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SYNCHROBETATRON RESONANCES*

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Introduction

At the 1975 Particle Accelerator Conference it was reported that a class of resonances were observed in SPEAR II that had not appeared before in SPEAR I.[1] These resonances occur when the betatron oscillation wave numbers ν_x or ν_y and the synchrotron wave number ν_s satisfy the relation $(\nu_{x,y} - m\nu_s) = 5$, with m an integer denoting the m^{th} satellite. While the existence of sideband resonances of the main betatron oscillation frequencies has been previously observed and analyzed, the resonances observed in SPEAR do not appear to be of the same variety.[2,3]

The main difference between SPEAR II and SPEAR I is the value of ν_s, which in SPEAR II is ~ 0.04, an order of magnitude larger than in SPEAR I. An ad hoc meeting was held at the 1975 Particle Accelerator Conference, where details of the SPEAR II results were presented and various possible mechanisms for producing these resonances were discussed. Later, experiments were performed at SPEAR to identify the mechanism we believe to be the most likely explanation. We have been aided immensely in arriving at our present interpretation by suggestions of Voss at the 1975 ad hoc meeting and by the theoretical work of Piwinski and Wurlich of DESY. The purpose of this paper is to present some of our current experimental knowledge and theoretical views on the source of these resonances.

General Experimental Observations

The size of the beam was observed to increase whenever $(\nu_{x,y} - m\nu_s) = 5$ with m integral. A survey of this growth in beam size showed that the satellite of the vertical integer resonance gave measurable growth up to m as large as 10, and that for $m \leq 5$ the vertical beam growth could fill the available aperture and cause beam loss; the satellites of the horizontal integral resonances are weaker than the vertical resonances; and the growth in betatron amplitude is incoherent.

Satellites of other resonances were also studied. We found that the satellites of the coupling resonance, $(\nu_x - \nu_y) = 0$, and the half-integral resonance, $\nu_{x,y} = 5.5$, are very weak as compared to the integral resonances. For these two cases, only the first two satellites have been observed. In all cases, the resonances are spaced by ν_s and we see no indication of resonance lines spaced at $\frac{1}{2}\nu_s$. The strength of the vertical integral resonances decreases with energy, increases with beam current, is strongly dependent upon vertical orbit distortions present in the ring, and is insensitive to changes in the chromaticity of the ring.

In order to understand the mechanism that produces the resonance, a careful study of the ring parameters which affect the resonance blowup was undertaken. For this purpose we studied predominantly, although not exclusively, the particular resonance $(\nu_y - 5\nu_s) = 5$.

FM Sidebands

It has been known for some time that because the transverse betatron oscillation frequencies are frequency modulated by the energy oscillations it is possible to excite resonances at frequencies which are sidebands of the main betatron-oscillation frequencies.[2,3] These sideband resonances are uniformly spaced at the synchrotron frequency; i.e., the m^{th} sideband occurs at a wave number equal to $(\nu_{x,y} \pm m\nu_s)$, with m an integer. Since the transverse force on a particle produced by the magnet elements in the lattice occurs at integral multiples of the revolution frequency, we would expect that whenever the sideband resonance frequency equals an integral multiple of the revolution frequency, i.e., for $(\nu_{x,y} \pm m\nu_s) = n$, we could have growth in the transverse motion.

The strength of the m^{th} sideband resonance is proportional to $J_m(\Delta\hat{\nu}_{x,y}/\nu_s)$, where J_m is the m^{th} order Bessel function and $\Delta\nu_{x,y}$ is the peak variation of the betatron tune and is equal to the average value of the chromaticity function times the peak relative momentum variation; i.e., $\Delta\hat{\nu}_{x,y} = \xi_{x,y}(\Delta\hat{p}/p)$. For these resonances, the strengths should depend very strongly upon ξ and vanish when ξ approaches zero. However, all experiments have shown clearly that the strength of the resonances is independent of ξ over the range $0 < \xi < 10$. In addition, the resonance strength should decrease with increasing values of ν_s. Since ν_s is higher in SPEAR II than in SPEAR I, these sideband resonances should be weaker in SPEAR II. It should also be easily possible to drive them by RF knockout techniques, which again was not the case. From all of these observations we have concluded that the resonances are not the usual FM sidebands.

Half-Integral Resonances

We have considered half-integral resonances driven by a periodic variation in the chromaticity function as a possible explanation of the sideband resonances. Since the 10th harmonic of the chromaticity function is rather insensitive to changes in the average chromaticity, and the strength of the half-integral resonances would be relatively independent of the value of ν_s, the experiments described above did not exclude this as a possible explanation. The 10th harmonic of the chromaticity function should drive the resonances at wave numbers $(2\nu_{x,y} - m\nu_s) = 10$, with m an integer which denotes the harmonic of the energy oscillation responsible for the resonance. The absence of resonances at odd values of m would tend to rule out the half-integral resonances as the mechanism, provided that the odd harmonics of the synchrotron oscillations are comparable to the even harmonics. An experiment was conducted in which the 10th harmonic of the chromaticity function was varied and indeed reduced to zero, independent of the average chromaticity, by powering several families of sextupoles. It was found that the strength of the resonances was not reduced as the 10th harmonic of the chromaticity function vanished, which led us to conclude that this was not an important contribution to the resonances found in SPEAR II.

Proposed Model

During the earlier studies of the integral resonances, it was difficult to obtain reproducible results on the resonance strengths. In particular, we originally found the resonance strength to be sensitive to the value of β_{max},[1] but this was subsequently traced to the fact that the closed orbit for fixed settings of the correcting elements was dependent upon the value of β_{max}. When the orbit errors were minimized separately for each value of β_{max}, much of the dependence of the resonance strength with β_{max} disappeared. This led to a systematic study of the effect of orbit errors on the resonance and to the current picture we have of the cause of these resonances.

There are many elements in SPEAR that can change the energy of a particle. These include various vacuum chamber discontinuities as well as the main RF cavities which supply the energy radiated into synchrotron light. Consider the case where there is no dispersion at one of these cavities; then the equilibrium orbits, y_e, for different energy particles coincide at the cavity, as shown in Fig. 1. Consider a

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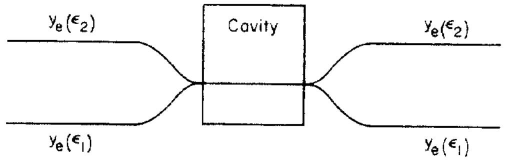

Fig. 1

particle with zero betatron amplitude but an energy deviation from the synchronous energy, ϵ_1. The particle will be traveling on the equilibrium orbit $y_e(\epsilon_1)$ before it enters the cavity and on the equilibrium orbit $y_e(\epsilon_2)$ when it leaves the cavity with energy ϵ_2. The betatron amplitude will be unchanged and remain zero.

On the other hand, if there is a dispersion at the cavity the equilibrium orbits for different energy particles do not coincide in the cavity, as shown in Fig. 2. When a particle

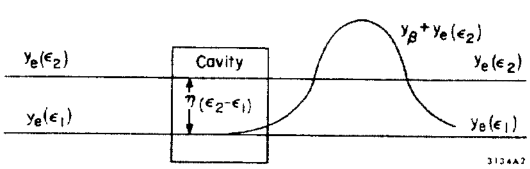

Fig. 2

with an initially zero betatron amplitude and energy deviation ϵ_1 leaves the cavity with a different energy deviation ϵ_2 it discovers that it is not on the proper equilibrium orbit and will start to execute betatron motion about the proper equilibrium orbit $y_e(\epsilon_2)$. Because the energy deviation of a particle oscillates about the synchronous energy at the synchrotron frequency, the betatron motion of a particle will be driven at the synchrotron frequency plus an integral value of the frequency of passage through the cavity, and a resonance will occur whenever this driving frequency is equal to the betatron frequency. Another important ingredient necessary to explain all of the observed resonances in SPEAR is the assumption that synchrotron motion is nonlinear so that energy change at the cavity oscillates at multiples of the synchrotron frequency. This leads to the resonance condition between the betatron wave numbers ν_x or ν_y and the synchrotron wave number ν_s

$$(\nu_{x,y} - m\nu_s) = p \tag{1}$$

with m and p integral.

One of the interesting observations of these resonances is that the vertical resonances are much stronger than the horizontal resonances. We note that the horizontal dispersion η_x is generally much larger than the vertical dispersion η_y, and in fact η_y is zero in a perfectly aligned ring. However, for the perfect ring the horizontal dispersion in SPEAR has only even harmonics and the cavities which produce the nonlinear energy changes are rather uniformly distributed around the ring. Thus, only even values of p in Eq. (1) can produce a resonance and, for a perfect ring with $\nu_x \approx 5$, we would not expect to observe strong horizontal resonances. The satellites of the $\nu_x \approx 4$ resonances have been observed to be noticeably stronger than those of $\nu_x \approx 5$.

Effects of Imperfections

In this section we will describe in detail how the resonances are excited by a variation of the closed orbit with momentum, i.e., the dispersion η. We will consider here the vertical resonance, noting that similar results apply for the horizontal resonances. The radial magnetic field seen by a particle can be written as

$$B_x(x,y,s) = a(s) + g(s)\,y + r(s)\,x + 2\lambda(s)\,xy \tag{2}$$

where x, y, and s are the horizontal, vertical, and longitudinal coordinates, and we define a(s) as the strength of the dipole field errors and correcting fields, g(s) the quadrupole strength, r(s) the skew quadrupole strength, and $\lambda(s)$ the sextupole strength.

For the synchronous particle, it is possible to define a closed orbit which we denote as $[x_0(s), y_0(s)]$. The function $y_0(s)$ must be periodic and satisfy the following equation.

$$y_0''(s) = \left(\frac{1}{B\rho}\right)_0 B_x(x_0, y_0, s)$$

or

$$y_0'' - \left(\frac{1}{B\rho}\right)_0 [g(s) + 2\lambda(s)x_0]\,y_0 = \left(\frac{1}{B\rho}\right)_0 [a(s) + r(s)x_0], \tag{3}$$

where $(B\rho)_0$ is the magnetic rigidity of a synchronous particle. The vertical motion of an off-energy particle is different from that for the synchronous particle because both the magnetic rigidity and the horizontal magnetic field experienced by the particle depend upon the particle energy. The difference between the vertical position of a particle y and the on-energy equilibrium orbit y_0 through first order in the energy deviation ϵ satisfies:

$$(y-y_0)'' - \left(\frac{1}{B\rho}\right)_0 [g(s)-(g(s)-2\lambda(s)\eta_x)\epsilon]\,(y-y_0)$$
$$= -\left(\frac{1}{B\rho}\right)_0 [a(s)+g(s)y_0 - r(s)\eta_x - 2\lambda(s)\eta_x y_0]\,\epsilon\ , \tag{4}$$

where we have assumed that the value of the horizontal dispersion η_x is much larger than the horizontal orbit distortion x_0, and we are neglecting the transverse coupling of the betatron motion. The equilibrium orbit $y_e(\epsilon)$ for a particle with energy deviation ϵ is related to the vertical dispersion function η_y to first order in ϵ by

$$y_e(\epsilon) = y_0 + \eta_y \epsilon\ . \tag{5}$$

The dispersion function η_y is the periodic solution to the following equation:

$$\eta_y'' - \left(\frac{1}{B\rho}\right)_0 g(s)\eta_y = -\left(\frac{1}{B\rho}\right)_0 [a(s)+g(s)y_0 - r(s)\eta_x - 2\lambda(s)\eta_x y_0]\,. \tag{6}$$

The chromaticity term on the left-hand side of Eq. (4) has been ignored, since we are considering the motion through first order in ϵ. We now can study the betatron motion for off-energy particles by denoting $y_\beta(\epsilon) = y - y_e(\epsilon)$ and combining Eqs. (4)-(6) to yield

$$y_\beta'' - \left(\frac{1}{B\rho}\right)_0 g(s)y_\beta = -(\eta_y \epsilon')' - \eta_y' \epsilon'\ , \tag{7}$$

where again we ignore the chromaticity terms. As was illustrated in Figs. 1 and 2, a change in the energy of $\Delta\epsilon$ produces a change in the betatron motion as given by

$$\Delta y_\beta = -\eta_y \Delta\epsilon\ , \quad \Delta y_\beta' = -\eta_y' \Delta\epsilon\ , \tag{8}$$

where η_y and η_y' are evaluated at the point where the energy change occurred. For the resonance $(\nu_y \pm m\nu_s) = p$ to be excited by η or η' there must be frequency $m\nu_s$ in the particle energy gain. For a short bunch the RF field is nearly linear and the fields that produce nonlinear energy gains must come from parasitic modes in cavities, chamber discontinuities, etc. Since these elements are rather uniformly distributed around the ring we assume that the portion of the energy change that oscillates at $m\nu_s$ is smoothed out around the ring, and the resonance $(\nu_y - m\nu_s) = p$ is driven by this portion of the energy oscillation, together with the p^{th} harmonic of η and η'. In SPEAR ν_x and ν_y are normally near 5 so the dominant harmonic of η and η' occurs for p=5 and is entirely due to imperfections and misalignments. The effect of these imperfections is probably worse vertically than horizontally. The p^{th} harmonic of η and η' is proportional to the p^{th}

harmonic of the function $F(\phi)$ given by

$$F(\phi) = \beta^{3/2}(\phi)\,[a(\phi)+g(\phi)y_0(\phi) - r(\phi)\eta_x(\phi) - 2\lambda(\phi)\eta_x(\phi)y_0(\phi)]. \quad (9)$$

A convenient measure of the resonance effect upon the beam size is the blowup factor B defined by the ratio of the beam height on-resonance to the height off-resonance. Since the portion of the beam height due to the resonance adds in quadrature to the normal beam height, it follows that, if the longitudinal particle oscillations are random in phase, the blowup factor is given by

$$B^2 = 1 + K|F_p|^2\langle\epsilon_m^2\rangle , \quad (10)$$

where K is a constant that depends upon the size of the beam off-resonance, the damping mechanism that restricts the resonance growth, etc. F_p is the p^{th} harmonic of $F(\phi)$ defined by Eq. (9), and $\langle\epsilon_m^2\rangle$ is the squared value of the $m\nu_s$ energy oscillation averaged over the particle distribution. While the exact value of $F(\phi)$ due to imperfection is not known, it is possible to determine the changes in the closed orbit resulting from change in dipole settings, and hence changes in F_p can be calculated quite accurately.

Experimental Results

The calculations presented in the previous section allow predictions to be made on how the beam blowup factor should vary with machine parameters in SPEAR. An experiment on the dependence of B^2 on y_0 was performed. The orbit correction program was used to determine the dipole fields in the ring necessary to change the value of y_0 at the two interaction regions in such a way as to produce only odd harmonics in $F(\phi)$. For small changes the value of $F_5(\phi)$ was proportional to the angle of the equilibrium orbit $(y_0')^*$ at the interaction region. The results shown in Fig. 3 demonstrate that B^2 does vary parabolically with $(y_0')^*$. When we compare the variation of B^2 found experimentally with that expected from Eq. (1), we find that the observed beam blowup can be explained by a 5th Fourier component of the synchrotron oscillation, which is about a factor of 500 smaller than the first Fourier component. We believe that this is a reasonable value to be expected from parasitic modes excited by the beam.

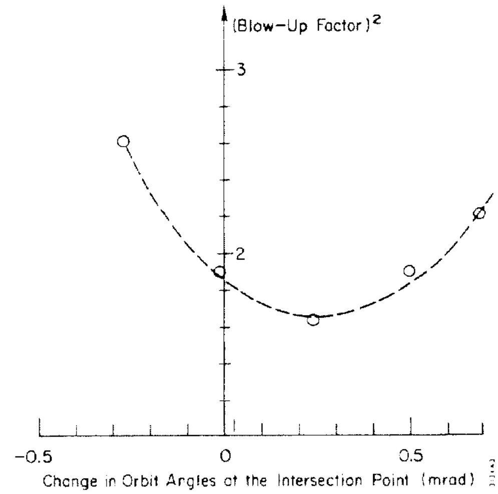

Fig. 3. B^2 as a function of $(y_0')^*$.

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