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BEAM-BEAM EFFECT SIMULATION FOR A MODEST COLLIDING-BEAM EXPERIMENT AT FERMILAB

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Summary

It has been proposed[1] to get the Fermilab main ring to collide with a small proton storage ring in one of the six straight sections. Proton-proton colliding beam experiments would be carried out parasitically during the main ring ordinary acceleration cycle so that the beam could also be extracted and used in the fixed target experimental area. The performance parameters are shown in Tables 1 and 2. This hybrid operation of the main ring caused some concerns about the deterioration and the survival of the main proton beam because of the very large beam-beam tune shift. We performed numerical simulations of the motion of the main ring protons. Since the number of revolutions is relatively small we could simulate also an entire cycle of acceleration. The results: surprisingly the main ring beam can tolerate quite a large tune-shift with little deterioration, provided few precautions are taken for both beams handling.

	Main Ring	Small Storage Ring
Interaction length (ℓ)	18 m	
Crossing Angle	0 mrad (head on)	
$\beta^*_x = \beta^*_y$	70 m	4.65 m
α^*_x	−0.7	0
α^*_y	+0.7	0
Dispers. Function (η^*)	2.3 m	0 m
$\epsilon_x = \epsilon_y$ (95% beam)	(-)	$0.35\pi\ 10^{-6}$ m
Current (I)	0.14 A	1.4 A
Energy (kinetic)	8-400 GeV	25 GeV

Table 1. Interaction Region Parameters

Energy (GeV)	$\epsilon^{(*)}_{MR}$ ($\pi\cdot10^{-6}$ m)	$\sigma^{(*)}_{MR}$ (mm)	$\Delta\nu^{(*)}_{MR}$	$\Delta\nu^{(*)}_{SR}$	L (10^{31}cm^{-2}s^{-1})
8	1.0149	3.441	1.985	0.0004	0.11
100	0.0894	1.021	0.174	0.005	0.90
200	0.0449	0.724	0.087	0.009	1.35
300	0.0300	0.592	0.058	0.0138	1.64
400	0.0225	0.512	0.044	0.0185	1.86

*The same value applies to the horizontal and to the vertical plane.

Table 2. Performance

The Equation of Motion

The equation of motion is

$$z'' + k(s)z = \frac{2e}{E}\,\xi(s)$$

where s is the longitudinal coordinate, z stands for either x or y, the two transverse coordinates, $' \equiv d/ds$, k(s) is the periodic linear lattice function, E the particle total energy, e the particle charge and $\xi(s)$ the beam-beam electric field which is zero everywhere and in the interaction region is[2]

$$\xi = \frac{2I}{c}\,\frac{1-e^{-\frac{x^2+y^2}{2\sigma^2}}}{x^2+y^2}\,z \tag{1}$$

where I and σ are the current and the rms beam size of the "other" beam which is round with gaussian distribution. We assume here that the small storage ring is not affected by the interaction with the main ring. The beam size σ is a function of s. Take s = 0 at the center of the collision length

$$\sigma^2 = \sigma^{*2}\,[1+(s/\beta^*)^2]$$

where $\sigma^{*2} = \epsilon\beta^*/6\pi$ and β^* are the values at s = 0. The phase advance across this interaction length is

$$2\psi = \mathrm{arctg}(\ell/2\beta^*)\ .$$

In the main ring, this quantity is reasonably small (~15°), so one can approximate tbe beam-beam effect as a lumped nonlinear kick which leaves the displacement z unchanged and changes z' by the amount

$$\Delta z' = \frac{4\pi}{\beta^*_{MR}}\,(\Delta\nu)\ zF\left(\frac{x^2+y^2}{2\sigma^{*2}}\right) \tag{2}$$

where β^*_{MR} is the main ring value of β^*,

$$\Delta\nu = \frac{6Ir_p\ \beta^*_{MR}}{ec\gamma\epsilon}\,\psi \tag{3}$$

is the tune shift in the limit of small amplitude, r_p is the classical proton radius, γ the relativistic energy factor, and

$$F(u^2) = \frac{1}{\psi}\int_0^{\ell/2\beta^*} \frac{1-e^{-\frac{u^2}{1+t^2}}}{u^2}\,dt\ . \tag{4}$$

Eq. (1) is valid for head-on collision and when the axis of the two beams coincide. When the two beams are separated by a distance x_o, one replaces x with $x - x_o$ in (1) and (2).

The Computer Simulation

We take one thousand particles for our simulation. To each particle we associate the four initial conditions x, x', y and y'. These are taken randomly with a four-dimensional distribution which describes the main-ring beam at the crossing location. Our simulation consists in applying simultaneously to all the particles a series of a large number of cycles. When the energy is constant all the cycles are identical. When the energy is varied (acceleration), at each cycle the energy-dependent parameters are properly scaled. Each cycle simulates one revolution in the main ring. Typically we start with a front porch of about 50,000 turns; we accelerate from 8 GeV to 400 GeV in 200,000 turns, and we end with a flattop of 50,000 turns. The beam takes about one second to make 50,000 revolutions, thus in our simulation the ramp speed is of about 100 GeV/sec. Each cycle is made of two steps. The first step consists in applying to the particles a linear transformation to their coordinates x, x', y, and y', from the crossing point back to the same point one turn around. The transformation is represented by a 4x4 matrix which describes the main-ring lattice. For its determination we supply β_x, β_y, α_x and α_y at the crossing point and the two phase advances per turn. In general it is sufficient to specify only the fractional part of the two betatron tunes ν_x and ν_y. The second step consists in changing both x' and y' of each particle by the amount (2). The function $F(u^2)$ is

*Operated by the Universities Research Association, Inc., under contract with the Energy Research and Development Administration.

calculated at the beginning of our simulation program at values of u^2 between 0 and 100 and equally spaced by 0.025. At the second step, the computer calculates first $u^2 = (x^2+y^2)/2\sigma^{*2}$ and then $F(u^2)$ by means of a linear interpolation between the calculated values. In the case $u^2>100$, the asymptotic expression

$$F(u^2) = \ell/2\beta^*\psi u^2$$

is used. When acceleration is applied the momentum is changed every cycle before the second step. The momentum receives the same increment every revolution. If we denote by δ the ratio of the new momentum value to the initial value, then, before performing the second step, the (actual) separation x_0 is multiplied by $\sqrt{\delta}$, the amplitude u^2 is divided by δ, and the tuneshift $\Delta\nu$ is calculated by dividing also the initial value by δ. With this procedure we always carry out momentum normalized beam sizes and divergences. Every 1,000 revolutions, four hystograms of 20 channels corresponding to the four coordinates are prepared and displayed. Then averages, standard deviations, minima and maxima are calculated and printed out. With exception of some special case, we always found that the hystograms fairly reproduce a gaussian distribution. Thus we take the standard deviation as a sort of the measure of the beam size and the average as the location of the main-ring beam center. Another form of output is a plotting of two coordinates, one against the other (usually x and x'). The beam can be "observed" at any location of the main ring.

Results

Runs with no acceleration

We made several runs at constant energy over a period of time coresponding to one second. We first applied a linear beam-particle kick by setting $F(u^2)$ at the right hand side of (2) equal to 1 for any value of u. The beam-beam effect in this case corresponds to a quadrupole defosussing on both planes. This causes the depression of the tunes and the alteration of the beta-function around the ring. The main-ring beam is then mismatched by an amount that can be also calculated analytically. The results are shown in Table 3. There is a reduction of the beam size at the crossing point and a corresponding increase at the point diametrically opposite. This was expected.

When we applied the actual nonlinear beam-beam kick the beam size variation was smaller (see Table 3). We believe that this is still due to the betatron phase mismatch. Since the other beam is now equivalent to some sort of nonlinear lens, the mismatch is now a function of the oscillation amplitude. The mismatch is larger for small amplitude oscillations and averages to zero for larger ones.

Of course, because of the rather short runs we performed, we did not expect to see any of the long time instabilities like Arnold diffusion.[3] But we are confident that we did not get any sign of "stochasticity", an instability that if occurs should occur fast.[3] We found though an instability when the main beam was moved on top of the other. We made several runs still at constant energy (100 GeV), starting with a separation of 5 mm, and for several displacement speeds. A typical result is shown in Fig. 1. We speculate the vertical size increase is caused by nonlinear coupling. The final beam growth depends on the displacement speed. If the displacement takes 20 msec the final growth is 50%. For 2 ms the growth is of only 10%. No appreciable closed-orbit distortion was ever found.

Runs with acceleration

In one run we started from 100 GeV, accelerated up to 400 GeV and ended with a one-second flattop. Apart from the initial increase due to the nonlinear mismatch we did not observe any further change during the rest of the cycle. In other runs we started from 8 GeV, keeping the beams separated by some distance x_0. At 100 GeV the main beam was moved on top of the other within 20 msec and then accelerated to 400 GeV with a flattop of one second. Fig. 2 shows the case for x_0 = 15 mm. It is obvious there is a horizontal size increase during the early part of the acceleration. The instability disappears once the two beams are on top of each other. Could this be stochasticity? By increasing x_0 the initial beam size growth reduces and practically disappears for $x_0 \gtrsim 3$ cm. In this case the only size increase noticed occurs during the beam displacement operation.

Tune dependence

In all the previous runs the fractional tunes were equal to 0.4, and the best we could get with a beam displacement at 100 GeV and acceleration, was a size increase of about 13%, With the same condition the increase became 36% by taking 0.1 for the fractional tunes, probably because this region is considerably denser in resonances or because one gets closer to a half-integer tune value. The effect of the tune splitting is more dramatic as one can see by inspecting the results in Table 4. The beam growth occurs only during the beam displacement provided the beam separation is initially kept to 3 cm. The addition of a 8 GeV, one-second long front porch did not change the results. As expected, we found also that the beam growth decreases inversely with the energy where the displacement occurs.

Simulation of the MR Beam Slow - Extraction

We considered first the possibility of using the first half of the flattop for the beam-beam colliding experiment and the second half for the slow-spill extraction. For this purpose we separated again the two beams to a final value of 7.5 mm in 20 msec. There was a horizontal size growth of a factor 1.7 which probably can be managed by the extraction system. Since the scheme adopted at Fermilab for slow-extraction is based on the half-integer resonance, we proceeded to retune the two existing extraction quadrupoles and eventually add a new one to get the proper beam ellipse orientation at the extraction system. The spill rate could be adjusted by varying the total horizontal tune including the beam-beam tune shift. We found a combination of parameters such that the beam could be extracted in the present transport channel with an efficiency better than 1%. We found actually that the extraction was easier in presence of the second beam, since this was acting as the nonlinear element enhancing a tune-spread across the main ring beam.

Periodic Oscillations of Betatron Tunes

These were taken into account to simulate synchrotron oscillation and chromaticity. The analysis nevertheless is not completed and we did not get really any firm conclusion. The oscillation frequency was typically 1 kHz and was first applied during the extraction process described above. We did not see any change on the shape of the extracted beam and on the spill rate for amplitudes up to 1%. The same kind of oscillations were also applied to a 400-GeV beam for about 1 second with fractional tunes of 0.42 and 0.38. The two beams were kept separated by 30 mm. The only change noticed was a small amplitude modulation of the beam size at the same oscillation frequency probably enhanced by a nonlinear modulated mismatch.

Conclusion

From the practical point of view we conclude that the hybrid operation to run the main ring as an accelerator and as a colliding ring is possible, provided the two beams are kept separated enough at low energy and then displaced on top of each other at a reasonable speed. In this case the design luminosity figures can be achieved. From the theoretical aspects, our computer calculations have shown three different beam behaviours: (i) A beam-size growth occurs due to the nonlinear mismatch enhanced by the second beam. (ii) There is a horizontal instability when the two beams are moved horizontally on top of each other. This instability depends on the speed of the displacement. (iii) Finally a second kind of instability exists when the two beams are not separated enough, which can be related to stochasticity.

For further details on this work see Ref. (4).

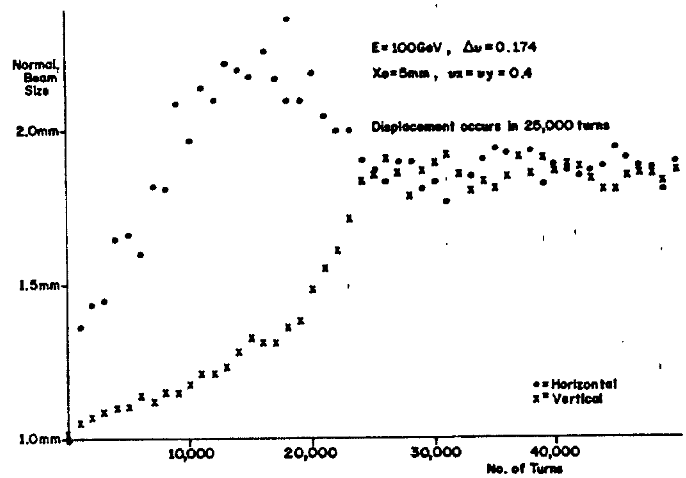

Fig. 1 Instability during beam displacement

	Linear Kick		Nonlinear Kick	
Energy	local	across	local	across
100 GeV	0.60	2.60	0.84	1.7
200 GeV	0.67	1.81	0.85	1.4
300 GeV	0.73	1.55	0.86	1.3
400 GeV	0.77	1.43	0.96	1.3

Table 3. Square ratio of average beam size to initial size

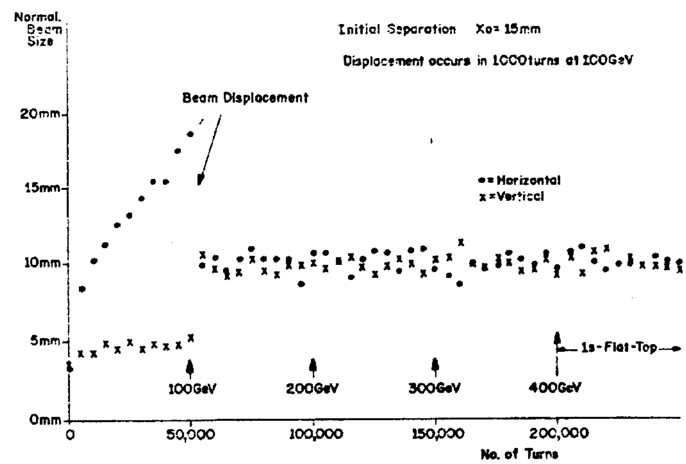

Fig. 2 Is this Stochasticity?

ν_x	ν_y	horiz	vertic.
.40	.40	1.1	1.1
.41	.39	1.9	1.9
.42	.38	1.8	2.0
.45	.35	1.8	2.4

Table 4. Relative size increase vs. tune separation

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