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COLLECTIVE ION ACCELERATION IN A TRAVELING VIRTUAL CATHODE*

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Summary

This paper describes a general class of collective acceleration schemes which rely on the controlled motion of a virtual cathode created in an intense relativistic electron beam. Preliminary numerical simulation results are presented for a specific scheme which does not depend on neutralization of the electron beam space charge. In particular, the position of virtual cathode formation appears to be adequately described by relatively simple equations. Also, electron beam turbulence due to a two-stream instability is shown to be substantially decreased by using a hollow beam and decreasing the external magnetic field strength.

Introduction

The results of experiments at the Air Force Weapons Laboratory[1,2] performed with the low-pressure neutral gas ion acceleration configuration have indicated that the primary acceleration mechanism is the collapse of the deep potential well associated with the virtual cathode due to neutralization of the electron space charge through ionization of the background gas. Since net acceleration occurs when the ions become untrapped, the process is largely uncontrolled and does not appear capable of generating large numbers of high energy ions.

A more desirable situation is that in which ions become trapped in a deep potential well and the motion of the well is then controlled to permit ion acceleration over substantial distances. In recent years, several acceleration schemes have been proposed to produce the desired goal. These schemes can generally be classified as either wave methods or net electron space-charge methods. Examples of the first class include the autoresonant accelerator concept[3] and the plasma wave scheme using a converging wave guide;[4] an example of the second class is the controlled ionization front scheme of Olson.[5] More recently, a class of net space charge acceleration schemes has been proposed.[6,7] The basic concept which involves creating a single deep potential well (a virtual cathode) and controlling its motion, arose from a number of observations concerning virtual cathode formation in unneutralized beam propagation experiments.[8] To summarize the significant features of these results: (1) the position of virtual cathode formation depends upon the ratio of the injected current to the space charge limiting current; and (2) this parameter varies over the electron beam pulse duration. It is therefore to be expected that the position of the virtual cathode should vary during the pulse, and it is speculated that by properly designing the drift tube geometry it should be possible to use the time-dependent generator-diode behavior (essentially diode impedance) to create the conditions for the controlled motion of a virtual cathode.

For a solid electron beam of radius r_b and kinetic energy $(\gamma_o-1)mc^2$ with fractional charge neutralization f_e injected into a long drift tube of radius R, the condition for virtual cathode formation is that the beam current exceed the space charge limiting current, i.e.,

$$I(z,t) \geq I_\ell(z,t) = \frac{\left[\gamma_o^{2/3}(z,t) - 1\right]^{3/2} mc^3/e}{1 + 2\ln\left[R(z,t)/r_b(z,t)\right]}\left[1 - f_e(z,t)\right]^{-1} \tag{1}$$

It has been assumed that slow adiabatic variations in the quantities are allowed so that the argument (z,t) denotes the values of the quantities at the axial position z and at time t.

An examination of this equation indicates five parameters which can be varied either singly or in combination to produce a virtual cathode whose position should vary with time.

(It should be noted that virtual cathode motion produced by variations in $f_e(z,t)$ is the essence of a particular embodiment of Olson's ionization front accelerator.[5] In addition, it is believed that this mechanism (changing $f_e(z,t)$) is responsible for the ion acceleration observed in experiments performed by Luce,[9] and Greenwald, et al.[10])

As an example of changing combinations of variables to produce the desired virtual cathode motion, consider the case in which a beam of constant voltage and radius, but variable current, is injected into an evacuated drift tube whose radius varies as a function of axial position. In particular, for the situation in which the current rise is linear, i.e.,

$$I(z,t) = \begin{cases} I_o(t-z/v_e)/t_r\ ; & 0 \leq (t-z/v_e) \leq t_r \\ 0\ ; & \text{otherwise} \end{cases} \tag{2}$$

where I_o is the peak injected current, t_r is the current risetime, and v_e is the electron velocity (assumed constant in z and t for the purpose of this example), it can be shown that the axial variation in drift tube radius required for uniform acceleration of the virtual cathode toward the anode is given by[6]

$$h(z) = h(z_o) + (\alpha v_e t_r)^{-1}\left\{\left[2v_e^2(z_o-z)/a\right]^{1/2} + (z_o-z)\right\} \tag{3}$$

where

$$h(z) = \left\{1 + 2\ln\left[R(z)/r_b\right]\right\}^{-1}$$
$$\alpha = (\gamma_o^{2/3} - 1)^{3/2} mc^3/eI_o \tag{4}$$

z_o and t_o (the initial position of virtual cathode formation) are related according to

$$z_o = v_e t_o - \alpha v_e t_r h(z_o) \tag{5}$$

with a being the acceleration of the virtual cathode. An illustration of this concept is presented in Figure 1.

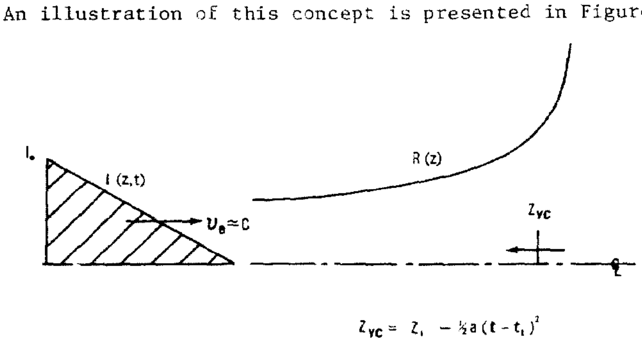

Figure 1. Schematic diagram of a moving virtual cathode resulting from time-dependent diode behavior and a flaring waveguide.

Much further analysis is obviously required for this particular scheme, in addition to several other schemes apparent from Eq. (1), but not considered here. Of particular concern are: (1) the shape of the potential well; (2) the effects of variations in electron beam velocity as the beam nears the virtual cathode region; (3) the effects of primary electrons reflected from the virtual cathode; and (4) the time dependent oscillations in well depth and position around the quasi-static values. An examination of these questions as pertaining to this concept has begun, supported in large part by numerical simulations using the CCUBE particle code developed by Godfrey.[11] While the calculations are still in an early stage the preliminary results are of some interest.

Numerical Simulation Results

The first problem to be addressed was the injection of a solid, constant current beam into a conical diverging waveguide for the purpose of examining the suitability of Eq. (1) for describing the position of virtual cathode formation. The configuration and the parameters used in the simulation are described in Figure 2.

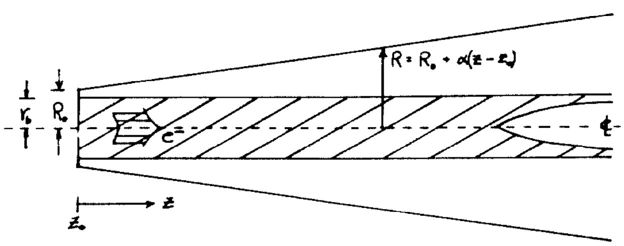

Figure 2
Schematic Geometry of the Numerical Simulations.

According to Eq. (1) for the parameter values used in the simulations, the virtual cathode should form at an axial position of approximately $\omega_p z/c = 127$. In Fig. 3 the kinetic energy $(\gamma-1)mc^2$ of the beam particles is presented as a function of axial position in the drift tube. The virtual cathode $(\gamma-1 = 0)$ forms at an axial position of $\omega_p z/c = 155$. Hence, it appears that the interpolation expression, Eq. (1), underestimates the space charge limiting current in this instance by approximately 14%.

The work of Voronin, et. al.,[12] has been recently extended to the case of a cylindrical drift cavity of length L for annular beams with arbitrary radial dimensions. In the limit of a solid beam with $(R/L) \to 0$, a rigorous upper bound for the space charge limiting current is obtained as

$$I_\ell \le \frac{mc^3}{4e} r_b^2 \lambda^2 (\gamma_o^{2/3} - 1)^{3/2} \qquad (6)$$

where λ is the smallest eigenvalue that satisfies the boundary condition

$$J_o(\lambda r_b) + \ln(r_b/R)\ (\lambda r_b)\ J_1(\lambda r_b) = 0 \qquad (7)$$

where J_0 and J_1 are the zero and first order Bessel functions of the first kind. Taking into account the slow axial variation in the drift tube radius, Eqs. (6) and (7) estimate that the maximum axial position at which the virtual cathode can initially form is $\omega_p z/c = 161$, in excellent agreement with the simulation result.†

Good agreement is also noted between the simulation results and the prediction of the simple electrostatic waveguide theory which gives the potential depression (far from endplates or the virtual cathode position) as

$$\phi(r,z) = \frac{I}{\beta c}\left[1 - (r/r_b)^2 + 2\ln(R(z)/r_b)\right] \qquad (8)$$

(The solid lines of Fig. 3 represent the extreme cases of $r = 0$, and $r = r_b$.) In addition, endplate effects and the sharp potential step on going from $I = I_\ell$ to $I > I_\ell$ (of order $(\gamma_o^{1/3}-1)$) are noted.

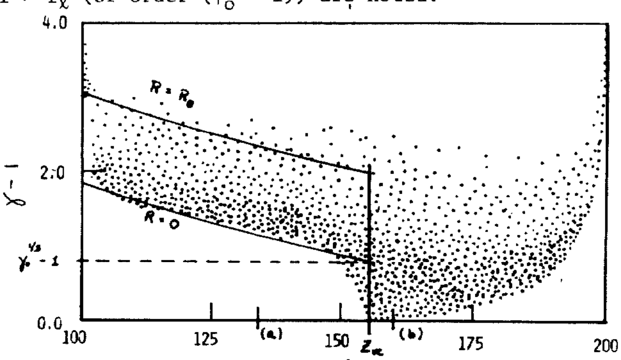

Figure 3. Electron kinetic energy as a function of axial position in the diverging conical waveguide. (a) estimated virtual cathode position as given by Eq. (1); (b) estimated upper bound for virtual cathode position as given by Eqs. (6) and (7).

Information concerning the evolution of the system of Fig. 2 can be obtained from the phase space diagram presented in Fig. 4 which exhibits Longitudinal beam particle velocity as a function of longitudinal position in the waveguide.

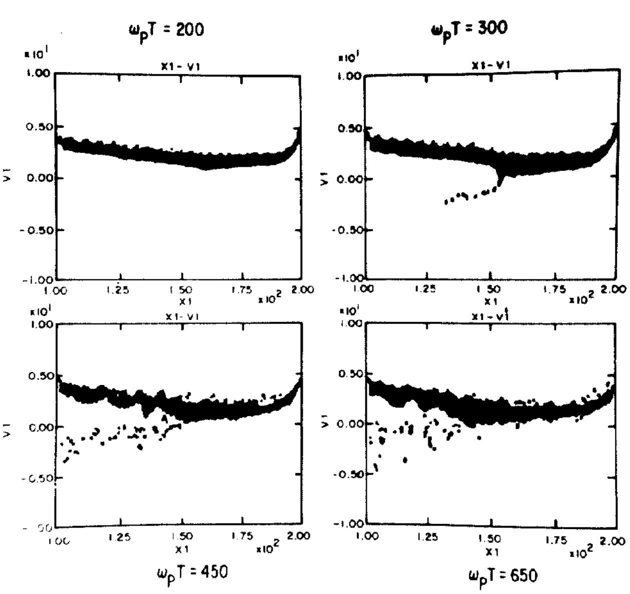

Figure 4. Evolution of longitudinal phase space. $(z = x_1,\ v_z = v_1)$.

Early in the simulation $(\omega_p T = 200)$ the magnitude of the electrostatic potential is increasing but virtual cathode formation $(v_1 = 0)$ has not occurred. By the time $\omega_p T = 300$, the virtual cathode has formed, and reflected particles $(v_1 < 0)$ in the center of the beam are apparent. The free energy available as a

†Approximate analytical expressions recently derived by J. R. Thompson and M. L. Sloan also appear to give excellent agreement with the simulations.

result of the streaming motion between the injected and reflected particles results in the growth of a two-stream instability ($\omega_p T = 450$). At late times ($\omega_p T = 650$) the instability has violently disrupted the beam and has essentially destroyed the virtual cathode structure. These remarks are indicated more clearly in the late-time contour plots of potential and longitudinal electric field strength presented in Fig. 5. The large magnitude potential contours are very choppy and the electric fields are small.

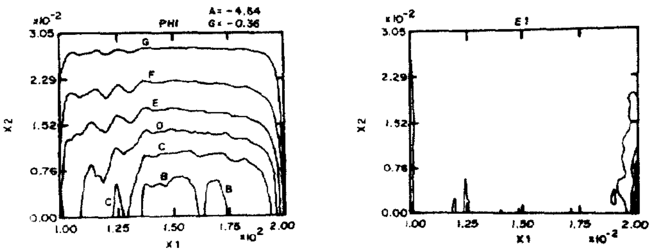

Figure 5. Solid beam potential and longitudinal electric field contours. $\omega_p T = 625$, $\Omega_c/\omega_p = 4.0$.

The problem of two-stream instability growth could be eliminated by having the reflected particles quickly pass to the wall of the drift tube. For the case of a solid beam, although the reflected particles did acquire a substantial radial component of velocity, the reflections occurred in the center of the beam. Consequently, the time over which the reflected particles could escape from the beam region was large, thereby giving rise to development of the catastrophic instability. It was speculated that by using a thin hollow beam and decreasing the external magnetic field strength this situation could be greatly improved. That this was indeed the case is shown in Fig. 6. For the hollow beam simulation, the reflected particles escape from the beam region across the decreased magnetic field much more quickly. As a result, the virtual cathode structure (and the large accelerating fields) exists for much longer times ($\omega_p T = 977$ for Fig. 6).

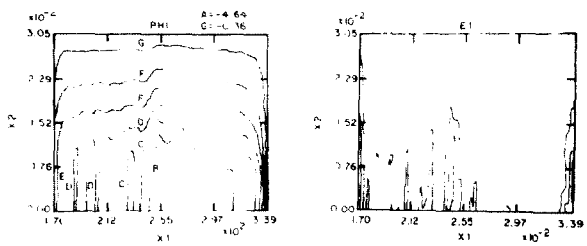

Figure 6. Hollow beam potential and longitudinal electric field contours. $\omega_p T = 977$, $\Omega_c/\omega_p = 2.0$.

Further simulation work will investigate the motion of the virtual cathode induced by a programmed increase in the injected current as a function of time. In addition, it should be noted that, as presented, the scheme indicated in Fig. 1 is more suitable for accelerating negative ions (or possibly electrons). The feasibility of accelerating positive ions will be further investigated. Finally, experimental results pertaining to this scheme should soon be forthcoming.

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