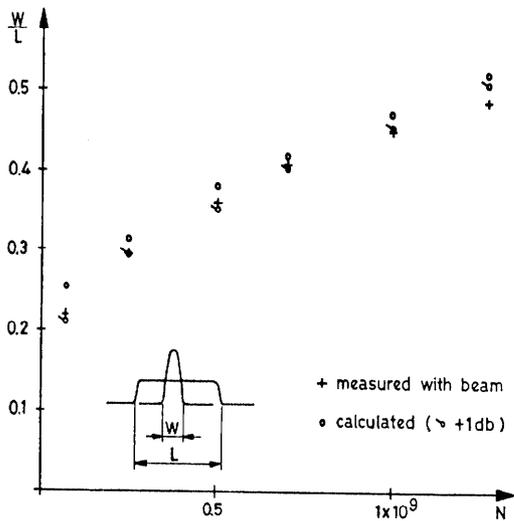


Fig. 14. Reduction of full width in  $\Delta p/p$ .

There is good agreement for the increase in peak density and reduction of full width.

The ratio between open loop and closed loop signals is given by (9). The Schottky signals of the 18th harmonic were photographed under both conditions for  $1.27 \cdot 10^9$  particles after eight minutes. They were also computed for eight minutes, for the two conditions.

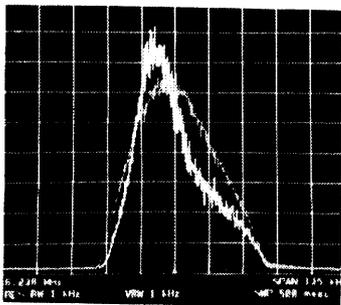


Fig. 15. Observed thin trace open loop, thick trace closed loop.

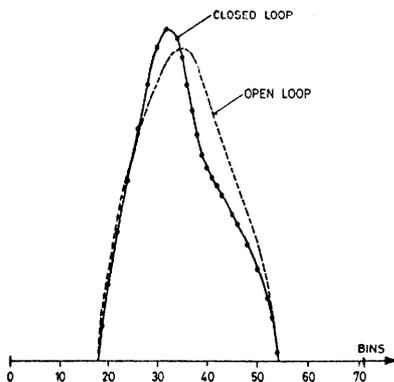


Fig. 16. Computed.

**7.2 Simultaneous stochastic cooling in all three planes** became an urgent goal when in early 1978 a suspicion surfaced that momentum cooling could exclude horizontal cooling or vice versa.

At that time only vertical and longitudinal cooling had been demonstrated, the horizontal pick-up and kicker being centred too far towards the inside of the machine acceptance. After modification the doubts were removed when a beam of  $6 \cdot 10^9$  protons was cooled simultaneously in all three planes, and its loss rate decreased from typically 100% per hour to a residual value of around 2% per hour, a loss rate compatible with single Coulomb scattering.

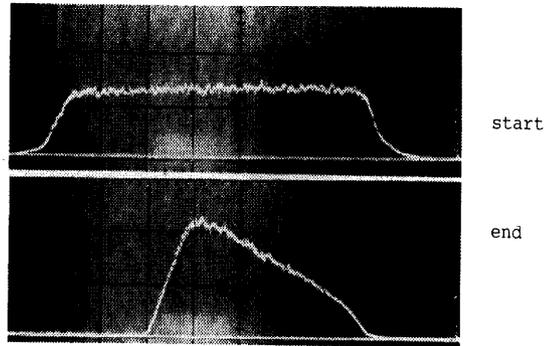


Fig. 17. Longitudinal Schottky scans before and after cooling in three planes for 30 min.

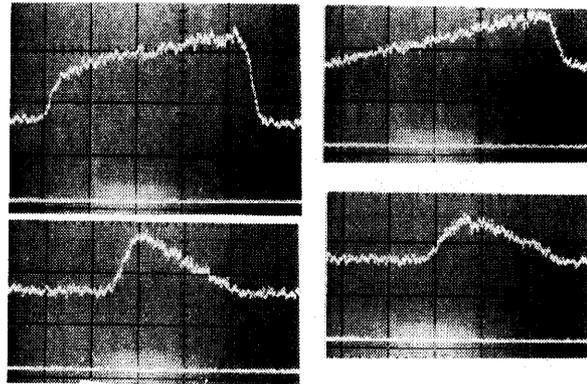


Fig. 18. Vertical and horizontal Schottky signals before and after 30 min of cooling. The rms betatron oscillation amplitude is  $\sim \gamma$ ,  $E \sim x$ .

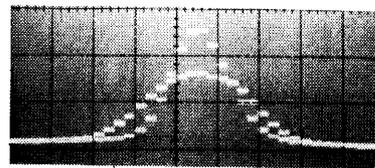


Fig. 19. Horizontal beam profile before and after cooling, as seen by a monitor based on beam-induced ionization electrons from the residual gas.

### 7.3 Antiproton lifetime experiment

Shortly afterwards it was decided to establish a new experimental lower limit for the lifetime of antiprotons. The existing lower limit was 120  $\mu s$ , far below the 24 h required to accumulate in the AA machine the nominal intensity of  $6 \cdot 10^{11}$   $\bar{p}$ . A tungsten target was installed in the transfer tunnel and bursts of around 250  $\bar{p}$  were accepted in ICE. Particles could be detected destructively independently of their momentum spread. After momentum cooling to a FWHH  $\Delta p/p = 10^{-5}$  their Schottky signal from a resonant cavity, tuned to the 35th harmonic and having a Q of 5000, became visible with as few as 50 antiprotons<sup>12</sup>.

With cooling in all three planes antiprotons were kept circulating for 86 hours<sup>9</sup>.

Stochastic cooling of bunched beams was investigated in order to stack antiprotons in ICE<sup>10</sup>. Bunching was done with the first harmonic and bunch

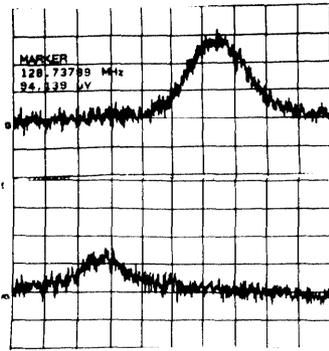


Fig. 20. Longitudinal Schottky signal from  $\sim 240 \bar{p}$ .

About 100 h later  $\sim 80 \bar{p}$  are left.

lengths  $1/3$  to  $1/2$  of the circumference, the remainder being kept free for injection (kicker rise, flat top and fall). The maximum RF bucket height (hardware limit) was  $\Delta p/p = \pm 5 \cdot 10^{-4}$ , whereas the injected particles had a  $\Delta p/p = \pm 3 \cdot 10^{-3}$ . With momentum cooling the beam progressively entered the bucket and accumulated in its centre until  $\Delta p/p$  approximately equalled the equilibrium value of  $\pm 2 \cdot 10^{-4}$ , observed with low-intensity unbunched beams. The beam bunching was clearly visible both in the time domain (oscilloscope) and the frequency domain where the lines of the first three harmonics grew to a value proportional to  $N$ . (Schottky signals are  $\propto \sqrt{N}$ .) In addition, the full aperture injection kicker, which could only kick in a limited particle-free interval of the RF period, did not cause beam loss.

With stochastic cooling in all three planes the lifetime of the bunched beam was equal to that of equally intense unbunched beams. This technique permitted the ICE team to collect 14,000  $\bar{p}$  from many pulses, each containing only a few hundred particles<sup>11</sup>.

#### 8. Acknowledgements

The ICE machine is the result of a successful collaboration between the EP, ISR, SB, PS and SPS divisions. The authors are grateful to F. Bonaudi, W. Schnell and P. Strolin for discussions and encouragement.

#### 9. References

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