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NEW WALL CURRENT BEAM POSITION MONITOR

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Introduction

When a beam pulse of charged particles moves through a beam duct, wall currents are induced. The current distribution on the wall depends on the beam position. Usually the measurement of the current distribution is done with resistors which are connected between two flanges insulated from eath other.[1-3] However with this method, the measurement of displacements for high energy bunched proton beam is difficult because of small magnitudes or fast decay constants of output signals. In this paper, a new type of the wall current beam position monitor is described. By inserting a segmented inner pipe into the beam duct, the low-frequency response of the monitor is improved. The current distribution is measured with four current transformers instead of resistors. Here the performance of this monitor is analysed by solving the electromagnetic field at the monitor, using a simple model.

Analysis of the monitor

The geometrical structureof the monitor is shown in Fig.1. The beam displacements in horizontal and vertical directions can be detected simultaneously. The wall currents, which are divided into four components by four slits in the inner pipe, flow through current transformers into the outer beam duct. To solve the EM field at the monitor, we adopt a simple model, in which the four slits are removed and the transformers are replaced with an azimuthally uniform resistor shown in Fig.2. The radius of the inner pipe is assumed to be equal to that of the circular beam duct. All pipes are assumed to be made of perfect conductor.

First we solve the EM field induced by the travelling charges in a circular beam duct.[4-7] In this case the EM fields are found by solving a wave equation only for the longitudinal electric field E_z in cylindrical coordinates

$$\left\{\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \psi^2} + \frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2}\right\} E_z = \frac{1}{\varepsilon}\frac{\partial \rho}{\partial t} - i\omega\mu J_z . \quad (1)$$

Here we have assumed a time dependence of $\exp(-i\omega t)$. The charge density is assumed to be expressed as $\rho = \rho_0 \delta_2(\vec{r}-\vec{a})e^{ikz}$ with $k = \frac{\omega}{v}$, in which velocity v of charges is uniform and the transverse components of the beam current are neglected. The displacement $\vec{a}$ is assumed to be (a, 0) in polar coordinates, that is, the displacement is in a horizontal median plane. Then E_z is expressed as

$$E_z(r,\psi) = \frac{1}{2\pi}\frac{ik\rho_0}{\varepsilon\gamma^2}\sum_{m=0}^{\infty}(2-\delta_{m0})\cos m\psi \frac{I_m(\frac{k}{\gamma}a)}{I_m(\frac{k}{\gamma}b)} \times \left\{I_m(\tfrac{k}{\gamma}b)K_m(\tfrac{k}{\gamma}r) - K_m(\tfrac{k}{\gamma}b)I_m(\tfrac{k}{\gamma}r)\right\} \quad (2)$$

with b the pipe radius, where I_m and K_m are modified Bessel functions. The wall current distribution is expressed as

$$J_w = -H_\psi(b,\psi) = -\frac{\omega\rho_0}{2\pi bk}\sum_{m=0}^{\infty}(2-\delta_{m0})\cos m\psi \frac{I_m(\frac{k}{\gamma}a)}{I_m(\frac{k}{\gamma}b)} . \quad (3)$$

In this paper we analyse mainly the EM fields for m=0 and 1, which are called a common mode and a difference mode respectively.

a) Low-frequency region —— In this region, conditions such as $\frac{\omega}{c}b \ll 1$ and $\frac{k}{\gamma}b \ll 1$ hold. Generally wall current monitors have poor position sensitivity for low-frequency components of the bunched beam. A decay constant of the sensitivity is calculated in this section with the model. When the wall currents flow through the resistor, a voltage difference is produced across it. This induced difference is cancelled by currents in by-passing circuits, and decays at a rate dependent on the resistance R and the inductances of the by-passing circuits.

For the common mode, the field in the coaxial region is assumed to be expressed with the TEM mode only. Furthermore we assume the following relation

$$\int_A^B E_z dz \simeq \int_C^B E_r dr . \quad (4)$$

Then H_ψ is expressed as

$$H_\psi(r,\psi,z) = -\frac{iE_0\, g}{c\mu \ln\frac{d}{b}\, \sin\frac{\omega}{c}\ell}\, \frac{1}{r}\cos\frac{\omega}{c}(z-\ell-g), \quad (5)$$

where g and ℓ are lengths of the resistor and the inner pipe respectively, and d is the radius of the outer pipe. E_0 is the longitudinal electric field at the resistor. In the low-frequency region, Eq.(3) is approximately reduced to

$$J_w(\psi) = -\frac{J_0}{2\pi b}\left\{1 + 2\frac{a}{b}\cos\psi\right\}, \quad (6)$$

where J_0 is the total beam current. From the continuity law of current, we obtain

$$J_w = J_R + J_C = \sigma E - H_\psi(b+,\psi\ ,\ g), \quad (7)$$

where J_R and J_C are currents flowing through the resistor and the coaxial region respectively, and σ is the conductivity of the resistor per unit azimuthal length. Taking terms for m = 0 in Eq.(7), the voltage difference V_0 across the resistor is approximately expressed as

$$V_0 = E_0 g = \frac{J_0}{\frac{2\pi b\sigma}{g} + \frac{2\pi}{-i\omega\mu\ell \ln\frac{d}{b}}} . \quad (8)$$

This represents that the monitor is a parallel circuit of the resistor and a inductor. The decay time is $\tau_0 = L_0/R$, where $R = \frac{g}{2\pi b\sigma}$ and $L_0 = \frac{\mu\ell}{2\pi}\ln\frac{d}{b}$.

For the difference mode, the field in the coaxial region is assumed to be represented with the H_{11}-mode only. The longitudinal magnetic field is written as

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$$H_z = A\{N_1'(\chi b)J_1(\chi r) - J_1'(\chi b)N_1(\chi r)\}\sinh\alpha(\ell+g-z)\sin\psi \quad (9)$$

with a factor A determined later. χb is the first zero of the equation

$$N_1'(\chi b)J_1'(\chi b\frac{d}{b}) - J_1'(\chi b)N_1'(\chi b\frac{d}{b}) = 0. \quad (10)$$

The first zero[8] is shown graphically as a function of d/b in Fig.3. Moreover, α is expressed as $\alpha^2 = \chi^2 - \frac{\omega^2}{c^2}$. Eq.(3) is rewritten in this case with a further approximation as

$$\int_C^B E_r dr \simeq E_r(b+,\psi,g)\int_d^b \frac{b}{r}dr \simeq \int_A^B E_z dz = E_1 g \quad (3a)$$

with this equation, H_ψ becomes

$$H_\psi(r,\psi,z) = \frac{\pi\ \alpha\ b\ E_1 g}{i2\mu\omega \ln\frac{d}{b}\ \sinh\alpha\ell}\ \frac{1}{r}\{N_1'(\chi b)J_1(\chi r) - J_1'(\chi r)N_1(\chi r)\}\ \cosh\alpha(g+\ell-z)\cos\psi\ . \quad (11)$$

In the difference mode, it is necessary to consider currents flowing azimuthally on the inside of the duct near the resistor, and to solve the EM field in the duct with boundary conditions such as

$$E_z(b,\psi,z) = E_1(\psi) \quad \text{for } 0 < z < g$$
$$\quad\quad 0 \quad \text{otherwise}$$
$$E_\psi(b,\psi,z) = H_r(b,\psi,z) = 0 \quad \text{for } z<0 \text{ or } z>g.$$

Then the EM field can be represented as a infinite summation of the cut-off E-modes. This implies that H_z is zero and no azimuthal current flows on the inside of the wall. Taking terms for m=1 in Eq.(7), the voltage difference V_1 across the resistor is

$$V_1(\psi) = E_1(\psi)g = \frac{J_0}{\frac{1}{R} + \frac{2\pi\alpha}{-i\omega\mu\ln\frac{d}{b}\ \tanh\alpha\ell}}\ \frac{2a}{b}\cos\psi. \quad (12)$$

The decay time is $\tau_1 = L_1/R$, where $L_1 = \frac{\mu\ln\frac{d}{b}}{2\pi\alpha}\tanh\alpha\ell$.

In the coaxial region, the wall currents in the difference mode flow azimuthally, and in the common mode flow longitudinary. The inductances L_0 and L_1 are of the same magnitude when ℓ is small, but L_1 has an upper limit when ℓ becomes large. If the inner pipe has slits, which interrupt the wall currents in the difference mode, the decay time τ_1 is expected to become long. It is difficult to take the effect of these slits in analysing the EM field.

b) High-frequency region —— In this region, we must calculate a rise time of the wall current and resonance frequency of the monitor.

With a condition a=0, Eq.(3) is reduced to

$$J_w = -\frac{J_0}{2\pi b}\ \frac{1}{I_0(\frac{k}{\gamma}b)}\ . \quad (13)$$

The total amount of the wall current is smaller than that of the beam current. The difference between them are transmitted as the displacement current through the beam duct, and can not be neglected in the high-frequency region. J_w and J_0 are considered to be an output and an input through a double stage RC-filter, when we take the first two terms in $I_0(\frac{k}{\gamma}b)$. Its rise time is

$$\tau_r \simeq 1.7 \times \frac{b}{\beta\gamma c}\ .$$

Next we conside the cavity resonance in the coaxial region. In both common and difference modes, R is very samll in comparison with the characteristic impedances of these modes in the coaxial region. This region can be seeemd as a cavity enclosed with perfectly conducting walls. In the common mode, using Eq.(5), a resonance condition $\frac{\omega}{c}\ell = \pi$ leads to the resonance frequency $f_0 = \frac{c}{2\ell}$. In the difference mode, Eq.(9) is rewritten as

$$H_z = A\{N_1'(\chi b)J_1(\chi r) - J_1'(\chi b)N_1(\chi r)\}\sin\eta(\ell+g-z)\sin\psi, \quad (9a)$$

where $\eta^2 = \frac{\omega^2}{c^2} - \chi^2$. The resonance condition is $\eta\ell = \pi$, and the resonance frequency is $f_1 = \frac{c}{2\pi}\sqrt{\chi^2 + \frac{\pi^2}{\ell^2}}$.

c) Position sensitivity —— In the real monitor, wall currents are divided into four components I_i and measured with current transformers. I_i's can be expressed approximately as

$$I_i = \int_{(2i-3)\pi/4}^{(2i-1)\pi/4} J_w(\psi) b d\psi \quad i=1,\ldots,4.$$

A ratio H defined as $(I_1-I_3)/(I_1+I_3)$ is calculated as

$$\Delta H = \frac{4\sqrt{2}}{\pi}\frac{a}{b}\{1 - (\frac{4}{\pi} - \frac{1}{3})(\frac{a}{b})^2\}, \quad (14)$$

when we take the first four terms in Eq.(3) with approximations in low-frequency region.

d) Effect of the inductance of a circuit through the current transformer —— Here we calculate effect of the inductance of a circuit, where wall current flows along a short wire through the current transformer, between the inner and outer pipe. The circuit of the wire has not only the impedance due to the transformer but also the self inductance. This is calculated as $2(d-b)\{\ln\frac{2(d-b)}{h} - 0.75\} \times 10^{-7}$, where h is the radius of the wire. Furthermore another inductance should be added in series because of the restricted wall currents streams on the inner and outer pipe near the wire. We would replace the transformers with a circuit of resistor and inductor in series shown in Fig.4. This correction changes the resistance R to $R/(1 + \frac{L}{L_{0,1}})$ in the equations of the voltage difference and the decay time for both common and difference modes, where L is the additional inductance. The effect of L is to decrease the magnitude of the output signal and to increase the decay time, and is not small when the inner pipe length is short.

Numerical example

A pair of monitor of this type, with and without the slits were bench tested. Its parameters were chosen so that the monitor can detect the beam position of a 12 GeV PS in KEK, where RF changes between 6∿8 MHz. b,d,h and ℓ are 69,79,2 and 150 mm respectively. The current transformer is a ferrite ring of the averaged diameter 9 mm with a 32-turn coil as the secondary winding, and its output is measured with a 50-Ω coaxial cable.

First we calculate the decay time and compare with experimental values. The resistance R is calculated as 1.22×10^{-2} Ω. L_0 and L_1 are determined as 4.1 and 1.9 nH respectively, with an approximation $\alpha \simeq \chi$. Then τ_0 and τ_1 is 3.3 and 1.6×10^{-7} s. But these values are small than the experimental τ_0 and τ_1, which are 4.5 and 2.6×10^{-7} s respectively for the monitor without the slits. If the difference is assumed to be due to the effect of the additional inductance L, L is about 1.5 nH, which is not an unreasonable value compared with the calculated inductance of the wires 0.8 nH. The expected decreace of the magnitude of the output by L was recognized experimentally in the common mode. In the monitor with the slits, τ_0 did not change but τ_1 changed to 3.3×10^{-7}s. This implies that L_1 increase to be about 1.6 times as much as that without the slits in this case.

Next the rise time τ_r is calculated as 0.3 ns for 500 MeV proton beam at the injection, and is short enough. The resonance frequency f_0 is 1 GHz, which was observed experimentally, But f_0 and f_1 ($>f_0$) are high enough compared with the frequency spectrum of the bunch in the 12 GeV PS, and have no effect on the output. The relation between the ratio ΔH and the displacement a explained well experimental results in the bench test.

Monitors of this type shown in Fig. 1 were tested in the 12 GeV PS in KEK, and the result was satisfactory. The results of the test with the simulated beam and the proton beam will be published elsewhere.[9]

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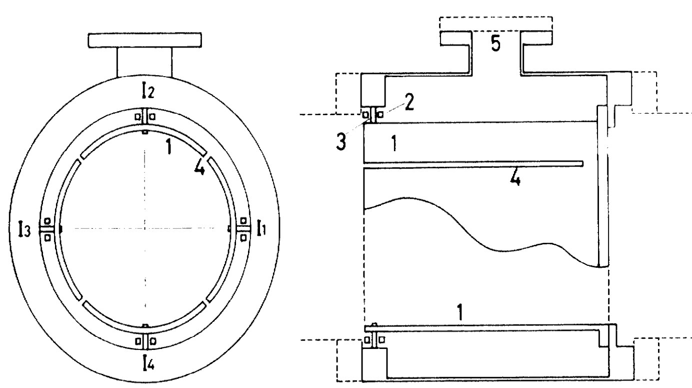

Fig.1 Monitor installed in 12 GeV PS in KEK
1: Inner pipe, 200 mm long, 2: Current transformer
3: Wire 4: Slit 5: Feedthrough

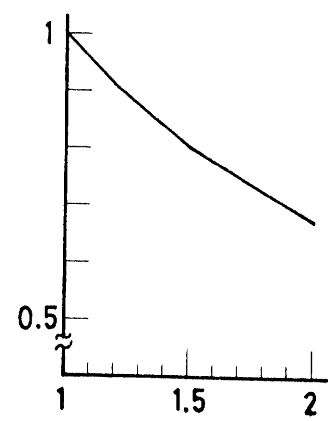

Fig.3 The first zero as a function of b/d. Vertical axis: χb
Horizontal axis: b/d

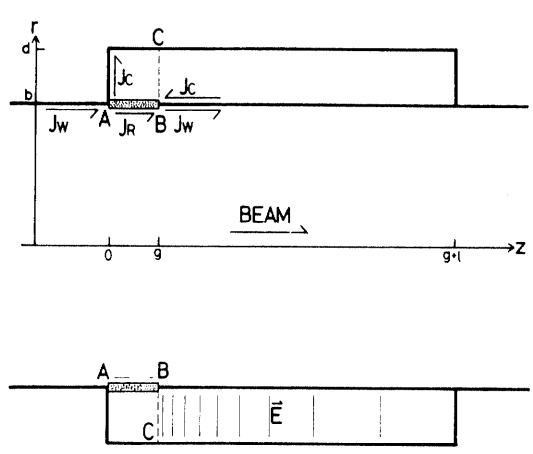

Fig.2 Model for the calculation.

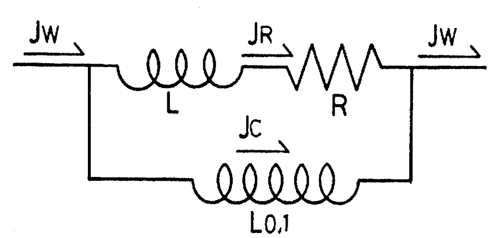

Fig.4 Equivalent circuit for the monitor including the additional inductance.