

TRANSPORT CALCULATIONS FOR VERY HIGH CURRENT BEAMS
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Abstract

A computer program has been developed to calculate the effect of the transverse space charge forces on a particle beam. The method is to divide the beam into a large number, M , of "beamlets," and compute the force on each beamlet by summing over the forces due to all the others. $M \geq 1000$ is needed to make spurious collisional effects negligible.¹ To the space charge forces, we add the forces due to the physical components of the system such as quadrupoles, solenoids or accelerating gaps. Using this program we have verified the prediction² that at high current, certain perturbations of the K-V distributions are unstable. In a FODO line tuned to 90° phase advance per cell at zero current, with the phase advance depressed to 30° by space charge, we found that the rms emittance of a perturbed K-V distribution grows by a factor of 2 in ~ 40 cells, and then remains stable, although no longer having the K-V form. For thin-lens focusing, this result is in good agreement with Haber's calculation. The use of finite quadrupoles rather than thin lenses has remarkably little effect.

Introduction

In a number of important applications of accelerator technology, it is necessary to transport particle beams of very high currents over extended distances. One such application is heavy-ion triggered inertial confinement fusion, where kiloampere beams of multi-GeV heavy ions ($A > 200$) must be brought to focus on a fusion pellet with dimensions of order one millimeter. Recent theoretical studies indicate that at sufficiently high current, a particle beam will be unstable to certain perturbations of its phase space distribution, at least in the case where the initial distribution is of the Kapchinsky-Vladimirsky (K-V) form. Presently available theoretical methods cannot answer the important questions of (1) how large the instabilities will become before saturating (or indeed if they will eventually saturate), or (2) whether these or similar instabilities exist for real beams which, in general, do not have the K-V form. The only method which appears suitable for answering these questions for this highly non-linear problem is numerical simulation using a large-scale computer program. In this straightforward approach the critical question is whether it is possible to represent a real beam (with a line density of 10^{10} - 10^{13} particles/cm) by a "reasonable" number, M , of representative particle trajectories and still obtain results which are applicable to the real beam. Reasonable here means that the calculations must be performed within acceptable computer running times, costs, and available memory size. We have developed a computer program which performs the required calculations including the effects of external focusing elements (quadrupole and solenoid magnets), and static accelerating fields (as in an induction linear accelerator), as well as the space charge forces. The calculation method has been described earlier.³ Results obtained with this program have been compared with theoretical predictions^{2,4} and another, completely independent, numerical simulation program^{1,5} and appear to be reliable. We have also performed a simulation of a planned experimental measurement. Eventual comparison of these calculations with experiment will provide a definitive test of the reliability of the simulation approach.

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Method

We represent a particle beam of total current I by a set of M line current of magnitude (I/M) . We choose a local cartesian coordinate system (x, y, s) , where s is the direction of propagation of the beam. The equation of motion of line current i is then

$$\frac{d^2 x_i}{ds^2} = -K_x(s)x_i + \frac{Q}{2N} \sum_{\substack{j=1 \\ j \neq i}}^M \frac{x_i - x_j}{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (1)$$

and similarly for y_i . The notation here is the same as in reference 3. The following approximations are made in obtaining equation (1):

a. The beam is continuous in the s direction or, more precisely, the longitudinal extent of the beam is large compared to x_{\max}/γ and y_{\max}/γ .

b. The beam is paraxial, i.e.

$$\left(\frac{dx_i}{ds}\right)^2 \ll 1 \text{ and } \left(\frac{dy_i}{ds}\right)^2 \ll 1 \text{ for all } i, \text{ and are}$$

neglected.

c. The beam is monoenergetic.

d. The external focusing forces are linear, i.e. $K_x(s)$ and $K_y(s)$ are not functions of x or y .

Of these approximations, only a is important. The others could be removed by straightforward modifications of the existing program.

The $2M$ second order differential equations of the type given by equation (1) are transformed (trivially) to $4M$ first order equations with the additional independent variables

$$(xp)_i = \frac{dx_i}{ds}, \quad (yp)_i = \frac{dy_i}{ds},$$

and solved simultaneously using a fast Adams method differential equation solver.⁶ This method was chosen because it was readily available as a fully tested and documented subroutine. In using this subroutine, initialization of the integration is trivial and accuracy is controlled automatically by specifying (with two constants) the required accuracy per integration step. Thus, using the Adams method integrator saved considerable time in writing and testing the program. It is not, however, the fastest integrating algorithm we could have used. Being a predictor-corrector type algorithm, it is about a factor of two slower for the same accuracy than the simple algorithm employed by Haber, in the case where K_x (and K_y) is a continuous function of s . Additional time is lost whenever K_x is discontinuous.

Our method of computing the space charge force is essentially the use of the Green's function, as implied by equation (1). We chose this method because of its simplicity and a (mistaken) belief that the number of trajectories needed to accurately simulate a real beam would be small enough that the Green's function calculation would be faster than the alternative method of using a Poisson solver technique. The computing time of our method is proportional to M^2 whereas the computing time of the Poisson solver method is proportional to $m \log m$ where m is the number of mesh points of

the grid on which Poisson's equation is solved. The remaining advantage of the Green's function method is that it can handle problems in which the beam size changes by large factors with no additional effort whereas in such cases the Poisson solver will be inefficient (due to unnecessarily large m values) unless a method for changing the mesh size as the beam size changes is built into the program.

The major difficulty in representing a real beam with a relatively small number of trajectories is the collisional effect. This effect requires that M be large and that the singularity in the space charge term of equation (1) be removed. We do this by replacing $(x_i - x_j)^2 + (y_i - y_j)^2 \equiv d_{ij}^2$ by r^2 whenever $d_{ij}^2 < r^2$. We choose $r^2 = \frac{C}{M} x_{\max}(s) y_{\max}(s)$, where C is an adjustable smoothing parameter of order unity. Our results in all cases studied are quite insensitive to the value of C . Note that the collisional effect is just as important in the Poisson solver technique as in the Green's function method for calculating the space charge force. As an example, we have calculated transport through a thin lens FODO system having a phase advance $\mu_0 = 90^\circ$ per cell depressed to $\mu = 30^\circ$ by the space charge force. With $M = 1500$ we find "secular" fractional emittance growths¹ of 4.6×10^{-3} per cell with $C = 1$ and 3.5×10^{-3} with $C = 3$. These values bracket the curve of figure 6 in reference 1, which gives the secular emittance growth rate for the same system in a Poisson solver calculation. The effective smoothing parameter in the latter calculation was quite close to our $C = 1$ case.

Most of the simulations we have performed thus far have been for cases where the input beam is matched to the system. Matching is done by an iterative technique using the K-V envelope equations as an option in our main program. The user guesses the initial beam envelope size x_{\max} and y_{\max} and the envelope slopes $x'_{\max} = \frac{dx_{\max}}{ds}$ and $y'_{\max} = \frac{dy_{\max}}{ds}$. For a given set of input parameters (energy, current, emittance = ϵ , etc.) and a defined structure for one cell of the transport system, the program then varies x_{\max} , y_{\max} , x'_{\max} , and y'_{\max} until all four parameters reproduce within the requested accuracy after one cell. The variation is restricted to a two-parameter search by making use of the symmetry of the system: the starting point of the system must be either a point where the phase ellipses are upright in both planes ($x_{\max}|x'_{\max}| = y_{\max}|y'_{\max}| = \epsilon/\pi$) or a point where $x_{\max} = y_{\max}$, $x'_{\max} = -y'_{\max}$, which always exists for a periodic system due to time-reversal invariance. Having obtained the matched beam envelope parameters, the M initial trajectory coordinates (x_i, xp_i, y_i, yp_i) are chosen at random from the K-V distribution using standard Monte Carlo methods. Other distribution functions are possible, but we have not found a unique procedure for beam matching for non-K-V distributions. In case the initial beam ellipse is not upright, we first populate an upright ellipse with semiaxes $\epsilon/|x'_{\max}| \pi$ in the spatial coordinates and $|x'_{\max}|$ in the momentum coordinates, and then perform the drift transformation

$$\left. \begin{aligned} x_i &\rightarrow x_i + Dx'_i \\ y_i &\rightarrow y_i - Dy'_i \\ x'_i &\rightarrow x'_i \\ y'_i &\rightarrow y'_i \end{aligned} \right\} \quad (2)$$

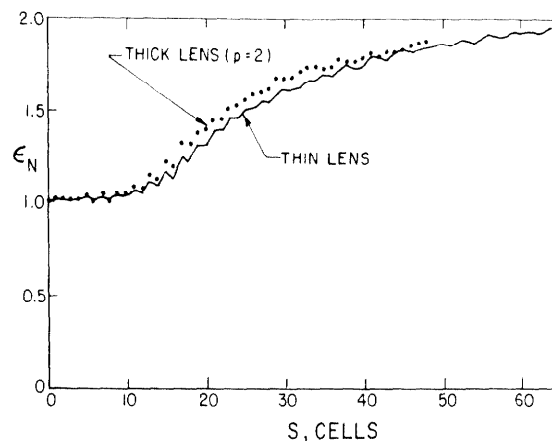
$$\text{where } D = \pm \frac{1}{|x'_{\max}|} \sqrt{x_{\max}^2 - (\epsilon/\pi x'_{\max})^2}, \quad (3)$$

which preserves the emittance while reproducing the beam envelope obtained in the matching process. We sometimes make use of the inverse of this transformation in displaying the phase-space plots resulting from a calculation. The transformation to an upright position increases the visibility of any distortions of a non-upright ellipse, and removes contributions to the moments of the beam distributions (i.e. quantities of the form $\langle (x_i)^m (xp_i)^m \rangle$) due solely to the non-upright position of the ellipse. The drift distance D in equation (2) can be obtained from equation (3) only in cases where the beam envelope is close to elliptical in shape. For non-K-V distributions the rms emittance (which is equal to the envelope emittance for a K-V distribution) is used and values of x_{\max} , x'_{\max} , etc. are found from the actual distribution in applying equation (3).

Results

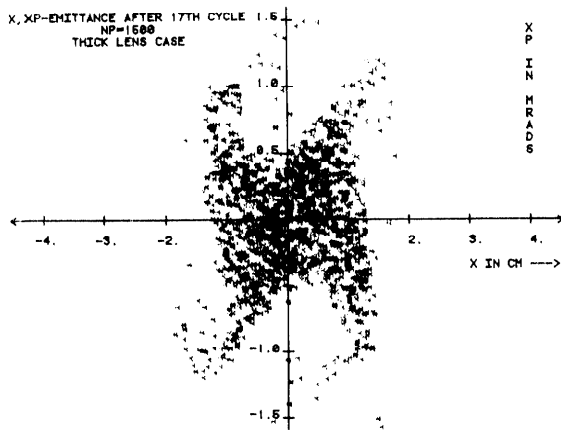
We present here a few examples of calculations performed with our program, in order to demonstrate the validity and power of the program. Although our calculation is done with physical variables (energies, currents, particle species and charge, etc.), we know that the space charge transport problem is scalable⁸ and thus fully described by the single-particle phase advance per cell at zero current, μ_0 , and the phase advance per cell, μ , with space charge.

Our first example is transport through a symmetric FODO lattice with $\mu_0 = 90^\circ$, $\mu = 30^\circ$. This case has been studied by Haber also.^{1,5} We regard the good agreement between his results and ours as verification of the validity of the numerical simulation approach, since the two calculations are completely independent and quite different. In figure 1 we show the mean emittance



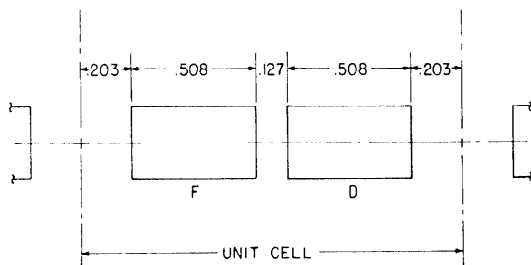
1. Emittance growth due to space charge effect in a FODO transport line. We plot the geometrical mean of the x-plane and y-plane normalized emittances, as a function of the number of cells traversed. The solid curve is obtained when the quadrupole magnets are approximated by thin lenses, the dots for finite quadrupoles filling one-half of the available space ($p = 2$).

$\epsilon = (\epsilon_x \epsilon_y)^{1/2}$, as a function of the number of cells traversed. For the first ten cells, we see a very slow increase in ϵ due to the (spurious) collisional effect



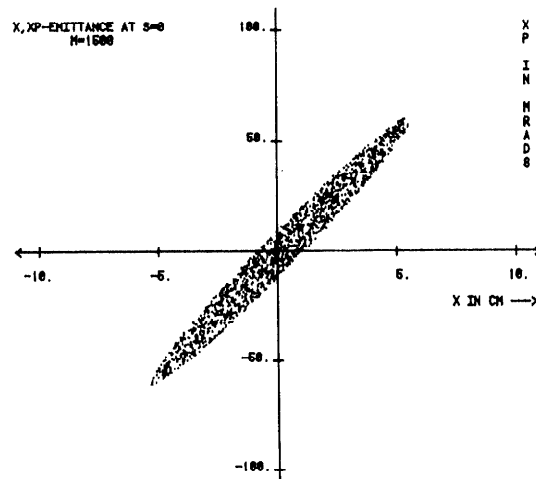
2. The x-xp projection of the beam phase space at the point $s = 17$ for the calculation of figure 1.

(here $M = 1500$), followed by a more rapid increase in cells 10 to about 40, and then a resumption of the slow collisional growth. The similarity to figure 1 of reference 1 is evident. In figure 2 we show the $x, xp \equiv x'$ projection of the phase space after 17 cells. The four-armed structure is quite similar to that displayed (for cell 20) in figure 2 of reference 5. With our program we are able to compare the thin-lens approximation used by Haber to the same calculation using finite-length quadrupoles. Figure 1 shows that the thin-lens and thick lens calculations are almost indistinguishable. Specifically, our thick-lens result is for the case where the quadrupole fields fill 50% of the available space. In this calculation the quadrupole fields terminate abruptly. It is quite feasible to insert in our calculation fields which fall off realistically, including the non-linear forces which result. Although we do not expect significant changes for the types of calculations presented here, we do intend to make this modification to the program in the future.

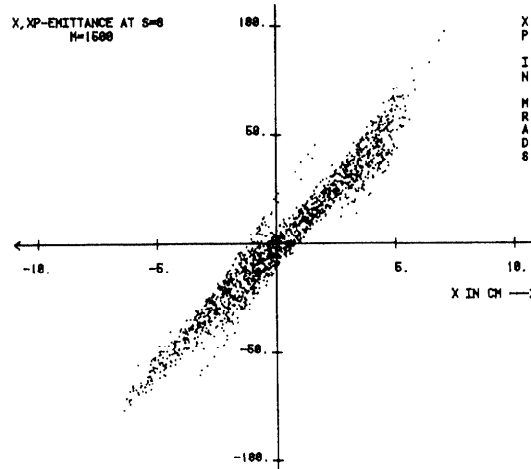


3. Geometry of the focusing structure for the planned cesium beam experiment. The field gradient in the quadrupoles is 0.823 kG/cm. Beam parameters are given in the text.

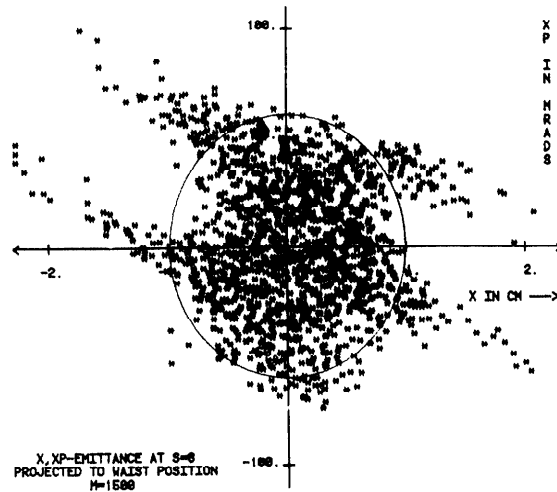
The second example is intended to verify our calculations experimentally, and to assist in the design of a planned experiment. This experiment⁹ consists of the transport of a high-current, one MeV Cs^{+1} beam through eight cells of a FODO system with a zero current phase advance of $\mu_0 = 90^\circ$ per cell. The asymmetric design of the focusing structure, shown in figure 3, and the limitation to eight cells is determined by available hardware. In the absence of a design for matching optics between the accelerator and the transport line, and between the transport line and the



(a)



(b)



(c)

4. Projections of the beam phase space in the x-xp plane for the cesium beam simulation. (a) Initial matched distribution. (b) Calculated distribution after eight cells. (c) Distribution after transformation of equation (2) is applied, and the horizontal scale expanded. The solid curve is the boundary of the initial distribution in this coordinate system.

final emittance-measuring instrumentation, we have calculated the matching condition at the center of the longer drift space, and carried the calculation only to the end of the eighth cell. We assume a beam current of 0.35 A which depresses the phase advance to $\mu = 19.3^\circ$, an emittance of 60π mr cm, and a K-V distribution. Representing the beam with $M = 1500$ particles, we observe a smooth rms emittance growth rate of 1.7 percent per cell for the first six cells, followed by a more rapid growth in the last two cells. We believe that most of the observed growth is the same spurious collisional effect seen in the first example because the rate is approximately doubled if we reduce M by a factor of two, and because of the larger tune shift in this example, together with some rough estimates of the scaling laws for the collisional effect as a function of time. The additional emittance growth of $\sim 12\%$ in the last two periods is probably real, but may be too small for definitive experimental verification. However, the distortion of the phase space distribution, shown in figure 4, is large and should be clearly observable in the experiment.

Future Work

The main goal of this work at present is to determine if the predictions of our numerical simulations are experimentally verified. To make the comparison with experiment definitive two modifications of the present version of the program are needed. The first is to include realistically the fringing fields of quadrupole magnets. The required modifications are straightforward. They should not increase the execution time of the program significantly, nor do we expect the non-linear effects of the quadrupole end fields to qualitatively alter our results. The second modification required is the use of non-K-V distributions. Specification of such distributions is not difficult, except that there is no unique definition of matching for arbitrary distribution functions.

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