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COMPUTER SIMULATION OF COLLECTIVE ION ACCELERATION BY DISCRETE CYCLOTRON MODES

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Abstract

Extensive analytical studies suggest that significant currents of high energy ions can be obtained by collective acceleration via large amplitude cyclotron waves in a non-neutral intense relativistic electron beam. We have already demonstrated this acceleration mechanism in fully self-consistent two-dimensional computer simulations for low ion current and energy. However, the simulations employed a packet of cyclotron waves created ad hoc upstream of the acceleration region. Acceleration was limited by phase-mixing and damping of the packet. Here, we shall present results of our ongoing effort to simulate, first, realistic growth of large amplitude, single frequency cyclotron waves in the relativistic electron beam and, second, acceleration of various ion currents with those waves.

Introduction

Utilization of intense relativistic electron beams to accelerate significant current of ions to high energy has been proposed by a number of researchers recently.[1-5] One of the most promising of these schemes, advanced by Sloan and Drummond,[5] is the Autoresonant Accelerator (ARA), involving self-consistent growth of discrete cyclotron waves in intense relativistic electron beams with subsequent trapping of ions in their potential wells. Net acceleration occurs then by increasing the phase velocity of the wave. There are several advantages of such a concept over conventional ion accelerators. First, relativistic electron beam energy in the range 1-10 MeV is readily available. The envisioned accelerating fields from such beams are of the order 10^5-10^6 V/cm. The cyclotron wave responsible for the fields is, moreover, a normal mode of the beam, with well understood propagation characteristics. Ions trapped in the wave potential well are assured of phase synchronism with the fields for long periods of time. Finally, the linear dispersion for the doppler-shifted slow cyclotron mode

$$\omega = kV_o - \frac{\Omega_o}{\gamma_o}\,\frac{(kV_o)^2}{(kV_o)^2+\omega_p{}^2} \tag{1}$$

where k is the wavenumber, V_o the beam velocity, $\Omega_o = \frac{eB_o}{m_e c}$ the cyclotron frequency, $\omega_p^2 = \left(\frac{4\pi n_o e^2}{m}\right)^{1/2}$ the local plasma frequency, and $\gamma_o = \left(1 - \left(\frac{V_o}{c}\right)^2\right)^{-1/2}$, indicates that it is negative energy, and so is susceptible to exponential amplification under certain conditions. On the other hand, the properties of large amplitude cyclotron waves, including propagation, stability, and sensitivity to nonideal beam states, are relatively unknown. Analytic study of these questions has been undertaken, but the validity of such models is unproven. To help resolve questions about the nonlinear wave properties, we have conducted a combined analytic and numerical investigation of the self-consistent electron beam/cyclotron wave system.

Numerical studies have been conducted primarily on a fully relativistic, electromagnetic, 2-dimensional, variable geometry particle simulation code, CCUBE.[6] This code is unique in its generality and flexibility. As such, it is well suited for study of not only the ARA scheme, but also such proposals as the localized pinch model (LPM)[2] and the traveling virtual cathode accelerator.[4] These studies will be reported elsewhere.

Linear Dispersion in Radially Inhomogeneous Beams

The usual procedure for finding a dispersion relation in an electron beam is to linearize the cold fluids equations[7,5]

$$\frac{\partial n}{\partial t} = -\nabla\cdot n\bar{V} \tag{2a}$$

$$\frac{\partial \gamma V}{\partial t} = -\bar{V}\cdot\nabla\gamma\bar{V} + \frac{q}{m}\left(\bar{E} + \frac{\bar{V}}{c}\times\bar{B}\right) \tag{2b}$$

and Maxwell's equations

$$\nabla\cdot\bar{E} = -4\pi en \tag{3a}$$

$$\nabla\cdot\bar{B} = 0 \tag{3b}$$

$$\nabla\times\bar{E} = -\frac{1}{c}\frac{\partial\bar{B}}{\partial t} \tag{3c}$$

$$\nabla\times\bar{B} = \frac{4\pi}{c}\bar{J} + \frac{1}{c}\frac{\partial\bar{E}}{\partial t} \tag{3d}$$

around an electron beam equilibrium. Two of the commonly employed equilibriums are the so-called "rigid-rotor"[7,8] with $\omega_e = \frac{V_o}{r}$ = constant and the radially uniform density model. Such idealized models are very useful for deriving analytic expressions. With uniform density, for instance, the radial dependence of the perturbed quantities is expressed in terms of simple Bessel functions. Further, the set of discrete modes is infinite and complete.

Simulation of Ion Acceleration

Computer simulation of electron beam propagation in waveguides has revealed that, although the simple models for equilibrium mentioned above describe the numerical results fairly well, the discrepancies can be significant. Shear in the beam kinetic energy, for instance, is an inevitable consequence of space charge effects, but is usually neglected when constructing analytic models. To examine such phenomena in more detail, we devised a numerical dispersion solver which included radially self-consistent beam equilibria. The results were somewhat surprising. Radial inhomogeneity in the background conditions led to branch cuts in the dispersion relation, which converts most discrete cyclotron modes into continuum, secularly decaying perturbations. To be sure, at least one discrete mode was always observed, but it is not always clear a priori where that mode would lie. Figure 1 shows the 8 normal modes of a cold, homogeneous beam as solid lines. Figure 2 shows that the actual radial structure of a typical cyclotron mode can have far from simple Bessel function dependence.

To test the stability of the wave during trapping itself, a spherical (ρ-θ), 2-dimensional particle simulation was conducted. The relativistic electron beam, $\gamma_o = 4.5$, was injected from the anode boundary ($\frac{\omega}{c}p\ \rho = 100$) and allowed to propagate to $\frac{\omega}{c}p\ \rho = 200$. The beam was artificially modulated as it was injected to give a cyclotron wave with $\frac{\omega}{\omega_p} = .023$, $\frac{kc}{\omega_p} = 0.46$. The amplitude of this launched wave was large but fixed, with a peak potential of $e\phi_{max} \approx 1$ MeV. No self-consistent growth of the wave was purported. The calculation was intended only to study the stability of a nonlinear cyclotron wave, loaded with a nominal number of ions($n_i/n_e \leq 10^{-3}$), and propagating in an inhomogeneous magnetic field. As Figure 3, a schematic depiction of the simulation geometry, shows, the transverse dimension of the waveguide, and so the magnetic field, flared in this conical simulation as a function of distance from the anode. The divergence half-angle, θ=.016 rad, was chosen so that this transverse dimension would double over the length of the tube. The magnetic field strength thus decreased by a factor of 4. Examination of the slow cyclotron dispersion relation, Eq (1), shows that the phase velocity $v_{ph} = \frac{\omega}{k}$, continually increases as the wave propagates down the waveguide. For these parameters, the phase velocity should have increased from v_{ph} =.05 c to v =.16 c, a 10-fold increase in the trapped ion energy if they remain trapped.

The simulation results were that once the ions were loaded into the potential wells, they remained stably trapped. The wave itself showed no destructive instability as it was accelerated, although it seemed to phase mix near the end of waveguide. Careful analysis of the simulation and comparison with numerical dispersion solutions indicates that the excited wave may have been a continuum, not discrete, wave, and so would be expected to decay secularly. It was otherwise similar enough to the discrete wave, that this latter's stability may also be presumed. The measured ion energy was found to be amplified by a factor of almost 10. These figures are modest, but they indicate the large amplitude cyclotron waves containing trapped ions can be accelerated without violent disruption of the waves. Some of the numerical results are shown in Figure 4, which has been replotted onto a rectangular grid for convenience of illustration.

Sheath Helix Growth Mechanism

Experimentally, it will not be as easy to excite a wave with predetermined characteristics as in the artificial technique described above. The wave must be self-consistently grown, extracting energy directly from the beam. The details of the wave growth mechanism must therefore be understood quantitatively. One of the most attractive techniques being considered at present involves a reactive sheath liner and a resonance instability between slow electromagnetic waveguide modes and the slow cyclotron waves. The use of a helical liner is the basis for the well understood traveling wave tube[9], used in broad-band amplification with electron beams. In those applications, though, the beam wave is a longitudinal Langmuir wave. Interaction with the cyclotron wave does not seem to have received any prior attention. As a first step, then, we have employed our radially inhomogeneous root solver, with a sheath helix, to investigate this instability.

Typical configuration parameters for reactively growing the cyclotron wave are beam radius $\frac{\omega}{c}p\ R_b$=2.65, sheath radius $\frac{\omega}{c}p\ R_s$=3.2, and waveguide radius of $\frac{\omega}{c}p\ R_o$=4.8. The dispersion of the guided electromagnetic waves is related to the helical pitch angle, ψ, approximately as

$$\omega = \pm kV \sin \psi \qquad (4)$$

When the frequency curves, Eqs (1) and (4), intersect, instability occurs, leading to simultaneous growth of the positive energy electromagnetic wave and the negative energy cyclotron wave. Figs. 5 and 6 show the frequency and associated growth rate curves for the above parameter with ψ=0.26 rad, γ_o=7, ν= 1.75, as a function of wavenumber, kc/ω_o. According to these simple arguments we expect the instability to be peaked in the vicinity of

$$k = \frac{\Omega_o}{\gamma_o c\ (\beta_o - \sin\psi)} \qquad (5)$$

As Figure 6 indicates the numerical growth peak is at kc/ω_p=.323. Full numerical simulation of the wave growth process to test the validity of analytic growth expressions and to observe saturation mechanisms have only recently commenced. Results of these calculations should be useful in determining the viability of the ARA concept as a practical ion accelerator, and will be presented at a later date.

In summary, we have studied the linear and nonlinear properties of the slow cyclotron wave in a relativistic electron beam with various numerical tools. We have found that modest acceleration of a small number of ions did not lead to any deleterious results. More exact modeling of the linear cyclotron wave has showed branch cuts in the dispersion relation. These lead to secularly decaying waves where discrete modes had been expected. Since excitation of such a mode would be unsuitable for accelerator applications, it is clearly important to understand which modes are discrete under given conditions. Numerical studies of self-consistent growth and saturation of the slow cyclotron wave are beginning and will be reported in the future.

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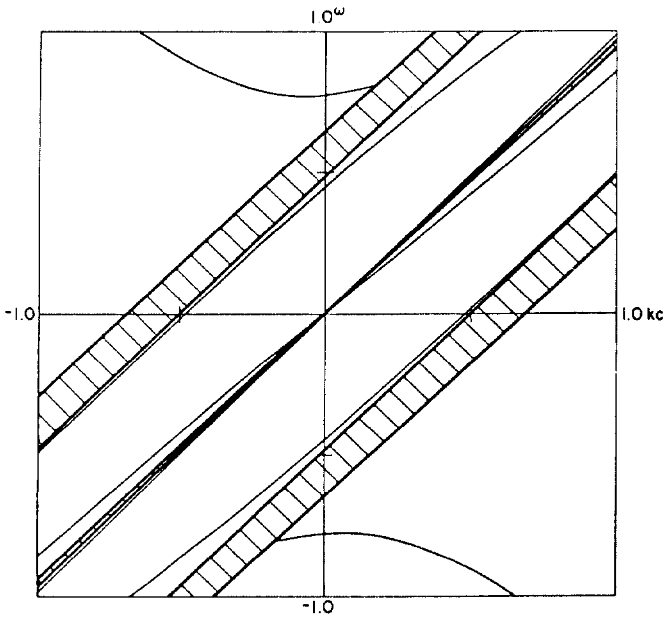

Fig. 1. Dispersion relation, ω/ω_p versus kc/ω_p for waves in a cylindrical electron beam, $\nu = 1$, $\gamma_0 = 5$, $\omega_c/\omega_p = 2.0$, $\omega_p R_b/c = 2.0$, $\omega_p R_0/c = 2.4$, waveguide wall grounded. Branch cuts are denoted by cross-hatched regions (c. f. Fig. 2 of Ref. 5).

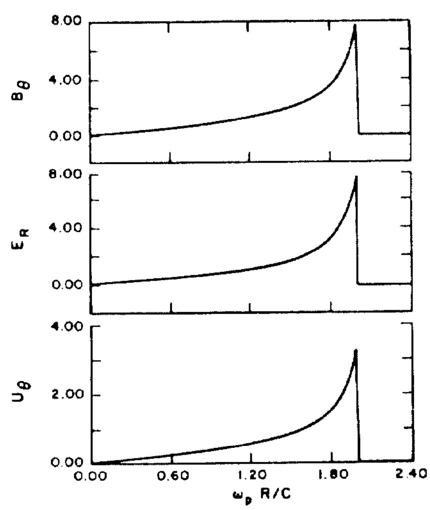

Fig. 2. Radial structure of perturbed wave quantities U_θ, E_R, B_θ in beam with $\nu = 1$, $\gamma_0 = 5$, $\omega_c/\omega_p = 2.0$, $\omega_p R$ $\omega_p R_b/c = 2.0$, $\omega_p R_0/c = 2.4$, $kc/\omega_p = 0.5$, $m = 0$. (Units are arbitrary for these perturbations.)

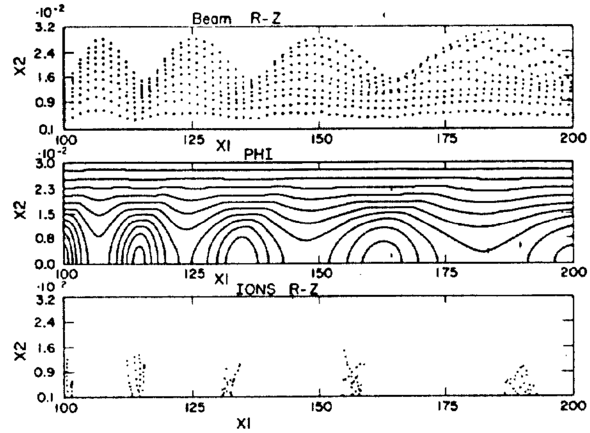

Fig. 4. Data of simulation of $\nu = 1$, $\gamma_0 = 5$, $\omega_c/\omega_p = 2$ modulated electron beam with ions propagating in a conical waveguide with open ends. Plots show (a) electron configuration space ($X1 = z$, $X2 = R$); (b) electrostatic field equipotentials; (c) in configuration space (X1-X2).

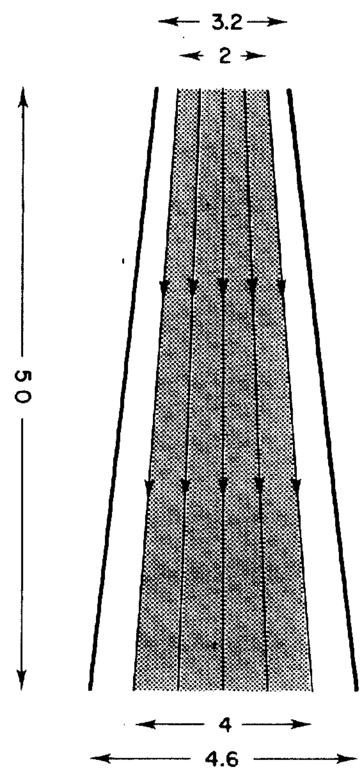

Fig. 3. Nominal experimental configuration corresponding to the simulations shown in Fig. 4. Arrows show diverging magnetic field lines. The electron beam, occupying the shaded area, propagates from left to right. Dimensions are in cm.

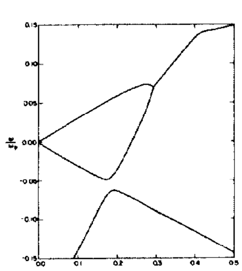

Fig. 5. Dispersion curves showing the intersection of the doppler-shifted slow cyclotron wave with the positive frequency waveguide mode, in a sheath helix liner. Although the dispersion curves separate at $kc/\omega_p = 0.44$, close proximity to the cyclotron branch cut has inhibited resolution of that curve; $\nu = 1.75$, $\gamma_0 = 7.0$, $\omega_c/\omega_p = 1.65$, $\omega_p R_b/c = 2.65$, $\omega_p R_s/c = 3.2$, $\omega_p R_0/c = 4.8$, $\psi = 15°$.

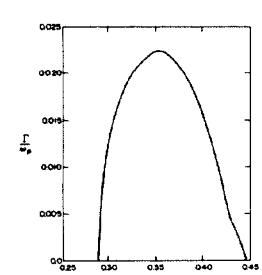

Fig. 6. Growth rate curve for same parameters as in Fig. 5.