IEEE Transactions on Nuclear Science, Vol.NS-24, No.3, June 1977

LONGITUDINAL INSTABILITIES OF BUNCHED BEAMS IN THE ISR

P. Bramham, S. Hansen, A. Hofmann, E. Peschardt

CERN,
Geneva, Switzerland

Summary

Microwave instabilities occur in bunched beams in the ISR leading to a dilution of the phase space density and limiting the longitudinal density of the stacked beams. According to D. Boussard this instability can be described as a coasting beam instability inside bunches. Experimental investigations of this microwave instability support this theory and give a high frequency impedance $|Z_L|/n \approx 14$ ohms. Injecting large currents in bunches of large area increases the threshold of this instability. The larger currents can produce coupled bunch mode instabilities which can be cured by a higher harmonic cavity.

1. Introduction

In the ISR the beams are accumulated by stacking in longitudinal phase space.[1] The injected bunches are matched to the RF voltage and accelerated towards their final orbit. Then the RF voltage is reduced and finally turned off and the beam debunches. During this process microwave instabilities occur which are similar to the bunch widening (turbulence) observed in electron rings. They lead to a blow-up of the bunch area and hence to a dilution of the phase space density. This effect limits the longitudinal density of the stacked beams and affects the performance of the ISR.[2] This microwave instability can be described as a coasting beam instability inside bunches.[3]

2. Microwave Instabilities (Bunch Widening)

2.1 Theoretical Description

Bunch widening or turbulence has been treated by several authors. We use here an approach first described by D. Boussard[3] and apply his results to our measurements. A longitudinal coasting beam instability can occur inside bunches if the frequency of the instability is so high that the corresponding wavelength is short compared to the bunch; and if the growth rate (calculated for the coasting beam) is large compared to the phase oscillation frequency Ω_s. In this case the coasting beam stability criterion[4] can be applied to the instantaneous values of momentum spread and current.[3]

$$\frac{|Z_L|}{n} \leqslant F\,\frac{m_o c^2 |\eta|}{e\gamma}\left(\frac{(\overline{\Delta\beta\gamma})^2}{I}\right)_{\text{inst.}} \qquad (1)$$

with F = form factor, $|\eta| = 1/\gamma_T^2 - 1/\gamma^2$, I = current, $\overline{\Delta\beta\gamma}$ = full spread in $(\beta\gamma)$ measured at half height and $|Z_L|/n$ = longitudinal impedance at high frequencies divided by mode number ($n = f_{\text{inst.}}/f_{\text{rev.}}$).

For a given $|Z_L|$, γ and $|\eta|$ the quantity $I/(\overline{\Delta\beta\gamma})^2$ has a threshold value beyond which a microwave instability will occur. For protons the phase space area A of the bunch is conserved and the above quantity is smaller for short bunches; (for electrons $\Delta\beta\gamma$ is fixed by synchrotron radiation and the above quantity becomes smaller for long bunches). We consider here protons with bunches in large stationary buckets having a parabolic "line" density with respect to the RF phase angle Φ:

$$\Lambda(\Phi) = \frac{3N_b}{4\Phi_o^3}(\Phi_o^2 - \Phi^2);\ (N_b = \text{particles per bunch}).$$

The phase space density g is then[3]

$$g(\Phi,\Delta\beta\gamma) = \frac{3N_b}{2A}\sqrt{1 - (\Phi/\Phi_o)^2 - (\Delta\beta\gamma/\Delta\beta\gamma_o)^2} \qquad (2)$$

with Φ_o = bunch length in RF phase angle, $\Delta\beta\gamma_o$ = half spread at the base of $(\beta\gamma)$ and $A = \pi\Delta\beta\gamma_o\Phi_o$ = bunch area. It has been shown[3,5] that for the distribution (2) the quantity $I/(\overline{\Delta\beta\gamma})^2$ is constant along the bunch (independent of Φ). Furthermore $\overline{\Delta\beta\gamma} = \sqrt{3}\Delta\beta\gamma_o$. This distribution is very realistic for the bunches in the ISR and has some nice properties which make calculations easy. The form factor F for this distribution is estimated to be $\sim$0.64. This estimate is based on the stability diagram of a coasting beam with the same momentum distribution allowing for a finite growth rate in the order of Ω_s.

For the quantity $I/(\overline{\Delta\beta\gamma})^2$ we find

$$\frac{I}{(\overline{\Delta\beta\gamma})^2} = \frac{\pi^3 I_o h\Phi_o}{2\,MA^2} = \frac{I_o h}{MA}\,\frac{\pi^3}{2}\sqrt[4]{\frac{2m_o c^2 h|\eta|}{\pi A^2\gamma eV^*}} \qquad (3)$$

with h = harmonic number, M = number of bunches, I_o = average current of all bunches. V^* is the effective RF voltage seen by a particle in the bunch. The large inductive wall impedance in the ISR gives a potential well distortion which for "parabolic" bunches has the same effect as a reduction of the external RF voltage.[6] The bunch area A can be calculated from the measured bunch length Φ_o.

$$A = \sqrt{\frac{\pi eV^*\gamma}{2h|\eta|m_o c^2}}\,\Phi_o^2 \qquad (4)$$

During debunching the property $I/(\overline{\Delta\beta\gamma})^2$ increases as

$$I/(\overline{\Delta\beta\gamma})^2_t = I/(\overline{\Delta\beta\gamma})^2_{t=0}\cdot\sqrt{1 + P^2t^2} \qquad (5)$$

with
$$P = \frac{\omega_o h|\eta|}{\sqrt{3}\,\beta\gamma}\left(\frac{\overline{\Delta\beta\gamma}}{\Phi_o}\right)_{t=0}$$

and $\omega_o = 2\pi \times$ revolution frequency. (The effect of the potential well has been neglected.)

2.2 Experiments

In a series of experiments we checked the expected dependence of the instability on the beam parameters and obtained a number for the longitudinal impedance at high frequency.

2.2.1 Blow-up of the bunch area. In one approach we measured the bunch length Φ_o as a function of injected current. The current was controlled by a vertical scraper which should not affect the longitudinal particle distribution. The effective RF voltage V^* was then determined using an inductance of 10 μH and correcting for space charge effects. This inductance is based on recent measurements and is slightly smaller than the value quoted earlier.[6] The quantity $I/(\overline{\Delta\beta\gamma})^2$ and the bunch area A are calculated from (3) and (4). Fig. 1 shows an example of such a measurement. The top curve shows the increase of bunch length with current. First this increase follows the dashed line expected for the potential well distortion. Above a certain threshold excessive bunch lengthening occurs. The bunch area A (centre curve) is at first independent of current but above the threshold a blow-up occurs. The last curve shows the quantity $I/(\Delta\overline{\beta\gamma})^2$ as a function of current. It increases as expected up to the threshold and decreases later due to an excessive blow-up of the bunch area (overshoot). If the initial (injected) value of $I/(\Delta\overline{\beta\gamma})^2$ is larger than the threshold value, its final value will be given by the Dory relation[7]

$$I/(\Delta\overline{\beta\gamma})^2_{initial} \cdot I/(\Delta\overline{\beta\gamma}^2)_{final} = (I/(\Delta\overline{\beta\gamma})^2)^2_{threshold}$$

which is indicated in Fig. 1 and agrees well with the measurements. From several such measurements we determined the threshold value of $I/(\Delta\overline{\beta\gamma})^2$ and calculated the impedances with the result $|Z_L|/n \approx 13.5$ ohms. In a second approach we injected a fixed current and reduced the RF voltage adiabatically. According to (3) the quantity $I/(\Delta\overline{\beta\gamma})^2$ increases in the process and can reach the threshold value. Such a measurement is shown in Fig. 2. These measurements were analysed the same way as the ones described before with the result $|Z_L|/n \approx 14.3$ ohms.

2.2.2 Observation of high frequency signals. During debunching $I/(\Delta\overline{\beta\gamma})^2$ is growing steadily (5). High frequency signals can be observed directly in time domain[2] or in frequency domain through a filter. The observed thresholds agree with the expectation but the results are probably less accurate since we cannot correct for the potential well. High frequency signals are also observed during voltage reduction (Fig. 3). At a certain time a signal appears suddenly. From the effective voltage at that time and the initial bunch we again determined the impedance $|Z_L|/n \approx 13$ ohms.

Finally, Fig. 4 shows high frequency signals appearing at injection for different frequencies and injected currents. Due to the sharp edges of the bunches we always observe some high frequency signals which are not relevant for our investigations. For small injected currents these signals decay soon. Above a certain threshold current we observe additional signals which are growing. They represent the microwave instability. Fig. 4 has only a qualitative value but shows that the instability seems to occur over a large frequency range.

2.3 Results

Measurements of the microwave instability carried out under quite different conditions show that the instability seems to depend for a given $|Z_L|$, γ and $|n|$ on $I/(\Delta\overline{\beta\gamma})^2$ as expected from the local stability criterion. The average value of the high frequency (0.3-1.8 GHz) impedance divided by mode number is for the ISR

$$|Z_L|/n \approx 14 \text{ ohms.}$$

2.4 Consequences for the ISR

According to (5) the instability will occur sooner or later during debunching. However if the debunching is carried out inside an already existing stacked beam, the effect can be reduced since the wall "sees" in first approximation only the difference between the density in the bunches and the density of the surrounding unbunched beam. This effect is called "shielding" and is used for high-density stacking.[8] We have now to prevent the bunches from blowing up before they are in tightly fitted buckets and can be debunched in an already existing stack. From (3) it is clear that by doubling the current I_o and the area A the phase space density is the same but the quantity $I/(\Delta\overline{\beta\gamma})^2$ is decreased by a $\sqrt{2}$. If this is done the bunched current is increased and the coupled bunch mode instability (which is otherwise harmless[9]) can present a problem.

3. Stabilization of Coupled Bunch Mode Instability with an Active Higher Harmonic Cavity (Landau Cavity)

Adding a higher harmonic voltage to the RF voltage V_o increases the phase oscillation frequency spread and can provide Landau damping.[10,11,12] We assume $\sin\Phi_s = 0$ and a total voltage

$$V = V_o\sin(\omega_R t) + V_N\sin(N\omega_{RF}t) = V_o(\sin\Phi + kN\sin(N\Phi))$$

The ratio $k = V_N/V_o$ between the two voltages is here either positive (increasing Ω_s) or negative (decreasing Ω_s); we do not consider other phase relations. From the equation of motion we get by integration

$$\dot{\Phi} = \sqrt{2}\Omega_o\left[\cos\Phi - \cos\Phi_o' + \frac{k}{N}(\cos(N\Phi) - \cos(N\Phi_o'))\right]^{\frac{1}{2}}$$

where Φ_o' is the amplitude of the phase oscillation and Ω_o the phase oscillation for small amplitudes in the absence of a Landau cavity

$$\Omega_o = \left(\frac{\omega_o^2 h|n|eV_o}{2\pi\ \beta^2 m_o c^2\gamma}\right)^{\frac{1}{2}}$$

The frequency $\Omega_s(\Phi_o')$ can be calculated from the period T_s

$$T_s = 4\int_o^{\Phi_o'}\frac{d\Phi}{\dot{\Phi}} \quad \text{and} \quad \Omega_s = \frac{2\pi}{T_s}$$

Numerical calculations give the exact solutions[11,12] but for small amplitudes $\Phi_o' \le \frac{3}{4}\frac{\pi}{N}$ we can develop the trigonometric functions to get some approximate solutions.

a) $1 + kN^3 > 0$

$$\Omega_s \approx \frac{\pi\Omega_o}{4}\sqrt{\frac{1+kN^3}{3}}\,\frac{q}{K(\Phi_o'/q)} \text{ with } q^2 = \frac{12(1+kN)}{1+kN^3} - \Phi_o'^2$$

and $K(\Phi_o'/q)$ = complete elliptic integral. If furthermore $\Phi_o'/q \ll 1$ we obtain[10]

$$\Omega_s \approx \Omega_o\sqrt{1+kN}\,\left(1 - \frac{1}{16}\,\frac{1+kN^3}{1+kN}\,\Phi_o'^2\right)$$

b) $1 + kN^3 < 0,\ 1 + kN > 0$

$$\Omega_s \approx \frac{\pi\Omega_s}{4}\sqrt{-\frac{1+kN^3}{3}}\sqrt{\Phi_o'^2 - q^2}\Big/K\left(\Phi_o'\Big/\sqrt{\Phi_o'^2 - q^2}\right)$$

The special case $1+kN = 0$ gives

$$\Omega_s \approx \frac{\pi\Omega_o}{2}\sqrt{\frac{N^2-1}{6}}\,\frac{\Phi_o'}{K(1/\sqrt{2})}\,,\quad K(1/\sqrt{2}) = 1.8541\ ,$$

which has in this approximation a frequency increasing linearly with amplitude.[12] The spread in frequency is very large in this last case.

Experiments have been carried out with a cavity operating at the 6th harmonic of the RF frequency (f_{RF} = 9.5 MHz) and with $k > 0$. It was driven with a 1 kW amplifier. Thanks to the rather high harmonic ($N = 6$) a large spread was obtained and bunched currents of 130 mA (in 20 bunches) could be stabilized. However, during the stacking cycle the bunch length exceeds the wavelength of this higher harmonic oscillation. In order to have clean conditions we reduced the resonant frequency at the Landau cavity to the 4th harmonic of the RF and operated close to $1+kN \approx 0$. This gives a large spread and leads to bunches with a flat top, see Fig. 5. Experiments to stack large injected currents with this cavity are under way and look promising.

Acknowledgement

We thank G. Dôme, H.G. Hereward, K. Hübner, W. Schnell and B. Zotter (CERN), and G.A. Voss (DESY) for stimulating discussions.

References

1. S. Hansen, M.J. de Jonge, Proc. VIIIth Internat. Conf. on High Energy Accel., CERN (1971), p. 505.
2. W. Schnell, IEEE Trans. on Nucl. Sci. NS-22, No.3, p. 1358 (1975).
3. D. Boussard, CERN-Lab II/RF/Int./75-2.
4. E. Keil, W. Schnell, CERN-ISR-TH-RF/69-48.
5. W. Hardt, private communication.
6. S. Hansen et al., IEEE Trans. on Nucl. Sci. NS-22, No.3, p. 1381 (1975).
7. R.A. Dory, MURA 654 (1962).
8. C. Fischer et al., CERN-ISR-OP/77-18.
9. P. Bramham et al., Proc. IXth Internat. Conf. on High Energy Accel., Stanford (1974), p. 359.
10. R. Avrill et al., Proc. VIIIth Internat. Conf. on High Energy Accel., CERN (1971), p. 301.
11. R. Gram, P. Morton, SLAC TN-67-30.
12. G.A. Voss, Proc. Internat. School of Particle Accelerators, Erice, 1976.

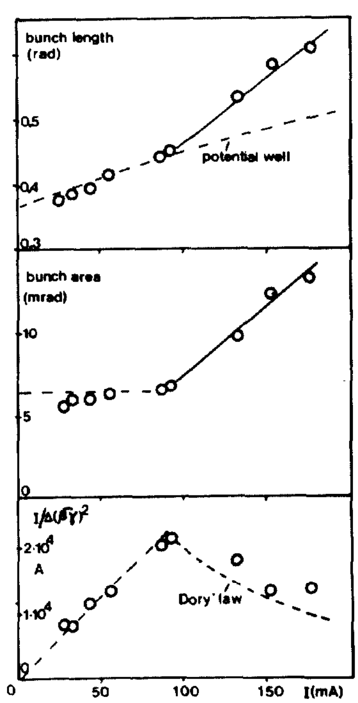

Fig. 1. Bunch length, bunch area and $I/(\Delta\overline{\beta\gamma})^2$ vs. beam current.

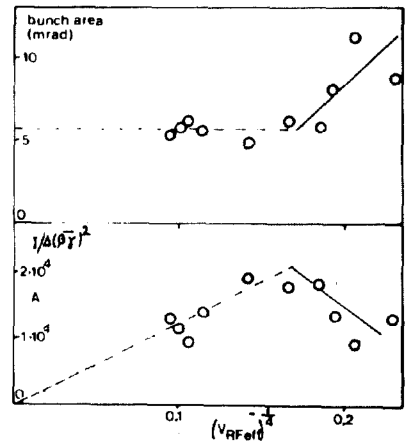

Fig. 2. Bunch area and $I/(\Delta\overline{\beta\gamma})^2$ as a function of RF voltage.

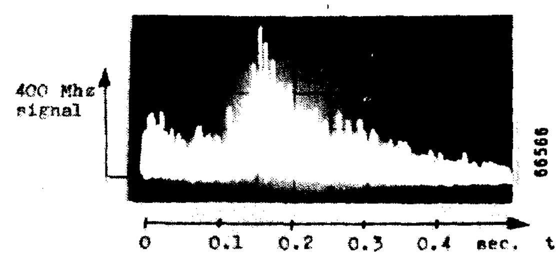

Fig. 3. High-frequency signal appearing during RF voltage decay.

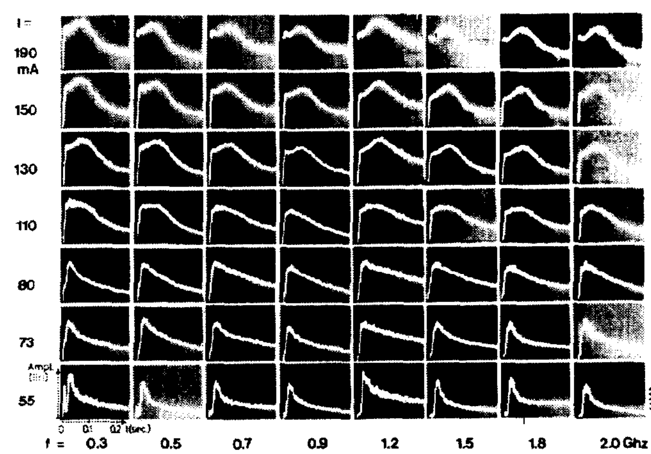

Fig. 4. High-frequency signals for different injected currents.

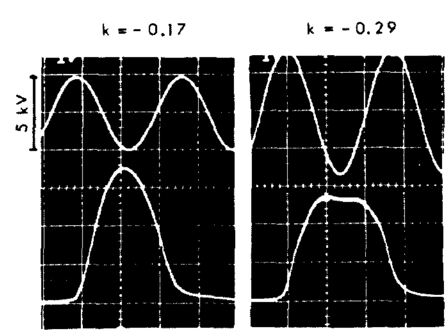

Fig. 5. Bunches (lower trace) stabilized by the higher harmonic voltage (upper trace). The phase of this voltage (not shown correctly on the picture) is chosen to reduce the phase focusing in the bunch centre.