

A METHOD FOR INCREASING THE MULTITURN INJECTION EFFICIENCY IN AG PROTON SYNCHROTRONS BY MEANS OF SKEW QUADRUPOLES

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Summary

An increase of the injection efficiency due to linear coupling [already known at the Cosmotron in 1953¹⁾] is shown to work in AG Proton Synchrotrons for $Q_H - Q_V = p$. The system is in operational use at the CERN Proton Synchrotron Booster (PSB) to reach intensities above 10^{13} ppp. An intensity increase of 20% is achieved at the expense of a slight vertical blow-up, which is however not noticeable for high-intensity beams as their emittance is already increased because of an integer stop-band. In this paper a comprehensive model is presented, which describes the efficiency as a function of several parameters, such as the coupling strength, injection geometry and Q-values. Provided enough vertical acceptance is available, the same scheme may be profitable for other accelerators using betatron stacking for $|Q_H - Q_V - p| < 0.1$; $\epsilon_H \gg \epsilon_V$.

Introduction

The mechanism leading to an increase of (horizontal) betatron stacking efficiency by skew quadrupoles comes down to this: horizontal oscillation energy is transferred to the vertical plane in the first few revolutions via linear coupling. The probability of missing the injection septum is thus increased for a significant part of the beam. This process has been found to work for strong focusing synchrotrons around $Q_H - Q_V = 0$ in the PSB²⁾. After separation of the Q-values by an integer, for various reasons³⁾, Gareyte suggested that this technique may also work around $Q_H - Q_V = -1$. Indeed, similar effects were observed on both difference lines, with a strong dependence on the phase of the 1st harmonic skew quadrupole component in the latter case. As "skew injection" (CERN PS jargon for this technique) seemed promising for reaching higher beam intensities, a more detailed study was made. This paper is a short version of a CERN internal report on the subject⁴⁾.

Linear Coupling

Skew injection needs a working point near a linear difference resonance and a set of skew quadrupoles for the generation of the relevant harmonic. (Longitudinal magnetic fields, though a possible alternative, are not considered here.) The reasoning of a recent paper⁵⁾ on the single particle motion with linear coupling around $Q_H - Q_V = k + \delta$, k integer, $\delta \ll 1$, will be used. The Courant-Snyder transformation is referred to the equations of motion which include the coupling field. Retaining only the k th harmonic coupling term $Q_C \exp(iq)$, substituting the unperturbed solution into these equations, and discarding fast amplitude variations, yields the following relations describing the motion in both transverse phase planes:

$$I_x = I_{x_0} + \frac{4Q_C \sin \Theta}{\delta^2 + 4Q_C^2} \left\{ \left[Q_C (I_{z_0} - I_{x_0}) + \delta \sqrt{I_{x_0} I_{z_0}} \cos \phi \right] \sin \Theta + \sqrt{I_{x_0} I_{z_0}} (\delta^2 + 4Q_C^2) \sin \phi \cos \Theta \right\} \quad (1)$$

$$I_x + I_z = \text{const} \quad (2)$$

$$\Theta = \sqrt{\delta^2 + 4Q_C^2} \theta \quad d\theta = \frac{ds}{R} \quad (3)$$

$$\phi = \phi_{x_0} - \phi_{z_0} - q. \quad (4)$$

I_x and I_z are the instantaneous values of the Courant-Snyder "invariant" (area/ π enclosed by space-plane trajectory of particle considered, not invariant in this

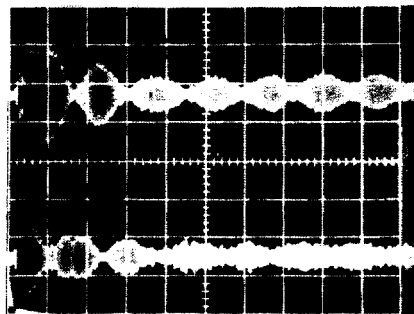


Fig. 1 Linear coupling observed on a position monitor (50 $\mu\text{sec/div}$) for $\delta = 0.004$, $Q_C = 0.01$. A full beating period lasts 95 machine revolutions.

case) with initial values I_{x_0} and I_{z_0} , respectively. ϕ_{x_0} and ϕ_{z_0} are the initial phases of the particle on these trajectories and R is the machine radius. The coupling term Q_C is a dimensionless quantity proportional to the skew-quadrupole strength. Equation (2) defines the constant of motion for linear coupling. The skew quadrupole phase term q is essential for the injection process.

Figure 1 shows the presence of coupling near $Q_H - Q_V = -1$. A single kick excites the beam in the vertical plane. Coupling leads to beating oscillations in both planes.

Skew Injection Mechanism

Equation (1) reveals the basic features of skew injection. The horizontal oscillation amplitude I_x decreases during the first few revolutions around the synchrotron if certain conditions are fulfilled.

In the case of a beam with zero vertical emittance, perfectly aligned with the vertical closed orbit, Eqs. (1) and (3) reduce to:

$$\Delta I_x = I_x - I_{x_0} = - \frac{4Q_C^2}{\delta^2 + 4Q_C^2} \sin^2 \Theta I_{x_0} \quad (5)$$

with

$$\Theta = \sqrt{\delta^2 + 4Q_C^2} \times 2\pi Q_H n.$$

$(\delta^2 + 4Q_C^2)^{1/2}$ is proportional to the beating frequency, a critical parameter of the process, and n is the number of machine revolutions after injection.

Leaving all other parameters unchanged, an increased I_{x_0} yields a large ΔI_x ; thus a beam slice with larger initial amplitude moves faster away from the septum. A simplified injection model suggests that for a given slice, the heaviest loss at the injection septum occurs after n_{crit} revolutions, with n_{crit} determined by the Q_H value. For skew injection δ and Q_C are chosen such that after n_{crit} revolutions ΔI_x is very different from zero.

Taking into account a finite vertical beam size and non-perfect vertical injection steering, reasonable estimates of the injected current can be obtained from the following model. A partial acceptance, defined by three or four "cuts" (loss boundary created by injection septum), is determined for each slice of incoming beam as described in a companion paper⁶⁾. The powering of the skew quadrupoles defines a multitude of partial acceptances linked to points in the vertical phase plane. Given the initial conditions of a particle in both phase

planes, one can predict whether or not it will hit the injection septum during the process. To avoid rather lengthy calculations, the mean movement of a point sitting on a septum cut is computed by an integration over all particles in the vertical phase plane, assuming a Gaussian density distribution

$$P(I_z) = (1/\bar{I}_z) \exp(-I_z/\bar{I}_z). \quad (6)$$

A further simplification consists in studying the motion of the cut's centre, with the assumption that this point is representative for the whole septum cut. The resulting change for a cut after i revolutions is

$$\Delta I_x = w_1 \left\{ \bar{I}_z + I_{z_0} - \bar{I}_{x_0} + \sqrt{I_{x_0} I_{z_0}} (w_2 \cos \phi_0 + w_3 \sin \phi_0) \right\} \quad (7a)$$

$$w_1 = \frac{4Q_c^2 \sin^2 \Theta}{\delta^2 + 4Q_c^2}, w_2 = \frac{4Q_c \delta \sin^2 \Theta}{\delta^2 + 4Q_c^2}, w_3 = \frac{4Q_c \sin \Theta \cos \Theta}{\sqrt{\delta^2 + 4Q_c^2}} \quad (7b)$$

$$\phi_0 = \phi_{x_0} - \phi_{z_0} - q,$$

where I_{z_0} and ϕ_{z_0} are the coordinates of the incoming beam centre in the vertical phase plane. Comparison with results coming from simulation programs⁷⁾ shows that this formula approximates well the intensity change of the injected beam caused by linear coupling.

Typical behaviour of skew injection computed by this approach for the PSB -- as a function of Q-split, strength of skew quadrupoles and vertical mis-steering -- is shown in Fig. 2. Losses due to vertical aperture limitation are not considered. Even for small vertical misalignments, appreciable changes in injected current are predicted.

Choice of Q-values; Vertical Emittance Increase

Results presented in Fig. 2 suggest that an optimum intensity gain is obtained with $\delta = 0$ (sitting right on the coupling line). However, Eq. (5) shows that in this case $[\Delta I_x]_{\max} = I_{x_0}$, and hence with Eq. (2) $[I_z]_{\max} = I_{x_0}$, the beam is round. As skew injection cannot betray Liouville's theorem, an improvement of the injected intensity has to be paid for with an increase of the vertical emittance ϵ_V .

Since the loss occurring when a slice of incoming beam returns to the injection septum after n_{crit} machine revolutions determines the injection efficiency, the best compromise between intensity gain and emittance increase is achieved when the injection parameters are adjusted such that I_x reaches a minimum just at the revolution n_{crit} (the point is that any unnecessary decrease of I_x results in an excessive enlargement of ϵ_V). Data for the vertical emittance increase (containing 95% of particles), computed for various values of δ and Q_c are summarized in Table 1 (with $\epsilon_{H_0} \approx 3\epsilon_{V_0}$):

Table 1
Emittance increase ratio $\epsilon_V/\epsilon_{V_0}$
as a function of Q_c and δ

$Q_c \backslash \delta$	0	0.03	0.06	0.09
0.01	2.15	1.40	1.13	1.06
0.02	2.15	1.78	1.40	1.22
0.03	2.15	1.95	1.63	1.38

The permissible vertical blow-up naturally depends on items like (i) the machine acceptance; (ii) the machine user's requirements; (iii) the blow-up the beam suffers from other causes. Typically $\epsilon_V/\epsilon_{V_0}$ can be kept below 1.40 if $\delta - 3Q_c > 0$.

Application to the CERN PS Booster

A few relevant features of the PSB injection scheme are as follows: up to 15 turns (lasting 25 μsec) of 80 to 90 mA incoming Linac beam (50 MeV; transverse 95% emittances $30 \times 10^{-6} \pi \text{ rad m}$ in both planes) are injected into each of the four rings. Four kicker magnets cause a local closed orbit deformation at the septum position. The deformation goes to zero linearly within $\sim 60 \mu\text{sec}$. Injection is stopped when the nominal horizontal emittance is reached (50 MeV emittances $\epsilon_H = 130$, $\epsilon_V = 40$, machine acceptances $A_H \approx 250$, $A_V \approx 95 \times 10^{-6} \pi \text{ rad m}$). Non-linear resonances, resistive wall instabilities, and space-charge effects³⁾ seriously affect beam quality on a working point near the main coupling line $Q_H - Q_V = 0$.

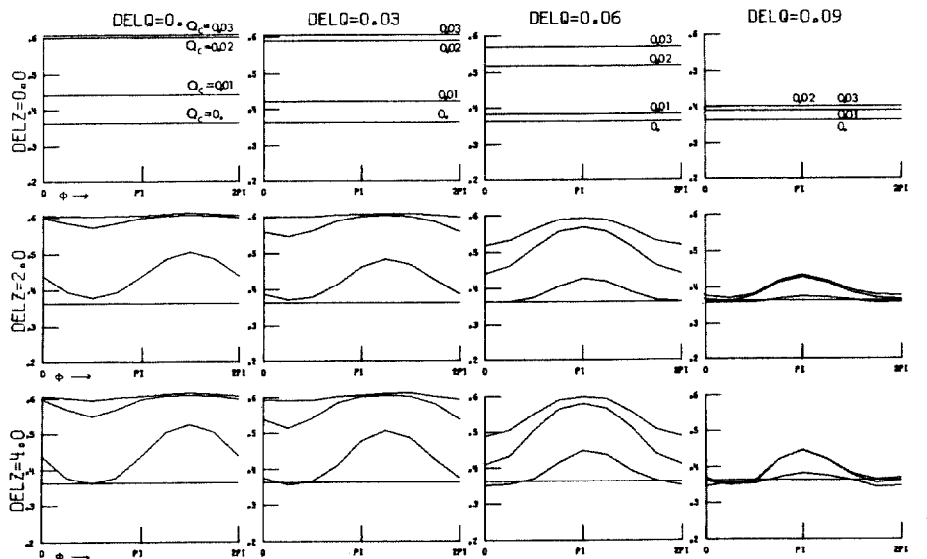


Fig. 2 Calculated multiturn injection efficiency (vertical axis) as a function of Q split DELQ, vertical mis-steering DELZ, and skew quadrupole's first harmonic phase in rad (horizontal axis) for coupling term values $Q_c = 0.01, 0.02, 0.03$ in the PSB.

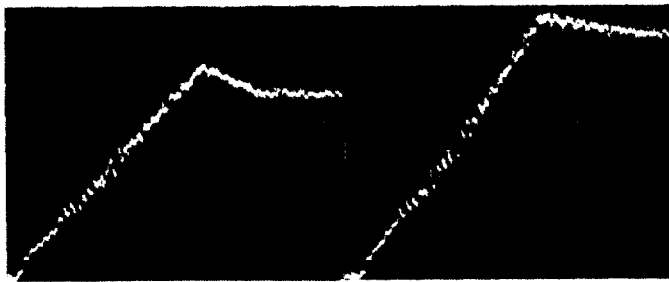


Fig. 3 Current build-up during 13-turn injection of a low-density beam (a) without and (b) with skew quadrupoles. (Vert. scale: 20 mA/div.; hor. scale: 5 μsec per div.)

Improved performance figures have been found by moving the working point to $(Q_H, Q_V) \approx (4.20, 5.30)$. However, the transverse beam density ($\sim 4.8 \times 10^{12}$ protons per ring at injection) is such that the betatron tune is moved downwards by almost half an integer, far beyond $Q_V = 5$, so the beam is blown up vertically by $\sim 50\%$ during RF trapping (it is not lost, as sufficient vertical acceptance is available). The main point of skew injection in the PSB is that higher injection efficiency can be obtained at almost no cost in vertical emittance, as the unavoidable increase of ϵ_V due to linear coupling is masked by the effect of the integer stop-band.

Typical zero-intensity tunes at injection are $(Q_H, Q_V) = (4.24, 5.34)$, so $\delta = 0.1$. Space charge lowers Q_V more than Q_H (because the beam is flat), hence the Q -split δ , relevant for skew injection, is a function of the particle's coordinates in the phase planes, and takes values between 0.05 and 0.08. A first harmonic skew quadrupole component can be generated by the four-lens arrangement originally foreseen for the compensation of the stop-band $Q_H + Q_V = 9$. Each lens has an integrated strength of 6×10^{-4} T/A. With $\sqrt{\beta_H \beta_V} = 8.2$ at the lens position, the relevant coupling constant Q_C is 0.03 at 50 MeV, provided all four lenses are powered with ± 30 A.

The efficiency increase for a low-density beam (with negligible space-charge effects) is illustrated in

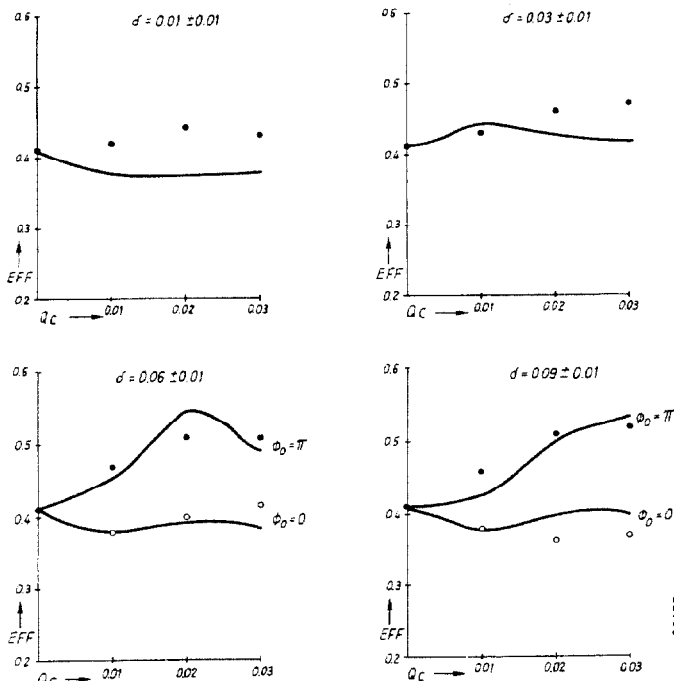


Fig. 4 Injection efficiency versus δ and Q_C measured with a low-density beam. Full lines are expected by theory. Note the dependence on skew quadrupole phase.

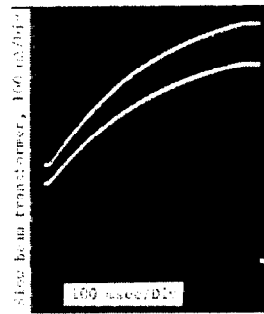


Fig. 5 PSB high intensity acceleration in ring 3 (top) with and (bottom) without skew injection.

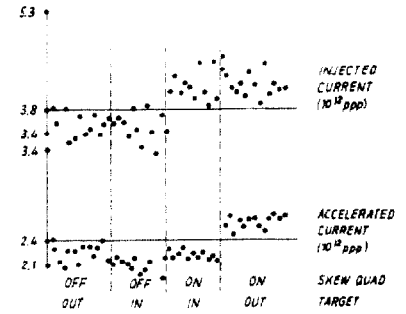


Fig. 6 Vertical density with skew injection (beam intensity, 12 pulses for each setting).

Fig. 3, showing the instantaneous beam intensity during the process, for $\delta = 0.09$ and $Q_C = 0.02$. The intensity gain of 30% is accompanied by an increase $\epsilon_V/\epsilon_{V_0}$ of 1.5. Note that the later the slice is injected, i.e. the larger its horizontal amplitude, the higher the efficiency gain, as suggested by theory. Experimental results with low-density beams fit theory reasonably well (efficiency versus δ and Q_C , Fig. 4). A further feature suggested by Eqs. (1) to (4) turned out to be valid: injection efficiency strongly depends on the skew quadrupole phase q around the ring and the vertical steering of the incoming beam. Unfavourable combinations of these parameters were found to cause an efficiency decrease (i.e. I_x goes up during the first few revolutions).

As high beam density leads to a spread in δ , the dynamics of the process becomes more complex; theory does not fit all experimental results. Nevertheless, an appreciable intensity increase of typically 15-20% is found for δ (coherent) ≈ 0.1 and $Q_C \approx 0.015$ (see Fig. 5). In contrast to the low-density beam, hardly any concomitant vertical blow-up is noticeable. This fact has been checked by vertically shaving the high intensity beam by means of horizontally plunging fork targets at the end of the acceleration cycle. Figure 6 indicates that, contrary to expectation, skew injection slightly increases vertical density.

Recent operational PSB performance figures are: 5×10^{12} protons per ring injected, with emittances (after fast blow-up) $\epsilon_H = 150$, $\epsilon_V = 65 \times 10^{-6} \pi$ rad m (at 50 MeV), leading to 1.3×10^{13} protons accelerated in all four rings.

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