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LONGITUDINAL BUNCH DILUTION DUE TO RF NOISE

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Summary

The effect of phase noise on a tightly bunched proton beam is investigated taking into account the frequency spread in the beam, the wall impedance and the phase feedback loop. Under normal conditions the measured dilution rates in the ISR correspond typically to a doubling of the bunch length in one to a few hours, and are consistent with the measured noise spectra. Amplitude noise is not investigated.

Theory

The phase ϕ_i of a single particle (measured from the RF phase ϕ_c) satisfies the equation

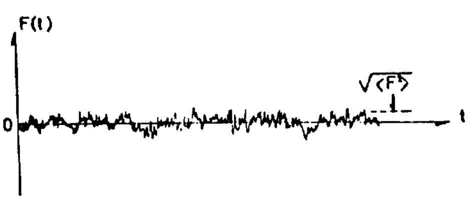
$$\ddot{\phi}_i + \omega_i^2\phi_i = \dot{\Omega}(t) \qquad (1)$$

where $\Omega(t)$ is the frequency deviation from its ideal value (harmonic number times revolution frequency). If frequency noise is present, the particle's amplitude A_i increases with time (see Fig. 1),

$$\frac{d}{dt}\langle A_i^2\rangle = 2\pi G_\Omega(\omega_i) \qquad (2)$$

where $G_\Omega(\omega_i)$ is the spectral density of the frequency error $\Omega(t)$ evaluated at the particle's synchrotron frequency ω_i. The frequency error is determined by the voltage $u(t)$ at the input to the voltage-controlled oscillator or VCO (see Fig. 2),

$$\tilde{\Omega} = k(\tilde{u}_B + \tilde{u}_e) = \beta(\tilde{\phi}_B + \tilde{\phi}_e) \qquad (3)$$



The rms value of F(t) is related to the spectral density $G_F(\omega)$ by

$$\langle F^2\rangle = \int_{-\infty}^{\infty} G_F(\omega)\, d\omega \quad ,$$

and if F(t) is filtered,

$$G_F = \langle F^2\rangle/2\Delta\omega$$

where $\Delta\omega$ is the effective noise bandwidth of the filter.

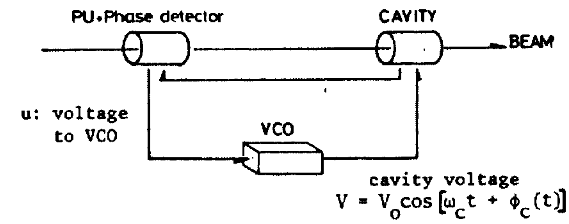
For a harmonic oscillator driven by F(t),

$$\ddot{x} + \omega_s^2 x = F(t) \quad ,$$

the amplitude $A = \sqrt{x^2 + (\dot{x}/\omega_s)^2}$ increases as

$$\frac{d}{dt}\langle A^2\rangle = \frac{2\pi}{\omega_s} G_F(\omega_s) \quad .$$

Fig. 1. Noise definitions.[1]



Ω = frequency error = $\dot{\phi}_c$ = ku
where k = $2\pi \times 10^3$ rad/Vs (= 1 Hz/mV)

u = $u_B + u_e$ = $M(\phi_B + \phi_e)$
where M is typically 0.6 V/radian

ϕ_B = bunch phase relative to bucket

ϕ_e = noise (referred to input of VCO)

Fig. 2. Phase loop.[2]

where the tilda indicates the Fourier transform and $\beta \equiv kM$ is typically $2\pi\times600$ rad/s. The noise voltage u_e (or phase noise ϕ_e) has contributions from the phase detector, VCO, and the magnetic field ripple. One can also add the term $2\omega\Delta\omega\phi_B$ to the RHS of (1), where $\Delta\omega$ is the coherent shift in synchrotron frequency due to coupling impedances. For the ISR, Re $\Delta\omega$ is typically $2\pi\times10$ rad/s due to inductive wall and space charge, while Im $\Delta\omega$ has a small damping contribution from the RF cavities (Robinson damping) plus perhaps antidamping contributions from other impedances. Thus (3) can be written as

$$\tilde{\Omega} = (\beta - 2j\Delta\omega)\tilde{\phi}_B + \beta\tilde{\phi}_e \quad . \qquad (4)$$

Finally, the bunch phase ϕ_B is given by the sum

$$\tilde{\phi}_B = \frac{1}{N}\sum_i \tilde{\phi}_i \qquad (5)$$

over single-particle phases, and therefore

$$\tilde{\phi}_B = j\,\frac{\tilde{\Omega}}{N}\sum_i \frac{\omega}{\omega_i^2 - \omega^2} \equiv j\tilde{\Omega}D^{-1}(\omega) \qquad (6)$$

where $D^{-1}(\omega)$ is related to the usual bunched-beam dispersion integral[3] (Fig. 3).

From the loop equations (3), (4) and (6) we find

$$\tilde{\phi}_B = \frac{j\beta\tilde{\phi}_e}{D(\omega) - (j\beta + 2\Delta\omega)} \rightarrow \frac{j\omega\beta\tilde{\phi}_e}{\omega_s^2 - \omega^2 - \omega(j\beta + 2\Delta\omega)} \qquad (7)$$

and

$$\tilde{u} = \frac{D(\omega)\tilde{u}_e}{D(\omega) - (j\beta + 2\Delta\omega)} \rightarrow \frac{(\omega_s^2 - \omega^2)\tilde{u}_e}{\omega_s^2 - \omega^2 - \omega(j\beta + 2\Delta\omega)} \qquad (8)$$

where the limits apply when all particles have the same synchrotron frequency, $\omega_i \equiv \omega_s$. In this limit, the feedback reduces the voltage $\tilde{u}$ to zero at the synchrotron frequency ω_s, regardless of the noise $\tilde{u}_e$, and since $G_\Omega(\omega_s) = k^2 G_u(\omega_s) = 0$, the particle amplitudes do not grow.[4]

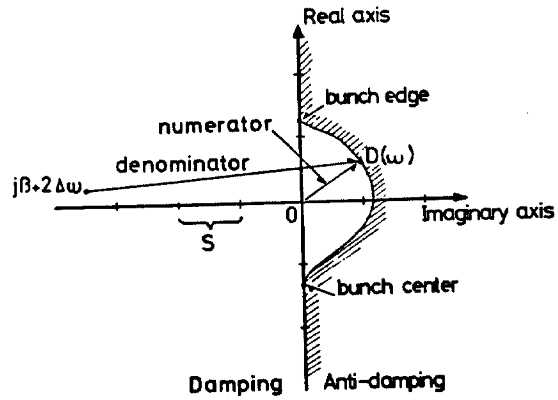


Fig. 3. The curve $D(\omega)$ follows the real axis except for values of ω within the band of incoherent synchrotron frequencies ω_i.

In general, the response $\tilde{u}(\omega)/\tilde{u}_e(\omega)$ at frequency ω is given by the ratio Numerator/Denominator indicated in Fig. 3. Here S is the total spread in synchrotron frequencies from bunch centre to bunch edge. As ω runs along the curve $D(\omega)$ (stability boundary) in Fig. 3, the curves in Figs. 4 and 5 are traced out, with a dip at the synchrotron frequency. The minimum value of $\tilde{u}$ in the dip is typically

$$\tilde{u}(\omega_i) \approx \frac{S}{\beta}\,\tilde{u}_e \tag{9}$$

since $|D(\omega_i)| \sim S$ within the band of single-particle synchrotron frequencies ω_i, and $\Delta\omega$ can be neglected in comparison with β. Therefore we expect

$$G_\Omega(\omega_i) = k^2 G_u(\omega_i) \approx k^2\,\frac{S^2}{\beta^2}\,G_e \tag{10}$$

where G_e is the spectral density of the noise voltage $u_e(t)$, which is assumed to be white noise.

For completeness, the rms bunch oscillation is given by (from eq. 7 and neglecting $\Delta\omega$)

$$\langle\phi_B^2\rangle = \int_{-\infty}^{\infty} \frac{\omega^2 k^2 G_e(\omega)}{(\omega_s^2-\omega^2)^2+\omega^2\beta^2}\,d\omega \tag{11}$$

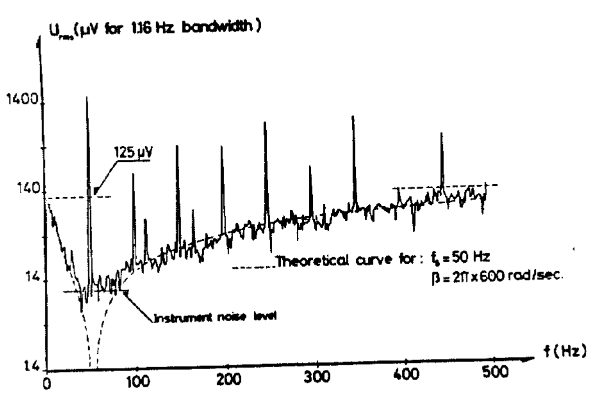


Fig. 4. Voltage at the input to the VCO and theoretical curve for no spread (S = 0). Measured curve averaged 16 times.

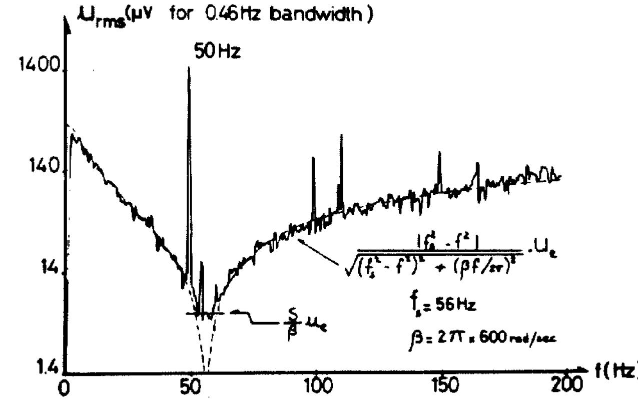


Fig. 5. Measured voltage with white noise added, and theoretical curve.

where for white noise, G_e can be removed from the integral, with the result

$$\langle\phi_B^2\rangle = \frac{\pi}{\beta}\,k^2 G_e\ . \tag{12}$$

Measurements

The noise voltage u_{rms} at the input to the VCO was measured with an FFT (fast Fourier transform) spectrum analyser (Figs. 4 and 5). There is good agreement with the theoretical values (eq. 8 with no spread), although the expected dip at the synchrotron frequency is normally below the instrument noise level. From Fig. 4 one finds $(u_e)_{rms} \approx 125\ \mu V$ in a bandwidth of 1.16 Hz, and this quantity squared and divided by twice the bandwidth in rad/s (see Fig. 1) gives the noise spectral density, $G_e = 1.07 \times 10^{-9}$ in $(\text{volts})^2/(\text{rad/s})$.

White noise was added (160 mV in 24 kHz bandwidth) to raise the dip at the synchrotron frequency to about the instrument noise level (Fig. 5). The measured value of S/β in this case is $\sim 1/80$, while the expected value is about 1/200 since $S \approx 2\pi\times 3$ and $\beta = 2\pi\times 600$ rad/s. This is reasonable agreement considering the uncertainty in the measurements. Finally, using eq. (10) with the measured value $S/\beta = 1/80$ and the spectral density $G_e = 1.07 \times 10^{-9}\ (\text{volts})^2/(\text{rad/s})$, we find

$$G_\Omega(\omega_i) \approx 6.6 \times 10^{-6}\ \text{rad/s}$$

for the normal loop gain, with $S \approx 3$ Hz and no noise added. Under these conditions, the dilution rate for a typical particle is expected to be (from eq. 2)

$$\frac{d}{dt}\langle A_i^2\rangle \approx 4.2 \times 10^{-5}\ \text{rad}^2/\text{s}$$

for phase noise. The measured dilution rates are usually somewhat faster, but generally within a factor of two of this expected value. The rms bunch oscillation from eq. (12) is 0.06 RF radian or 3.4°.

To measure the longitudinal dilution of bunches we work with only one injected bunch or operate with a higher harmonic cavity to avoid coupled bunch mode instabilities. The bunch is observed on a scope and its half-width A_h at half height measured in RF phase angle as a function of time (Fig. 6). The quantity $A^2{}_h$

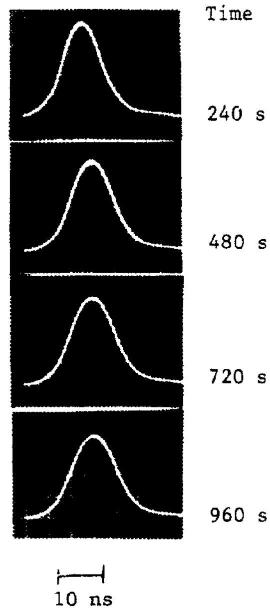


Fig. 6. Bunch dilution under normal conditions.

$$\frac{d^2A_h^2}{dt} = 3.7 \times 10^{-5}\,\mathrm{rad^2s^{-1}}$$

is then plotted against time and fitted by a straight line which gives the dilution rate dA_h^2/dt from its slope. Fig. 7 shows a particular example of such a measurement. The results of many such measurements are shown in Table 1 for different rms bunch lengths $\sqrt{\overline{<A_h^2>}}$, phase oscillation frequency spread S_h (measured at half height) and phase loop gain η ($\eta = B/\omega_s$, normal value $\sim$10). We also varied the beam current per bunch (2-6 mA), the RF voltage (8-16 kV) and the beam energy (11-26 GeV) without seeing any significant effect.

The dilution rate does not seem to depend on the spread S_h (as long as this is small) or on the gain η. The average dilution rate from the first group of measurements in Table 1 taken under "normal" conditions gives

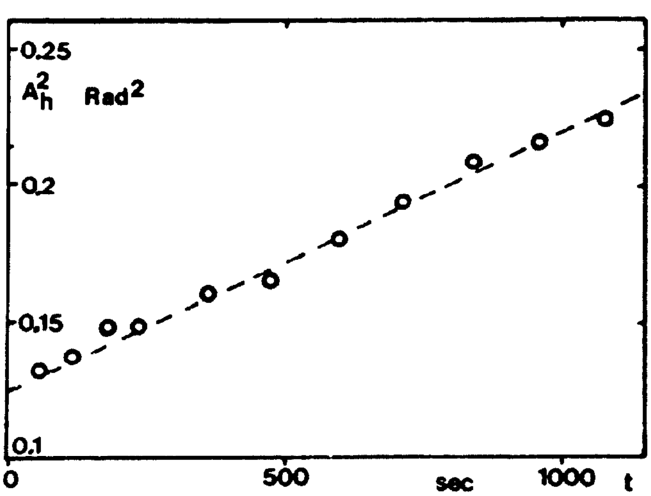


Fig. 7. Increase of the bunch length A_h with time

Table 1

bunch length $\sqrt{\overline{<A_h^2>}}$ rad	spread S_h rad/s	loop gain η	injected noise mV	dilution dA_h^2/dt $\mathrm{rad^2s^{-1}}$
0.54	6.5	10	-	1.7 10^{-5}
0.32	2.4	10	-	5.2 10^{-5}
0.40	3.6	1	-	2.4 10^{-5}
0.45	4.6	10	-	3.8 10^{-5}
0.44	4.3	10	-	2.8 10^{-5}
0.44	3.0	0.1	-	8.4 10^{-5}
0.54	6.1	0.1	-	1.6 10^{-5}
0.48	5.1	10	-	14.0 10^{-5}
0.53	small	10	-	5.1 10^{-5}
0.46	9.3	10	-	8.5 10^{-5}
0.44	2.1	10	-	8.5 10^{-5}
0.45	4.5	$\sim$100	-	8.1 10^{-5}
0.47	4.9	1	-	5.3 10^{-5}
0.41	4.0	10	-	9.5 10^{-5}
0.37	3.1	10	-	4.6 10^{-5}
0.42	4.2	10	160-(0-24)[a]	7.9 10^{-5}
0.37	3.1	10	160-(0-0.4)	2.7 10^{-5}
0.35	2.7	10	160-(0.2-24)	3.1 10^{-5}
0.41	$\sim$18.8[b]	10	-	11.2 10^{-5}
0.49	$\sim$74.0	10	-	37.0 10^{-5}
0.47	$\sim$66.0	10	-	25.0 10^{-5}
0.45	$\sim$61.0	10	-	31.0 10^{-5}
0.36	$\sim$21.0	10	-	41.4 10^{-5}
0.41	$\sim$28.0	10	-	42.8 10^{-5}
1.30	$\sim$30.0	10	-	$\sim$0
1.22	$\sim$37.0	10	-	3.7 10^{-5}

[a] rms voltage for indicated frequency band (kHz).

[b] Higher harmonic cavity was used to increase the spread.

$$dA_h^2/dt \approx 6 \times 10^{-5}\ \mathrm{rad^2s^{-1}}$$

with a statistical error of about 60%. In some cases we injected noise into the phase lock loop (next group in Table 1). With small noise the effect on the dilution rate is hardly observable. If the injected noise is too large we start to lose particles out of the tails of the bucket. It is difficult to describe this effect quantitatively. Finally in the last group we list some measurements with large spread S_h which have large dilution rates as expected.

References

1. H.G. Hereward and K. Johnsen, CERN 60-38 (1960).

2. S. Hansen and M.J. de Jonge, Proc. VIIIth Internat. Conf. on High Energy Accelerators, CERN (1971), p. 505.

3. F. Sacherer, 1973 Particle Accelerator Conf. (IEEE Trans. Nucl. Sci. NS-20, 825, 1973).

4. E. Hartwig, V.K. Neil and R.K. Cooper, 1973 Particle Accelerator Conf. (IEEE Trans. Nucl. Sci. NS-20, 833, 1973).