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COMPUTER SIMULATION OF THE ELECTRON-BUNCH WIDENING AS DUE TO SELF-BUNCHING

A.G. Ruggiero
Fermi National Accelerator Laboratory*
Batavia, Illinois 60510

Summary

Several theories[1-10] have been proposed to explain anomalous bunch lengthening and widening in particle storage rings.[11-13] All these theories either assume a modification of the accelerating potential well or a high frequency self-bunching mechanism similar to the one proposed for proton bunches.[8,10,14,15] Some of the theories assume interaction between several modes of self-bunching which leads then to "turbulence". In most cases it is possible to derive a stability condition in terms of bunch current and impedance, but then the assumption is made that the bunch will blow up its width to a value which matches the stability condition. To check how valid is this assumption we carried out a computer simulation of the behaviour of a high-intensity electron bunch.

We adopted a very special case of impedance model which is a constant and real impedance for any frequency. In this case the space charge effect is simply proportional to the local current.

The Equations of Motion

They are

$$\dot{\phi} = h\Omega' w \tag{1a}$$

and

$$\dot{w} = eV_c\,\delta_p(\theta)\,(\sin\phi-\sin\phi_r) + - 2eI\,\delta_p(\theta)\sum_{n=1}^{\infty} A_n G_n \cos(\tfrac{n}{h}\phi+\alpha_n+\beta_n) + - \frac{\Omega_o}{2\pi} Dw - q\delta_p(\theta) \tag{1b}$$

where e is the electron charge, I the bunch average current, Ω_o the angular revolution frequency, θ the angular coordinate, $\delta_p(\theta)$ a periodic delta function with a period $2\pi/M$, M being the number of identical, lumped and equispaced RF cavities in the ring. MV_c is the total peak RF voltage and h the harmonic number. The damping factor[17]

$$D = J_e U_r/E_r$$

where J_E is the energy oscillation radiation repartition factor, and U_r the average energy loss per turn at the reference energy E_r.

$$\Omega' = -\Omega_o^2\,\alpha/E_r$$

α being the momentum compaction factor. The quantum excitation effect[17] is lumped at the location of each RF system. The effect is expressed by the random number q with zero average and rms value

$$\langle q^2\rangle = 2D\,\sigma_E^2/M$$

where σ_E is the equilibrium (zero-current) rms energy distribution width.[17] The canonical variables which describe the motion of the particle are the RF phase angle $\phi = (\theta-\Omega t)$, Ω being the angular revolution frequency of particle with energy E and the canonical momentum difference

$$w = \int_{E_r}^{E} dE/\Omega(E)$$

ϕ_r is the synchronous phase angle.

We assumed that at the location of the RF cavities there is also a lumped impedance which at the angular frequency $\Omega_n = n\Omega_o$ has the complex value

$$Z_n = A_n e^{i\alpha_n}$$

with A_n and α_n real. The current distribution is described by the complex function

$$g_n(t) = \frac{1}{N}\sum_{s=1}^{N} e^{-i\frac{n}{h}\phi_s} = G_n e^{i\beta_n}$$

where G_n and β_n are real, N is the total number of particles in the bunch and ϕ_s the angle value of the s-th particle when crossing the RF system.

In the particular case $\alpha_n = 0$ and $A_n = Z/M$ ($n = 1,2,3\cdots$) the summation at the right hand side of (1b) reduces to

$$\frac{Z}{2M}\{2\pi h g(\phi)-1\}$$

where $g(\phi)$ is the longitudinal particle distribution normalized to unit.

The Computer Simulation

It is obviously convenient to break down the system of equations (1a and 1b) in a series of difference equations which describe the various steps between two consecutive RF passages. The steps are: (i) the drift between two groups of RF cavities, which takes also into account the radiation damping, (ii) the quantum fluctuation, where the variable w is added a random number q, (iii) the RF kick which is a function of the angle variable ϕ, and (iv) the space charge kick. Here the longitudinal distribution $g(\phi)$ is calculated in the form of a hystogram made up to 40 bins. The kick is proportional to the number of particles in the bin which include the particle under consideration.

One computer run is completely determined by assigning the collowing parameters:

- E_r, reference energy (19 GeV),
- ρ, bending radius (192.05 m),
- ν_s, number of phase oscillations per turn (0.08),
- α, momentum compaction factor (0.0024),
- h, RF harmonic number (3840),

$M = 2$ and $J_E = 2$. The values in brackets apply to PETRA[16] and have been used as reference for our calculations. One more parameter is given by

$$Q = 2IZ\,\pi h/MV_c\,N\sigma \tag{3}$$

where N now is the number of particles in the simulation and σ the actual rms bunch length.

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Typically one-thousand particles are taken in the simulation. The computer makes plots of the particle distribution in the (ϕ, w) phase space, calculates the phase and energy distributions separately in the form of hystograms, averages and standard deviations. The initial conditions are taken randomly according to a gaussian distribution with zero averages and standard deviations equal to a factor λ times the natural rms values. Usually $\lambda = 1$. A run lasts at least three energy radiation damping times.

The Computer Results

We checked the dependence of the bunch width and length versus various parameters. For this purpose we took the averages σ and δ of the last 6-10 outputs of the bunch rms respectively length and width. But we first observed that in all cases we ran there was a bunch shape and size change during a fraction of the first damping time. For the remaining period of time the bunch size remained about constant apart from some relatively small amplitude oscillations. We convinced ourselves that these oscillations were not merely due to statistical fluctuations. We could clearly observe centre-of-mass oscillations aswell as bunch thumbling. In one case we follow this constant pattern for a very long period of time, up to thirty damping times. For very large value of the current we observed initially a fast overshoot which was then damped with a characteristic radiation damping time. When the overshoot was too large it caused beam loss. The loss could have been reduced and even eliminated by blowing up the initial beam size ($\lambda > 1$). This, we believe, is a typical indication that initially the synchrotron radiation is not important and that an electron bunch behaves initially as a proton bunch.

Typical bunch distributions are shown in Fig. 1. The longitudinal distribution is bell-shaped with a long backward tail and a sharper front edge. The energy distribution looks to be gaussian within the limit of the hystogram resolutions.

Denote with σ_0 and δ_0 the natural value of the bunch rms length and width then

$$S = \sigma/\sigma_0 \quad \text{and} \quad R = \delta/\delta_0$$

are respectively the lengthening and widening factors. We show the dependence of S (black circles) and R (white circles) on various parameters (λ, Q, ν_s, α, h, ρ and E_r) in Figs. 2 to 8. To show more explicitly the power of the dependence we plotted the logarithm of all the parameters involved. The straight lines are our interpolation. From their inclination we worked out the power dependence. The fluctuations are probably due to our way of averaging for σ and δ.

From Fig. 2 we see that the lengthening and widening do not depend on the initial beam size as we would have expected. The dependence with the other parameters can be summarized with the following two formulae

$$S = 1.7\times10^3 \left(\frac{Q\nu_s}{\alpha h}\right)^{\frac{1}{2}} \frac{\rho^{\frac{1}{3}}}{E_r} \tag{3a}$$

$$R = 1.9\times10^3 \left(\frac{Q\rho}{\alpha h}\right)^{\frac{1}{2}} \frac{\nu_s}{E_r} \tag{3b}$$

(E_r in GeV and ρ in meters) where the two factors have been obtained by averaging over all our computer runs. Observe also that there is a threshold current below which S and R are about unit as one can see from Fig. 3.

Discussion

As one can see from (2), Q is inversely proportional to S. Henceforth, unfolding this dependence and using standard formulae,[17] Eqs. (3a and b) change to

$$S = 0.15 \left(\frac{IZ\rho^{7/6} \cos\phi_r}{\alpha E_r^{4}}\right)^{1/3} \tag{4a}$$

and

$$R = 1.1\, \nu_s^{\frac{1}{2}}\, \rho^{1/6}\, S \tag{4b}$$

where I is in Ampères, Z in ohms, ρ in meters and E_r in GeV. The dependence of the factors S and R are more or less in agreement with some of the theoretical predictions. The SPEAR II experimental data[13] could be fitted by Eqs. (4a and b) if one takes Z = 8 kΩ, which is a reasonable value. The impedance value for a machine like PETRA, based on a 5-cell cavity measurement,[18] is of about 60 kΩ. At 19 GeV with a peak voltage of 102 MV one has $\nu_s = 0.08$ and

$$S = 1.7 \quad \text{and} \quad R = 1.3$$

for a bunch of 20 mA and $\alpha = 0.0024$. It is convenient to operate at low RF voltage, compatible with a reasonable long quantum lifetime, and at larger value of α. At the lower energy of 7 GeV one can take $\nu_s = 0.01$ and $\cos\phi_r = 0.5$, then

$$S = 5.5 \quad \text{and} \quad R = 1.5.$$

Thus, according to our computer results not much bunch widening is expected for a storage ring like PETRA.

For the near future we plan to repeat the same numerical calculation employing a more physical impedance model.

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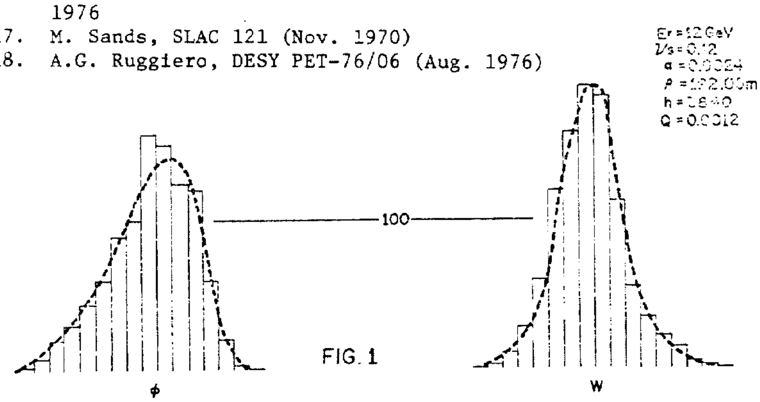

FIG. 1

Fig. 1 Typical Bunch Shape

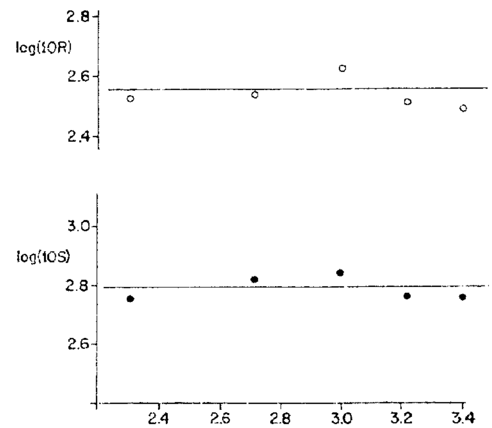

FIG.2

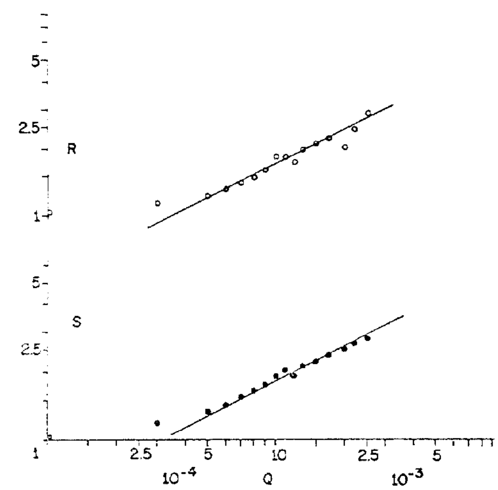

FIG.3

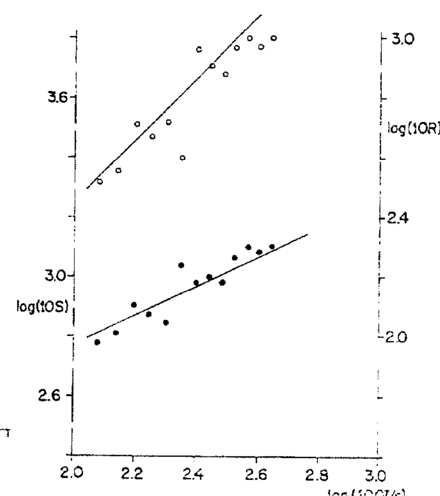

FIG.4

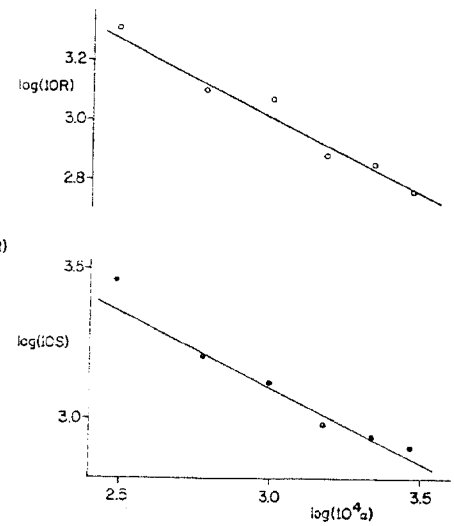

FIG.5

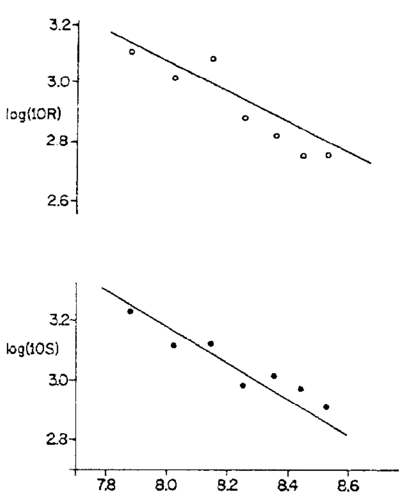

FIG.6

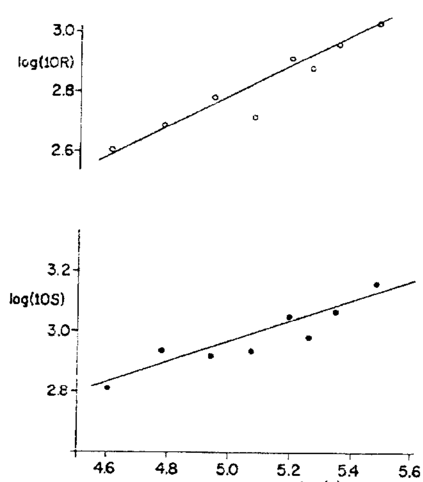

FIG.7

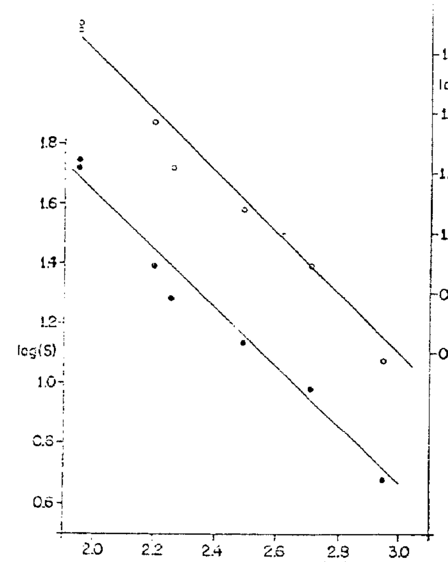

FIG.8

Figs. 2-8 Dependence of S and R on various parameters