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COMPARISON OF MEASURED AND COMPUTED LOSS TO PARASITIC MODES IN CYLINDRICAL CAVITIES WITH BEAM PORTS*

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Introduction

The importance of parasitic mode losses for the design and operation of electron-positron storage rings is now well recognized. These losses at present set the limit on allowable beam current in the SPEAR II ring under some operating conditions.[1] Parasitic mode losses and their potential deleterious effects are a prime consideration in the design of the PEP vacuum chamber. Too high a loss impedance can lead not only to overheating of individual components but to a reduced threshold for bunch instabilities. It is important therefore to have available adequate measurement and computational methods, both as an aid in the design of specific vacuum chamber components and to provide a better understanding of the nature of the loss impedance.

Measurement Technique

The basic measurement method has been described by Sands and Rees.[2] Figure 1 shows a diagram of the instrumentation. In this method a wire is stretched along the axis

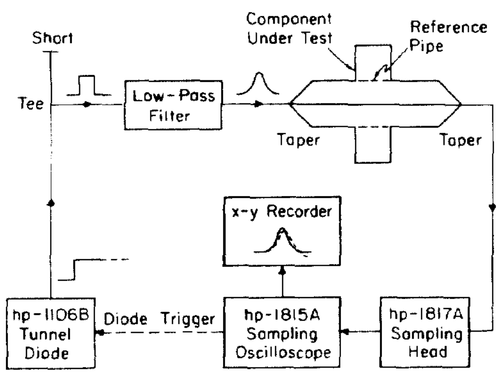

Fig. 1. Diagram of the instrumentation for measuring parasitic mode loss.

of the component under test. Transitions taper from the beam port apertures at each end of the component down to 50 ohm coax lines. A nearly Gaussian pulse is generated by passing the step output (20 ps risetime) from the tunnel diode through an appropriate combination of a shorted tee and low-pass filter. The pulse passes through the test component to the sampling oscilloscope and the output is recorded on an x-y recorder. As the incident pulse $I_0(t)$ passes through the test component, resonant electromagnetic modes which have a longitudinal electric field on the beam axis are excited. These excited modes in turn induce a secondary pulse, $I_s(t)$, in the axial wire. A perturbed pulse $I_1(t)=I_0(t)-I_s(t)$ then emerges from the system. The incident (unperturbed) pulse $I_0(t)$ is obtained by replacing the test component with a reference pipe having the same cross section as the beam apertures of the component. Following the theory in Ref. 2, a loss parameter k (having dimensions volts/Coulomb) is obtained from

$$k = \left(2Z_0/Q^2\right)\int I_0(t)\cdot I_s(t)\,dt \qquad (1)$$

where Z_0 is the characteristic impedance of the reference pipe and Q is the charge in the pulse, $Q=\int I_0(t)\,dt$.

Figure 2 shows a typical recording for $I_0(t)$ and $I_1(t)$ for the cavity shown. Note the crossover of I_0 and I_1, indicating that the latter part of a bunch gains energy from the fields

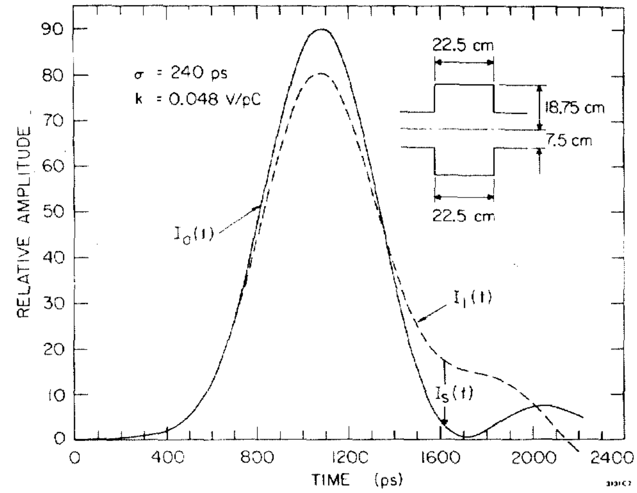

Fig. 2. Recorded output pulses $I_0(t)$ for the reference pipe and $I_1(t)$ for a cylindrical cavity having the dimensions shown.

induced by the preceding particles. In making this recording, the waveforms for I_0 and I_1 were carefully superimposed in the knee region (t=400–600 ps). In more recent measurements the curves are not superimposed during the measurement but are recorded with a horizontal separation, then digitized and shifted by a computer program to obtain proper coincidence. It can be shown in any case that to first order k is not sensitive to small relative shifts between I_0 and I_1.

There are some subtle points in the theory of the method, and some difficulties in the measurement technique as described here. For example, the axial wire would seem to short out the very modes that the pulse is trying to excite. The wire used in our measurements (2.4 mm diameter) does indeed lower the Q's of the modes drastically, and in addition the frequencies of the lowest few modes are perturbed somewhat. The success of the technique is, however, based on the reasonable supposition that the wire does not substantially change the R/Q of a mode, defined as $R/Q=V^2/\omega W$ where eV is the energy loss for a particle passing along the axis and W is the stored energy. The total loss depends only on the R/Q's of the modes through[3]

$$k = \sum_n (\omega_n/4)(R/Q)_n \exp\left(-\omega_n^2\sigma^2\right) \qquad (2)$$

Here the exponential factor takes into account the effect of the bunch length σ, assuming a Gaussian bunch.

As an example of one of the difficulties in the measurement technique, the method relies on the stability of the sampling oscilloscope during the time it takes to exchange the test component and reference pipe. In particular, the stability of the time base (~1 ps) limits the resolution to (for example) $\Delta k \approx .005$ at $\sigma=75$ ps. There are ways around this problem (e.g., by splitting the incident pulse and recording the output pulses from the test component and reference pipe simultaneously), but this and alternative measurement schemes have problems of their own. Because of these uncertainties in theory and technique, we have felt it to be important to compare the measured results with both computations and with experimental results from SPEAR when possible. Agreement would build confidence in both the measurement and the computations.

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Comparison of Computation and Measurements

Our basic computational tool at present is E. Keil's program KN7C.[4] This program calculates the resonant frequencies and R/Q's for the longitudinal modes in a periodic chain of cylindrical cavities coupled by cylindrical beam pipes. Such a structure cannot accurately model a single cell for those modes which can propagate in the beam pipe. However, in many practical cases, because of the exponential factor in Eq. (2), the loss for bunch lengths of interest is almost all to the nonpropagating cavity modes.

Table I below shows a comparison between the computed and measured values for total loss for three cylindrical cavities. In each case, the radius of the beam port is 7.5 cm and the height (gap length) of the cavity is 22.5 cm.

Table I

Outer Cavity Radius (cm)	k Computed from KN7C (V/pC)	Measured k (V/pC)
30	.156	.162 - .185
18.75	.144	.153
12.5	.070	.069

These results are for a bunch length $\sigma_z = c\sigma = 3.3$ cm. For a bunch length of 7.25 cm, the computed and measured values were .054 V/pC and .048 V/pC respectively for the 18.75 cm radius cavity. The recorded curves in Fig. 2 show $I_0(t)$ and $I_1(t)$ for this latter case.

A more detailed comparison between the theory and the bench measurements can be made by considering the function $I_s(t)$ in Fig. 2. To compute I_s, we require the wake potential $w(\tau)$ for the cavity. This is the potential seen by a particle crossing the cavity at time τ behind a charge impulse of unit amplitude. The wake potential can be obtained[3] from the cavity modes through

$$w(\tau) = 2 \sum_n (\omega_n/4)(R/Q)_n \cos \omega_n \tau \qquad (3)$$

In practice, it requires too much computer time to compute more than about 100 modes. Equation (3) has therefore been extended to include the effect of higher frequencies (giving a more accurate wake at small τ) by adding an integral based on an optical resonator model of the periodic structure. Details of the calculation are given in Ref. 3. In Fig. 3 the relative contributions to the wake are shown for a cylindrical cavity having the dimensions given in Fig. 2. Note that the form of the wake at small τ is dominated by the analytic extension, but that the characteristic length and integrated strength of the wake is mainly determined by the sum over modes (in this case 50 modes). By comparing Eqs. (2) and (3), note also that the intercept of the wake at τ=0 is related to the loss parameter k by $w(0)=2k(\sigma=0)$.

As was mentioned previously, if results from KN7C are used in Eq. (3) the wake function will include the effect of the interaction of the bunch with traveling wave modes in an infinite periodic structure for those frequencies high enough to propagate in the beam port cylinders. There is a second, more subtle, difficulty. Equation (3) is correct for a current impulse crossing the cavity, but any contribution to the wake due to the scalar potential arising from free charges in the cavity has been ignored. The question of the relative contribution to the wake from the scalar potential has not yet been satisfactorily resolved theoretically, but good agreement with the results from bench measurements, where it would seem that the scalar potential should contribute to $I_s(t)$, would place a limit on this contribution.

The total potential at any time t within the bunch due to all charges passing through the cavity or system previous to time t is given by

$$k(t) = \frac{1}{Q} \int_0^\infty w(\tau)\, I(t-\tau)\, d\tau \qquad (4)$$

For a Gaussian bunch, Eq. (4) becomes

$$k(t) = \frac{1}{\sqrt{2\pi}\sigma} \int_0^\infty w(\tau)\, e^{-(t-\tau)^2/2\sigma^2}\, d\tau \qquad (5)$$

The total loss per unit charge is obtained from k(t) for the general and Gaussian cases as

$$k_{tot} = \frac{1}{Q} \int_{-\infty}^{\infty} k(t)\, I_0(t)\, dt = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} k(t)\, e^{-t^2/2\sigma^2}\, dt \qquad (6)$$

It can be shown[3] that the function k(t) is related to the experimentally measurable function $I_s(t)$ in Fig. 2 by $k(t) = 2Z_0 I_s(t)/Q$. For a Gaussian bunch this becomes

$$\frac{I_s(t)}{I_p} = \sqrt{\frac{\pi}{2}}\, \frac{\sigma}{Z_0}\, k(t) \qquad (7)$$

where $I_p = Q/(\sqrt{2\pi}\,\sigma)$ is the peak current. In Figs. 4, 5 and 6, measured values of $I_s(t)/I_p$ are compared with computed values obtained starting with the R/Q's and ω_n's from KN7C, then applying Eqs. (3), (5) and (7). The computed curves have not been normalized in any way; it is seen that the agreement is quite good, in spite of the uncertainties in both theory and measurement.

Note that the total loss parameter k can be obtained in two ways: directly from the sum of modes using Eq. (2), and from the integral in Eq. (6). A comparison of the two values provides a check on the computation. This

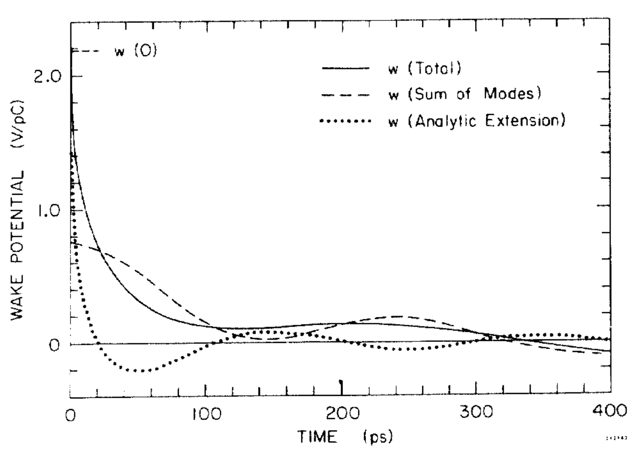

Fig. 3. Relative contributions to the wake functions from the sum over modes and the analytic extension for the cavity of Fig. 2.

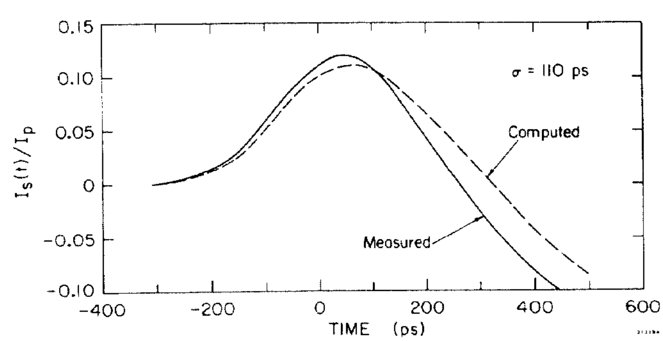

Fig. 4. Computed and measured functions $I_s(t)/I_p$ for the cavity shown in Fig. 2 for a bunch length $\sigma = 110$ ps.

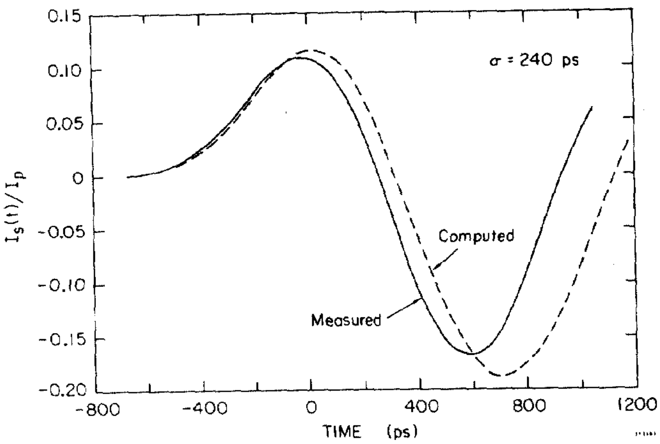

Fig. 5. Computed and measured functions $I_s(t)/I_p$ for the cavity shown in Fig. 2 for a bunch length $\sigma = 240$ ps.

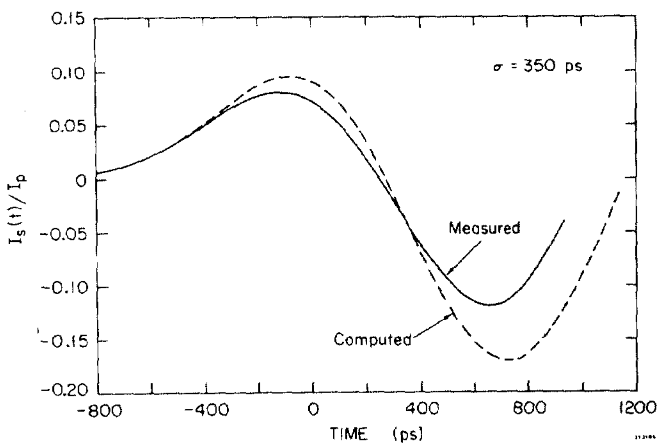

Fig. 6. Computed and measured functions $I_s(t)/I_p$ for the cavity shown in Fig. 2 for a bunch length $\sigma = 350$ ps.

comparison is given in Table II below, together with measured values of k, for the same cavity and bunch lengths as in Figs. 4-6.

Comparison with Measured Loss in SPEAR

There have been several instances where a comparison has been possible between the parasitic mode heating produced in a component by the SPEAR beam, and the heating calculated from the result of a bench measurement on the same component. The most precise results have been obtained from heating and cooling curves for a flange pair in SPEAR.[5] In this flange, a radial gap several centimeters deep by several millimeters in length presents a discontinuity which is a source of power dissipation. For a 30 mA beam with a bunch length of 3.5 cm, the power dissipation obtained from the temperature-time plots was 12.5 W. The bench measurement gave a value for the loss parameter of $k = .014$ V/pC at this bunch length. The loss resistance is related to k by $R = kT_0$, where T_0 is the revolution time. For SPEAR $T_0 = 0.78\ \mu s$, giving $R = 11\ k\Omega$. For a 30 mA beam $P = I^2R = 10$ W, in good agreement with the power dissipation obtained directly from the temperature rise data.

Conclusion

Good agreement has been obtained between computed values and results from a bench measurement technique for the total loss to parasitic modes in several cylindrical cavities with beam ports. The measurement of loss as a function of time within the current pulse also gives results which are in good agreement with computed functions, especially considering the fact that there are questionable points concerning both the theory and the measurement technique. Within measurement errors, there is also agreement in a few cases where a comparison is possible between a bench measurement result and the heating produced directly in a component by the SPEAR beam.

Acknowledgments

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References

1. J. McE. Paterson, private communication.
2. M. Sands and J. Rees, Stanford Linear Accelerator Center report PEP-95 (August 1974, unpublished).
3. P. B. Wilson and K.L.F. Bane, Stanford Linear Accelerator Center report PEP-226A (March 1977, unpublished).
4. The mathematics of the program is described in E. Keil, Nucl. Instr. Methods 100, 419 (1972).
5. J. Jurow and P. Wilson, Stanford Linear Accelerator Center report SPEAR Note 200 (March 1977, unpublished).

Table II

σ ps	k_{tot} V/pC	k (Σ modes) V/pC	k (measured) V/pC
110	.146	.144	.153
240	.0551	.0543	.048
350	.0183	.0177	.0174