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STRUCTURE RESONANCES IN PROTON LINEAR ACCELERATORS

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1. In high-current proton linear accelerators aimed for studying the mean energy particle physics (meson factories) the particle losses during the acceleration process must be very small because of the radiation safety[1].

The particle losses in linear accelerators are mainly due to small random perturbations of the focusing channel elements slowly increasing the transverse oscillation amplitude. The particle losses can be also considerably contributed by the regular perturbations of focusing forces with the frequency multiple to that of the focusing structure Ω. The oscillation amplitude increases in this case at definite resonant values of transverse oscillation frequences Ω_x, Ω_y connected by the known relation with Ω:

$$n\Omega_x + m\Omega_y = K\Omega \tag{1}$$

If one introduces the dimensionless frequences $\nu_{x,y}$ instead of frequences $\Omega_{x,y}$, the relation (1) takes the form

$$n\nu_x + m\nu_y = K \tag{2}$$

In (2) n + m is the resonance order, K is the number of the resonant perturbation mode.

It is evident that in the linear accelerators we may choose the working point only in the first stability region, i.e. $\nu_x < 1/2$ and $\nu_y < 1/2$, thus we shall be interested only in the structure resonances on the nonlinearities of the lense field and of the eigen field of the beam.

The lense magnetic field accounting for the pole symmetry has the form

$$H_y = G(t)x[1 + a_5(x^4 - 10x^2y^2 + 5y^4)a^{-4} + \ldots] \tag{3}$$

The expression for H_x is obtained from (3) by replacing X by Y. In (3) G(t) is the lense magnetic field gradient, $2a$ is the diameter of the focusing channel aperture, a_5 is one of the coefficients of nonlinearity of magnetic field.

Let us consider a one-dimensional model of the beam believing the particle losses at a given place of the focusing channel to occur in the direction in which the envelope of the beam is maximal, the n-order nonlinearity determines the resonance $\nu_{res} = K/(n+1)$ n may take, according to (3), the values 5,9 etc.

The structure resonances on the nonlinearity of the eigen field (Coulomb resonances) result from the fact that the beam envelope and consequently, the space charge force repeat regularly with a frequency of the focusing structure.

The expression for E_c inside the beam within the given model has the form ($G_c(t)$ is the gradient of the field E_c, $2b$ - is the beam diameter):

$$E_c = G_c(t)x\left[1 + \sum_{\ell=1}^{\infty} c_{2\ell}\left(\frac{x}{b}\right)^{2\ell}\right] \tag{4}$$

Coulomb resonances may be excited at frequences $\nu = K/2(\ell+1)$. In the range of ν -values from 0.1 up to 0.2 which is characteristic for proton linear accelerators, the nonlinear resonances correspond to the values ν_{res} 1/6, 1/8, 1/10, 2/10 etc. The width of these resonances is, as a rule, small, so the working point on the plane of transverse oscillation frequences may be chosen sufficiently far from the resonances from (2) which could be nondesirable at permissible level of the particle losses. In the strong-focusing channel the optimal value of ν is near 0.2 but at the beginning of the acceleration of protons (low $\beta = v/c$) the values of ν are practically not more than 0.1. At acceleration ν gradually increases, thus the beam passes through the structure resonance with inevitable loss of a part of the beam.

To estimate the number of lost particles ΔN_{res} let us introduce the particle distribution function in the phase space of transverse oscillations F(u,t), $\int du F(u,t) = 1$, $u = A^2/A^2_{max}$, A is the oscillation amplitude.

Let Δu_{res} is the maximal u increase when the beam passes through the resonance, then the approximate expression for $\Delta N_{res}/N$ is of the form

$$\frac{\Delta N_{res}}{N} \cong \frac{1}{4} \int_{1-\Delta u_{res}}^{1} du\, F(u,t) \qquad (5)$$

The coefficient 1/4 approximately takes into account the oscillation phase particle distribution at the moment of passing through resonance. Analysing the nonlinear perturbations of transverse oscillations [2], [3] by the averaging method, we get an approximate expression for small Δu_{res}:

$$\Delta u_{res} \cong \frac{g_{kn}}{\sqrt{n \frac{d\nu}{dM}}} \left(1 + \frac{n-1}{2} \frac{g_{kn}}{\sqrt{n \frac{d\nu}{dM}}} \right)^{-1} \qquad (6)$$

$$g_{kn} = \left(\frac{1}{2}\right)^{n-1} \frac{\sin \pi k \xi}{\pi k} |f|^2_{max} \left(\frac{4\pi\nu}{\xi} a_{n-1} + \frac{8\lambda^2 e I_{max}}{\beta \gamma^3 r^2 mc^3} C_{n-1} \right) \qquad (7)$$

In (6) and (7) g_{kn} is the resonance intensity ($\nu_{res} = k/n$),

ξ is the lense length relative to the focusing period length,

M is the number of periods of the focusing structure,

λ - is the accelerating field wave length,

I_{max} - is the maximal peak current of the beam

$|f|$ - beam envelope (Flocket function modulus),

r - is the effective radius of the beam, (r, the linear density of the beam λ_c and gradiant G_c are connected by relation $G_c = 2\lambda_c / r^2$).

The character of the time variation of function F(u,t) is determined by two processes: 1) F(u,t) slowly varies under the action of small random perturbations of focusing forces 2) F(u,t) rapidly varies when passing the structure resonance. During the acceleration process the time interval between passing of two successive structure resonances is large enough, so we put that at the moment of approach to the next resonance the evolution of F(u,t) is conditioned by random perturbations of the focusing channel elements. This means that F(u,t) obeys the Einstein-Fokker equation

$$\frac{\partial F}{\partial t} = -\frac{\partial}{\partial u}(B_1 F) + \frac{1}{2}\frac{\partial^2}{\partial u^2}(B_2 F) \qquad (8)$$

with the boundary and initial conditions:

$F(1,t) = 0, \quad F(u,0) = F_0(u)$

In (8) $B_1 = \frac{d\overline{\Delta u}}{dt}$, $B_2 = \frac{d\overline{\Delta u^2}}{dt}$, $\overline{\Delta u}, \overline{\Delta u^2}$ are correspondingly the mean increase and the mean square increase of u due to the random perturbations of the focusing forces.

In the high-current linear accelerators of protons the particle losses must be small [1] and the most part of particles is far from the absorbing wall u=1, thus the boundary conditions may be given on ∞. The latter assumption seems to overestimate the losses ΔN_{res}, but it enables one to get expression for ΔN_{res} having a simple physical meaning. If one takes $F_0(u) = \delta(u)$ as an initial condition, then F(u,t) will be of the form [3]

$$F(u,t) = \frac{1}{\overline{\Delta u}(t)} \exp\left(-\frac{u}{\overline{\Delta u}(t)}\right) \qquad (9)$$

The particle losses due to the random perturbations up to t_0 will be ($\Delta u_{stoch} = \Delta u(t_0)$):

$$\frac{\Delta N_{stoch}}{N} = \exp\left(-\frac{1}{\Delta u_{stoch}}\right) \qquad (10)$$

and the particle losses when passing through the resonance at t_0 will be

$$\frac{\Delta N_{res}}{\Delta N_{stoch}} = \frac{1}{4}\left[\exp\left(\frac{\Delta u_{res}}{\Delta u_{stoch}}\right) - 1\right] \qquad (11)$$

3. Let us consider as an example the beam passing through the structure resonance ν_{res} =1/6 in the first section of the linear accelerator of protons for a meson factory [1] (λ =1.5 m, M=100, a =1.5 cm, ξ =0.25, I_{peak}=50 mA, I_{max}=400mA). We put that ν varies evenly from 0.1 at injection up to 0.2 at the end of acceleration, $\frac{d\nu}{dM}$ =10^{-3} and the resonance ν =1/6 is passed at β = 0.3, a_5 is connected with the lense magnetic field nonlinearity tolerance $\frac{\Delta H}{H}$ which is usually given at a distance of 0.7a from the lense axis; at $\frac{\Delta H}{H}$ =10^{-2}, $a_5 \cong 5 \cdot 10^{-2}$.

With the account of the parameters of the focusing channel and of the beam (with F(u,t) from (9) $r = a\sqrt{\Delta u_{stoch}}$, $C_{2\ell} = \frac{(-1)^\ell}{(\ell+1)!} \Delta u^\ell_{stoch}$) for the resonance intensity g_{16} we get

$$g_{16} \cong 0{,}15\, a_5 + 10^{-5} (\Delta u_{stoch})^{-3} \qquad (12)$$

Table

$\frac{\Delta N_{stoch}}{N}$	Δu_{stoch}	Δu_{res} I	$\frac{\Delta N_{res}}{\Delta N_{stoch}}$	Δu_{res} II	$\frac{\Delta N_{res}}{\Delta N_{stoch}}$	$\frac{\Delta N_c}{\Delta N_{stoch}}$
10^{-5}	0,09	0,16	1,3	0,18	1,7	3,0
10^{-4}	0,11	0,12	0,5	0,15	0,75	0,6
10^{-3}	0,14	0,09	0,2	0,12	0,35	0,2
10^{-2}	0,22	0,07	0,1	0,11	0,15	0,02

I $\frac{\Delta H}{H} = 7.10^{-3}$

II $\frac{\Delta H}{H} = 1,4.10^{-2}$

The table gives relative losses due to the passing the structure resonance ν =1/6, $\Delta N_{res}/\Delta N_{stoch}$. Separately, the table gives the losses due to the Coulomb nonlinearity ΔN_c on two resonances ν =1/8, ν =1/6 at the ideal field in the lenses.

Before we analyse the calculation results it should be emphasized that the assumed model of the particle distribution is valid when the coherent oscillations of the beam transform into noncoherent. Let us also notice that the behaviour of transverse oscillation frequences should be chosen so that either to exclude the passing through the resonance ν =1/6 at all or to pass it fastly at the beginning of the acceleration when the particle energy is yet small, as the structure resonances of higher order in n give a smaller contribution to the particle losses.

It follows from the table that the particle losses due to the passing through nonlinear resonances depend on the value of nonlinearity of the focusing fields at a given aperture of the channel. In practically interesting cases these losses are comparable in their value with those due to stohastic processes resulting from the errors of elements of the focusing channel.

Note that at the given current in the pulse the particle losses in the passing through the Coulomb structure resonances comprise the 10^{-4} even at very hard tolerances to the regular nonlinearity of magnetic lenses and to the random perturbations of the focusing forces. This is due to the fact that in the high-current accelerators of protons the lower limit for the particle losses in passing through the structure resonances is determined by the nonlinearity of the eigen field of the beam.

References

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