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A MINUS-I QUADRUPOLE SYSTEM FOR CONTAINING ABERRATION-CORRECTION OCTUPOLES*

Stanley Fenster†

Abstract

Octupoles may be used to correct the third order spherical aberration of quadrupole transport systems. Crosstalk in the coupling of an octupole placed at a given point causes it to add a term with the wrong sign in the y-channel if it has the right sign in the x-channel, thus severely reducing efficiency. It is often convenient to utilize a special correcting section insertion which is seen as a +I transfer matrix by the first order focussing. Within point-to-point thin lens optics we give two-parameter systems with 16 magnets having locations with large S_x where $S_y = 0$ and vice versa for octupole placement.

The problem is to set up a -I quadrupole system which contains two special locations. At the first, S_x is large and S_y is zero; while at the second the reverse is true. Octupoles are placed at these locations and make corrections free of channel crosstalk. The layout is:

-I System

Section 1

- subsection 1A
  - part 1Aa
  - part 1Ab
- subsection 1B
  - part 1Ba
  - part 1Bb

Section 2

- subsection 2A
  - part 2Aa
  - part 2Ab
- subsection 2B
  - part 2Ba
  - part 2Bb

Note: Part B is the same as A but with the order of the elements reversed. Subsection 2 is the same as 1 but with the polarities of the quadrupoles reversed. Section 1 is X = -I, Y = +I. Section 2 is X = +I, Y = -I.

The basic module for the 1Aa part is a five element OFODO sequence of drift spaces and thin lenses and part 1Aa consists of two such modules notated in English and Greek respectively as

$$\text{1Aa:}\quad \underbrace{\ell\ P\ m\ Q\ n}_{\substack{\text{one module}\\ \text{1 Aa}}}\qquad \underbrace{\lambda\ \pi\ \mu\ \tau\ \nu}_{\substack{\text{one module}\\ \text{1Ab}}}$$

For example, the second element (P) after the source has transfer matrix

$$\begin{pmatrix} 1 & 0 \\ -P & 1 \end{pmatrix}$$

in the X-channel. Each module is one of the modes:

$$\text{PP = point-to-point} \quad = \begin{pmatrix} x & 0 \\ x & x \end{pmatrix} R_{12} = 0$$

$$\text{LP = parallel-to-point} \quad = \begin{pmatrix} 0 & x \\ x & x \end{pmatrix} R_{11} = 0$$

$$\text{PL = point-to-parallel} \quad = \begin{pmatrix} x & x \\ x & 0 \end{pmatrix} R_{22} = 0$$

$$\text{LL = parallel-to-parallel} = \begin{pmatrix} x & x \\ 0 & x \end{pmatrix} R_{21} = 0$$

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†Argonne National Laboratory, Argonne, IL 60439 USA

For the present model we have chosen

X_{1A}			PL	and	LP
Y_{1A}			PP	and	LL
X_{1Aa}	LL	Y_{1Aa}	LP	and	PL
X_{1Ab}	LP	Y_{1Ab}	LP	and	PL

This decomposition makes the problem tractable because the one LL zero of X_{1Aa} and the two (PL and LP) zeroes of Y_{1Aa} are easy to solve. By matrix multiplication

$$\text{(LP and PL)} \cdot \text{(LP and PL)} = \text{PP and LL}$$

$$Y_{1Aa} \cdot Y_{1Aa} = Y_{1A}$$

$$\text{LP} \cdot \text{LL} = \text{LP}$$

$$X_{1Ab} \cdot X_{1Aa} = X_{1A}$$

Thus Y_{1A} is automatically PP and LL, while X_{1A} needs to have one condition satisfied $((X_{1A})_{22} = 0)$ to become LP and PL. We should also like to specify, at the end of subsection 1A, the value of S_x $(=[X_{1A}]_{12})$. A count shows that 3+3+1+1 = 8 restrictions have been put on the 10 parameters in the two modules of 1Aa leaving two (mQ and πμ) free. One finds, with notation

$$a = m\,Q$$

$$b = \pi\,\mu$$

the expressions

$$\ell = \frac{b(1-a^2)\,(2-b^2)\,S_x}{(1+b)\,(1-b^2)\,(1+a+2\,a^2)}$$

$$P = \frac{(1+a+2\,a^2)\,(1-b^2)\,(1+b)}{2ab\,(1+a)\,(2-b^2)\,S_x}$$

$$m = \frac{2\,a^2 b\,(2-b^2)\,S_x}{(1+a+2\,a^2)\,(1-b^2)\,(1+b)}$$

$$Q = \frac{(1+a+2\,a^2)\,(1-b^2)\,(1+b)}{2ab\,(2-b^2)\,S_x}$$

$$n = \frac{b(2-b^2)\,(1+2\,a)\,S_x}{(1+a+2\,a^2)\,(1-b^2)\,(1+b)}$$

$$\lambda = \frac{(1-b-b^2)\,S_x}{(1+b)^2\,(1+a)}$$

$$\pi = \frac{(1+b)\,(1+a)}{b\,S_x}$$

$$\mu = \frac{b^2\,S_x}{(1+a)\,(1+b)}$$

$$\tau = \frac{(1+a)}{b(1-b)\,S_x}$$

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$$\nu = \frac{(1-b)\ S_x}{1+a}$$

$$S_y = (Y_{1A})_{12} = \frac{b(1+a)\ (2-b^2)\ S_x}{(1-b^2)\ (1+a+2\ a^2)}$$

where S_x is the peak of the X-channel sinelike function. The system allows all a, b choices within the limits

$$0 < a < 1$$

$$0 < b < \frac{\sqrt{5}-1}{2} = 0.618034$$

to keep all drift distances positive.

For a trial system it was found

$$(S_x)_{max} = (S_y)_{max} = 30 \text{ m}$$

was adequate to keep the octupole strength near 1 T/m. As an example we have chosen

$$a = 0.85 \quad b = 0.20$$

in an attempt to get a small total length

$$L = 4(\ell+m+n+\lambda+\mu+\nu).$$

We find (MKS units)

ℓ = 0.85973117	λ = 8.5585586
P = 0.10263132	π = 0.37000000
m = 4.4767956	μ = 0.54054056
Q = 0.18986795	τ = 0.38541667
n = 8.36494947	ν = 12.972973

The resulting cosinelike and sinelike trajectories are listed below

s	C_x	S_x	C_y	S_y
0.0	1.0	0.0	1.0	0.0
0.85973117	1.0	0.8597311	1.0	0.85973117
5.3365268	0.54054056	4.9415152	1.459	5.7315382
13.701476	0.54054056	20.416672	0.0	5.7315382
22.260035	0.54054056	36.250005	-1.4932394	5.7315382
22.800575	0.43243243	30.000000	-1.8861971	6.8778458
35.773548	0.0	30.000000	-1.8861971	0.0

$$X_{1A} = \begin{pmatrix} 0 & 30 \\ \frac{-1}{30} & 0 \end{pmatrix}$$

$$Y_{1A} + \begin{pmatrix} -1.8861971 & 0 \\ 0 & -0.53016733 \end{pmatrix}$$

Octupoles are placed at 1A and 2A. A similar type of problem was solved for a beam rotator.[1]

Alternate solutions are possible. One needs to consider only subsection 1A, which is divided into parts 1Aa, 1Ab, and repetitions as specified above. A part <u>a</u> or <u>b</u> may contain two or three magnets. There are sixteen cases, based on the following matrix multiplication table:

PL·LP = LL	LL·PL = PL
PL·PP = PL	LL·LL = LL
LP·PL = PP	PP·LP = LP
LP·LL = LP	PP·PP = PP

In the list below, N indicates the number of magnets per module. The example[2] given above corresponds to the first line of this list.

X_{Ab}	X_{Aa}	Y_{Ab}	Y_{Aa}	N_{Ab}	N_{Aa}	X_A	Y_A
LP	LL	(PL & LP)		2	2	LP	(LL & PP)
LL	PL			2	2	PL	
PL	PP			3	3	PL	
PP	LP			3	3	LP	
PL	PP	(LL & PP)		2	2	PL	
PP	LP			2	2	LP	
LP	LL			3	3	LP	
LL	PL			3	3	PL	
PP	LP	(PL & LP)		3	2	LP	
LP	LL			3	2	LP	
LL	PL			2	3	PL	
PL	PP			2	3	PL	
LL	PL	(LL & PP)		3	2	PL	
PL	PP			3	2	PL	
PP	LP			2	3	LP	
LP	LL			2	3	LP	

<u>References</u>

1. S. Kowalski and H. Enge, "Beam Rotator", <u>Proc. Fourth Int'l. Conf. on Magnet Technology</u>, CONF-720908, Brookhaven, 1972.

2. Karl Brown has emphasized +I sections as sextupole sacs.