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SECONDARY YIELD ENHANCEMENT FROM CURRENT-CARRYING TARGET*

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Summary

Monte Carlo calculations of relative yield of π^-, K^- and $\bar{p}$ secondaries from cylindrical current-carrying production targets in high energy proton beams are presented. The results show that the expected focusing effect can increase the secondary intensity within the acceptance of a secondary beam by significant factors--in some cases two or more--with currents in the 15-to-25 kA range. These currents are readily obtainable from pulsed power supplies. For dc operation currents of this magnitude appear possible from existing power supplies and for targets of small transverse dimensions such as 2 mm radius Ag.

Introduction

In accelerator operations the machine performance is usually measured by the intensity of particles delivered to the production target with minimum size and time structure. From the point of view of the experiment, however, the figure of merit is often the signal-to-background ratio which limits the number of events per unit time which can be accepted. Efforts to optimize this ratio are usually directed at design of secondary beams of maximum acceptance for the desired particle and momentum. In the present paper we explore the problem of production target efficiency which is concerned with the number of secondaries per incident particle within the secondary beam acceptance. As a model we used the solid angle acceptance $\Delta\Omega \approx 13.3$ msr about $\theta = 0^o$ and momentum acceptance $\Delta p/p \approx 0.05$ for 800 MeV/c K^- secondaries from the new Low Energy Separated Beam (LESB II) now under construction at the AGS.[1] We report the results of a Monte-Carlo calculation CTARGET[2] in which the focusing effects of a target carrying a longitudinal current are included. A lens based on this principle was discussed by Panofsky,[3] Luckey[4] and Collins[5] and more recently a target was proposed by Budker[6] using this principle for positron production for an e^+e^- colliding beam machine. The calculation also includes the effect of energy degradation of secondaries and low energy primaries within the target (hereafter referred to as a Budker target) and loss of secondaries by strong interactions. Multiple coulomb scattering and nuclear scattering of the primary and secondary particles are ignored, and the nucleon-meson cascade is not included in CTARGET. The incident beam is assumed to be a pencil beam along the axis of a cylindrical target and the secondary angle and momentum are selected at random from a Sanford-Wang distribution.[7] The parameters varied include the target length, radius and composition as well as the current. In all cases 10^5 incident protons were sampled. We also calculated the relative yield enhancement of low energy (200 MeV/c) π^- to be expected from the LESB II, and the increase in 12.4 GeV/c $\bar{p}$ intensity anticipated in an operating 0^o AGS medium energy unseparated beam[8] of solid angle acceptance ~ 0.3 msr and momentum acceptance $\Delta p/p = 0.06$. The calculation was also applied to 800 MeV/c K^- production for an incident proton momentum of 13 GeV/c such as at the ZGS and KEK accelerators and the enhancement of 200 MeV/c π^- production at the LAMPF proton momentum of 1.46 GeV/c. In the latter case, no attempt was made to adjust the Sanford-Wang distribution to conform with the production data at this energy.[9]

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Relative Yield vs Current, Target Length and Target Radius for p_{Beam} = 28.5, 13.0 and 1.46 GeV/c

The CTARGET program selects a random interaction point Z_R along the target symmetry axis with the expression $Z_R = -\lambda_{int}\,\ln(1-RN)$ where λ_{int} is the interaction length for the target material and RN is a random number uniformly distributed in the interval $0 \le RN \le 1$. Similarly, the random secondary momentum P_R and random polar angle θ_R are chosen in the interval $P_s(\min) \le P_R \le P_s(\max)$ and $\theta_s(\min) \le \theta_R \le \theta_s(\max)$. The forward yield of secondary particles Y_s for the momentum P_s and angle θ_s is evaluated from the Sanford-Wang expression[6]

$$Y_s = \frac{d^2N_s}{d\Omega dp_s} = C_1 P_s^{C_2}\left(1 - \frac{P_s}{P_o}\right) \times \exp\left\{-\frac{C_3 P_s^{C_4}}{P_o^{C_5}} - C_6\theta\,(P_s - C_7 P_o \cos^{C_8}\theta)\right\} \quad (1)$$

where p_o is the momentum of the incident particle. For secondaries used in this report the constants are given in Sanford and Wang's report.[7] The radial and longitudinal equations of motion of the particle in cylindrical coordinates are

$$\left.\begin{aligned}\ddot{r} + K_1\dot{z}r &= 0\\ \ddot{z} - K_1\dot{r}r &= 0\end{aligned}\right\} r < R \qquad \left.\begin{aligned}\ddot{r} + K_2\dot{z}/r &= 0\\ \ddot{z} - K_2\dot{r}/r &= 0\end{aligned}\right\} r > R \quad (2)$$

where $K_1 = q\mu_o I/2\pi MR^2$ and $K_2 = q\mu_o I/2\pi M$. We assume no azimuthal motion, i.e. $\dot{\phi} = 0$. M is the total relativistic mass of the secondary. Equations (2) are integrated along the length L of the target from $Z = Z_R$ to $Z = L$. The particle Y_s is then added to the contents of a bin in an angle and momentum grid and the total yield Y_{tot} is proportional to the sum of the contents of bins within a specified angle range $\theta_{max} - \theta_{min}$ and momentum interval $P_{max} - P_{min}$

$$SUM = \sum_{i=1}^{i_{max}} (2i-1)\,Y_s(i) \quad . \quad (3)$$

We first calculated Eq. (3) for 800 MeV/c K^- for various currents and lengths in a Budker target of silver and platinum of radius R = 1 mm. The results are summarized in Tables I and II and show an increase in relative yield with current up to approximately 25 kA and a broad plateau thereafter. The yield is also an increasing function of target length up to the largest values of L in the table, 35 cm for Ag ($\sim 2.24\,\lambda_{int}$) and 25 cm for Pt ($\sim 2.72\,\lambda_{int}$). For comparable values of length in units of λ_{int} the yields are approximately equal for Ag and Pt. For $L \approx 2\lambda_{int}$ and I = 25 kA the yield increase is $\sim 60\%$.

In Tables III, IV and V we show the relative K^-, $\bar{p}$ and π^- yield vs current for a 10 cm Pt target of several radii. The results show a decrease in yield with increasing radius as would be expected from absorption of secondaries within the target. For small radius R = 0.1 mm and, to a lesser extent, R = 1 mm and low momentum secondaries such as 200 MeV/c π^- we see a sharp peak near 20 kA. The drop in yield at higher current is the result of over-focusing where the secondary is bent out of the acceptance of the secondary beam. The enhancement in yield of 200 MeV π^- with I = 15 kA is ~ 2.8.

TABLE I

800 MeV/c K^- Relative Yield vs Target Length for 28.5 GeV/c Protons on Ag

I(kA)	L=5cm	10cm	15cm	20cm	25cm	30cm	35cm
0	236	389	478	599	678	700	758
5	251	448	570	662	714	749	833
10	294	486	617	733	813	891	933
15	306	518	717	876	923	1015	1063
20	280	559	727	950	1027	1046	1101
25	305	625	868	983	1032	1127	1163
30	283	661	899	1013	1142	1169	1141
35	338	731	899	1110	1153	1190	1235
40	377	710	938	1077	1170	1252	1221
50	429	728	920	1048	1152	1124	1120
60	443	788	905	1094	1124	1130	
70	440	734	965	1078	1057	1064	
80	457	776	894	984	969		
90	448	761	852	923	892		

TABLE II

800 MeV/c K^- Relative Yield vs Target Length for 28.5 GeV/c Protons on Pt

L =	5 cm	10 cm	15 cm	20 cm	25 cm
0	326	519	620	658	696
5	363	556	656	737	772
10	388	618	725	826	917
15	441	705	862	916	1006
20	420	719	959	1033	1047
25	453	851	1030	1041	1062
30	451	879	1036	1086	1164
35	527	883	1078	1110	1091
40	522	908	1070	1100	1129
50	589	937	1115	1095	985
60	620	946	1083	1120	1074
70	612	956	1088	1045	
80	688	964	1046	912	
90	673	944	929	833	

TABLE III

800 MeV/c K^- Relative Yield vs Target Radius for 28.5 GeV/c Protons on Pt

I(kA)	R=.1mm	1mm	2mm	3mm
0	614	519	419	416
5	705	556	466	464
10	826	618	567	478
15	1072	705	566	500
20	1187	719	610	529
25	1315	851	659	604
30	1330	879	672	621
35	1263	883	668	638
40	1228	908	732	610
50	1134	937	778	641
60	1044	946	890	638
70	1080	956	889	726
80	1035	965	851	768
90	908	944	934	842

TABLE IV

12.4 GeV/c $\bar{p}$ Relative Yield vs Target Radius for 28.5 GeV/c Protons on Pt

I(kA)	R=.1mm	1mm	2mm	3mm
0	45.7	29.8	28.8	30.1
5	44.3	30.0	28.6	29.4
10	53.0	31.0	29.2	28.6
15	54.0	32.2	30.0	28.7
20	58.3	34.0	30.2	29.2
25	55.5	33.5	30.4	30.0
30	57.6	34.1	31.4	29.3
35	57.8	35.9	30.7	29.6
40	58.8	36.4	31.9	30.9
50	59.0	36.1	31.6	30.7
60	58.2	36.4	31.4	30.9
70	56.7	37.0	32.6	31.4
80	56.2	38.8	33.9	30.6
90	54.8	42.6	34.9	31.2

TABLE V

200 MeV/c π^- Relative Yield vs Target Radius for 28.5 GeV/c Protons on Pt

I(kA)	R=.1mm	1mm	2mm	3mm
0	3825	2727	2232	2356
2.5		3432		
5	8199	4885	3381	2865
7.5		5527		
10	8564	5464	3859	3596
12.5		5311		
15	10579	5586	4937	4706
17.5		6126		
20	8714	6448	5241	4190
25	7717	5602	4856	4656
30	8481	5485	4944	5770
35	8399	4443	5097	5794
40	5775	4428	4838	4826
50	5614	4810	4911	5033
60	6368	5243	4685	4479
70	5022	4166	5035	4059
80	4649	5149	4470	4282
90	4064	4616	4583	4210

At lower incident proton momentum the effects are similar to those at 28.5 GeV/c but less pronounced. In Table VI for 13 GeV/c protons on an L = 10 cm Pt target we see that for I = 25 kA the yield enhancement of K^- for R = 0.1 mm is ~ 1.7. We assume in these calculations that the R = 0.1 mm behavior is the expected yield when the beam and target radii are comparable.

TABLE VI

800 MeV/c K^- Relative Yield vs Target Radius for 13 GeV/c Protons on Pt

I(KA)	R=.1mm	1mm	2mm	3mm
0	334	278	221	219
5	331	277	238	239
10	364	291	277	242
15	463	320	270	248
20	515	324	281	256
25	573	377	296	283
30	592	389	300	285
35	555	386	294	290
40	542	395	321	273
50	503	408	338	282
60	464	414	388	280
70	480	424	390	316
80	465	432	376	333
90	413	429	416	366

Also of interest is a possible yield enhancement from a Budker target at a meson factory such as LAMPF. Although no attempt has been made to fit the Sanford-Wang expression at these low beam and secondary momenta[10] we nonetheless performed the calculation to give a qualitative indication of the enhancement. The results for 200 MeV/c π^- from 1.46 GeV/c protons on an L = 10 cm Pt target are given in Table VII and show a large (> 3) effect for 25 kA and a thin target.

TABLE VII

200 MeV/c π^- Relative Yield vs Target Radius for 1.46 GeV/c Protons on Pt

I(KA)	R=.1mm	1mm	2mm	3mm
0	321	170	119	122
5	344	196	181	166
10	530	284	182	173
15	812	307	171	147
20	1066	273	229	165
25	1172	264	270	203
30	1124	278	252	240
35	911	322	320	240
40	447	277	271	256
50	426	345	259	274
60	392	332	267	269
70	273	299	314	
80	245	357	333	
90	264		277	

The Budker target is obviously most applicable to targeting a fast extracted beam where the target current can be pulsed. Septum magnets used in fast extraction attain the ~ 20 kA currents[11] of interest here. For slow extraction, however, the ~ 50% duty factor requires essentially dc operation for external beam components. We therefore evaluated the temperature profile resulting from a dc current in a cylindrical conductor by numerical integration of the time-independent temperature equation[12] $\nabla^2 T + q(\vec{r})/K = 0$ using the computer program TEMPER.[2] In the temperature equation $q(\vec{r})$ is the source function in energy per unit volume per unit time and K is the thermal conductivity. The source function in this application is $q = J^2\rho$ where J is the current density and ρ the resistivity. We further assume that the thermal conductivity and electrical conductivity σ are related by the Wiedemann-Franz ratio[13]

$$\frac{K}{\sigma T} = \text{constant} = C_{WF} = \frac{\pi^2}{3}\left(\frac{k}{e}\right)^2 \qquad (4)$$

where k is the Boltzmann constant and e the electron charge. The temperature equation then reduces to

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{c}{T} = 0 \qquad (5)$$

where $c = \mathcal{E}^2/c_{WF}$ and $\mathcal{E}$ is the constant electric field in the conductor. In solving Eq. (5) we use the symmetry condition $dT/dr\ (r = 0) = 0$. With an initial guess for C and T (r = 0) program TEMPER iterates to obtain the value of c satisfying $T\ (r = R) = 100^\circ C$ and ultimately the value of c also satisfying $T\ (r = 0) = T_{melting}$. We then obtain the current density function $J(r) = \mathcal{E}/\rho$ and the total current $I = \int J dA$ and the power dissipation per unit length $P_\ell = \int J^2\rho dA$, given the resistivity function $\rho(T)$ obtained from a fit of the quadratic function $\rho(T) = \rho_0 + \rho_1 T + \rho_2 T^2$ to the data. The results are summarized below.

TABLE VIII

Maximum Current in Cylindrical Conductor

Tgt	R (mm)	I_{max} (kA)	Power (W/mm)	$\rho_0(\Omega\text{-mm}) \times 10^5$	$\rho_1(\Omega\text{-mm}/^\circ K) \times 10^5$	$\rho_2(\Omega\text{-mm}/^\circ K^2) \times 10^5$
Pt	1	4.06	2075	-2.09	0.0455	-5.69×10^{-6}
Ag	1	20.32	5914	0.0166	0.00484	1.47×10^{-6}

The calculation also shows that the maximum current is proportional to radius and the associated voltage is inversely proportional to radius; thus, the peak power dissipation remains constant. To obtain the $\sim$ 20 kA current indicated in Tables I-VII would require for Pt an R = 5 mm target and power dissipation of $\sim$ 400 kW for an L = 20 cm target. For Ag we could obtain 20 kA for R = 2 mm and power dissipation $\sim$ 450 kW for an L = 30 cm target. These values are within the capability of existing commercially available magnet power supplies.[14] The large power dissipation will, however, pose a challenging heat transfer problem.[15]

Cascade Production for Ag and Pt Targets

The nucleon-meson cascade was simulated by Ranft[16] in the Monte-Carlo program KASPRO. We used this program to obtain estimates of absolute yields expected for low energy π^- and $\bar{p}$ for 28.5 GeV/c protons on Ag and Pt targets. The results are given in Table IX for two momentum bins of average value $p_{av} = 0.85$ and 1.42 GeV/c. The low momentum $\bar{p}$ yield appear to have inadequate statistics; the 1.4 GeV/c group indicates an increasing yield vs target length out to the longest targets considered here ($\sim 3\lambda_{int}$). The increase is larger than the results of Tables I and II using program CTARGET and gives confidence that long Budker targets can be used without loss of yield from secondary absorption effects.

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TABLE IX

KASPRO Results. Secondary Yield vs Target Length for 28.5 GeV/c Protons on R = 2 mm Radius Target. 5×10^5 Protons Sampled per Run. Beam = ± 0.5 mm rms, ± 1 mr rms. $\Delta p_s = 0.57$ GeV/c, $\Delta\theta = 3$ mr.

Negative Pion Yield $\times 10^{-9}$ for 10^{12} Protons on Ag

L(cm)	$Y_{\pi^-}(p_{av} = .855)$	$Y_{\pi^-}(p_{av} = 1.425\,\text{GeV/c})$
6	5.015	4.388
10	6.481	5.558
14	7.947	4.788
18	8.659	6.775
22	5.721	5.834
26	4.913	6.278
30	5.098	7.360
34	5.642	7.107
38	5.249	7.777
42	5.177	7.714
46	4.848	8.001

Antiproton Yield $\times 10^{-6}$ for 10^{12} Protons on Pt

4	0.336	2.214
7	0.484	2.361
10	0.421	2.967
13	0.639	7.054
16	0.560	7.424
19	0.306	7.543
22	0.306	8.285
25	0.306	7.543
28	0.414	7.440
31	0.414	7.440

Antiproton Yield $\times 10^{-6}$ for 10^{12} Protons on Ag

6	0.159	1.910
10	0.110	2.464
14	0.057	8.091
18	0.057	4.483
22	0.218	5.110
26	0.498	6.850
30	0.699	6.266
34	0.699	6.842
38	0.761	6.632
42	0.755	7.204

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