IEEE Transactions on Nuclear Science, Vol.NS-24, No.3, June 1977

PENETRATION OF THE FIELD OF A BUNCHED BEAM THROUGH A CERAMIC VACUUM CHAMBER WITH METALLIC COATING

A. Piwinski

Deutsches Elektronen-Synchrotron DESY
Hamburg, Germany

Summary

The penetration of the electro-magnetic field of a bunched beam through a ceramic vacuum chamber with a metallic coating on the inside is investigated. It is shown that the field will only penetrate the chamber if the thickness of the metallic coating is small as compared to the square of the skin depth in the metal, divided by the thickness of the ceramics. If, in addition, the chamber is surrounded by ferrite, as used in beam monitors, the field penetration is further reduced. The field outside of the chamber and the power losses in the metallic layer are calculated. It turns out that these losses can be larger by orders of magnitude than the losses in a pure metallic vacuum chamber.

Introduction

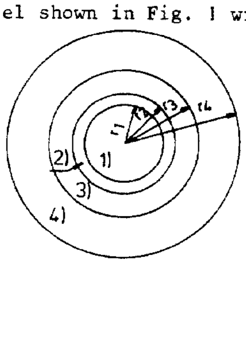
Ceramic vacuum chambers are used for beam monitors and for injection and feedback systems. To prevent an accumulation of static charge and to avoid discontinuities in the chamber, which can lead to beam instabilities, a conductive coating is provided on the inside of the chamber. With respect to instabilities, the thickness of the coating should be as large as possible. On the other hand, the coating reduces the sensitivity of a beam monitor. To find a compromise one has to know the attenuation of the electromagnetic field of the beam by the coating. Another important problem are the ohmic power losses which are caused by a bunched beam in a metallic coating.

The condition, that the field will penetrate the coating if the thickness of the coating is smaller than the skin depth, is not valid for relativistic particle beams. Since the velocitiy of light in the ceramics is smaller than the velocity of the circulating particles, the transverse electric field of the beam is rotated, and a longitudinal field component appears. The longitudinal field drives currents in the coating which shield the field of the beam and which lead to ohmic power losses. Additionally they act on the beam and change the synchrotron oscillation potential[1].

The calculation shows that the field will only penetrate the chamber if the thickness of the coating is small as compared to the square of the skin depth, divided by the thickness of the ceramics. If the thickness of the coating is larger than that critical value, the total mirror current will run in the coating, also if the thickness is much smaller than the skin depth. In that case the power losses can be larger by orders of magnitude than the losses in a pure metallic vacuum chamber.

Calculation of the Electromagnetic Fields

For the calculation of the fields we assume the model shown in Fig. 1 with 4 different regions:



1) Vacuum
2) Metal
3) Ceramics
4) Air or Ferrite

Fig. 1

Outside of region 4 we assume a shielding with an infinite conducticity. The calculation shows that the results are not changed, within our approximation, if the shielding is removed.

The bunch may be represented by a line current which has a gaussian distribution in longitudinal direction. The density distribution of the charge is then given by

$$\rho(t - \frac{s}{v}) = \frac{N_b e}{\sqrt{2\pi}\sigma_s} \exp\left\{ - \frac{(s-vt)^2}{2\sigma_s^2} \right\} \qquad (1)$$

with N_b = number of particles per bunch, e = electron charge, s = longitudinal coordinate, σ_s = standard deviation, v = velocity of the bunch

The distribution of a single bunch can be expressed by a Fourier integral

$$\rho(t - \frac{s}{v}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{\rho}(\omega) \exp\left\{ -i\omega(t - \frac{s}{v})\right\} d\omega \qquad (2)$$

where the Fourier transform is given by

$$\tilde{\rho}(\omega) = \frac{N_b e}{\sqrt{2\pi} v} \exp\left\{ - \frac{\omega^2\sigma_s^2}{2v^2} \right\} \qquad (3)$$

The field components can now be calculated for each frequency ω, and the total field can be expressed by a Fourier integral. The results will show that the fields drop very fast behind a bunch so that a Fourier integral is permissible instead of a Fourier series.

The electromagnetic fields in the 4 different regions of Fig. 1 can be expressed in the following form ($r_o = 0$):

$$r_{k-1} < r < r_k: \ \tilde{E}_r = Z_k \tilde{H}_\phi = A_k K_o'(a_k r) + B_k I_o'(a_k r) \qquad (4)$$

$$\tilde{E}_s = i \frac{v}{\omega} a_k (A_k K_o(a_k r) + B_k I_o(a_k r)) \qquad (5)$$

with

$$a_k = (\frac{\omega^2}{v^2} - \omega^2 \epsilon_k \mu_k - i\omega\mu_k\sigma_k)^{1/2} \qquad (6)$$

$$Z_k = \frac{\omega}{v} (\omega\epsilon_k + i\sigma_k)^{-1} \qquad (7)$$

σ_k is the conductivity and I_o and K_o are the modified Bessel functions. The constant A_1 is determined such that for small r the field is that of a line current:

$$A_1 = - \frac{a_1}{2\pi\epsilon_o} \tilde{\rho}(\omega) \qquad (8)$$

The other constants A_k and B_k are determined such that $\tilde{E}_s$ and $\tilde{H}_\phi$ are continuous at the boundaries $r = r_k$. To simplify the results we introduce the following approximations

$$d_s \ll r_1 , \ \sigma_s ,$$

where d_s is the skin depth in the metallic coating

$$d_s = \sqrt{\frac{2}{\omega\mu_2\sigma_2}} , \qquad (9)$$

$$\frac{r_2 - r_1}{r_1} \ll 1 , \quad 1 - \frac{v^2}{c^2} \ll 1 , \quad \frac{\epsilon_{3r} - 1}{3} (\frac{r_3 - r_2}{\sigma_s})^2 \ll 1 ,$$

$$\frac{r_2(r_3-r_2)(\varepsilon_{3r}-1)}{2c_s^2\varepsilon_{3r}} \ll 1\ , \qquad \frac{\mu_{4r}-1}{3}\left(\frac{r_4-r_3}{\sigma_s}\right)^2 \ll 1\ ,$$

$$\frac{r_3(r_4-r_3)(\mu_{4r}-1)}{2\sigma_s^2\mu_{4r}} \ll 1$$

with $\varepsilon_{3r} = \varepsilon_3/\varepsilon_o$ and $\mu_{4r} = \mu_4/\mu_o$. The last 4 relations make sure that the bunch spectrum has no frequencies which lead to resonances in the chamber and chamber walls.

With these approximations one obtains for the longitudinal field in region 1)

$$\tilde{E}_s = \frac{i\omega}{2\pi}\,\frac{Z_o S\tilde{\rho}(\omega)}{1+a_2 r_1 S/\mu_{2r}\tanh a_2(r_2-r_1)} \qquad (10)$$

with

$$Z_o = \sqrt{\mu_o/\varepsilon_o} = 376.7\ \Omega \qquad (11)$$

and

$$S = \left(1-\frac{1}{\varepsilon_{3r}}\right)\ \ln\frac{r_3}{r_2} + (\mu_{4r}-1)\ \ln\frac{r_4}{r_3} \qquad (12)$$

The voltage $\tilde{U}$ used for a beam monitor is measured in a loop with the length ℓ and the height r_4-r_3 in region 4). The absolute value of the voltage is given by

$$|\tilde{U}| = \left|\mu_4\int_{r_3}^{r_4}\int_0^{\ell}\frac{dH_\phi}{dt}\,ds\,dr\right|$$

$$= \frac{|\sin(\pi\ell/\lambda)\tilde{\rho}|\mu_{4r}}{\pi\varepsilon_o|\cosh a_2(r_2-r_1)+a_2 r_1 S/\mu_{r2}\sinh a_2(r_2-r_1)|}\ , \qquad (13)$$

where $\lambda = 2\pi c/\omega$ is the wave length.

We can now distinguish two cases:

a) $r_2-r_1 \gg d_s$

This case represents a pure metallic vacuum chamber. Eq.(10) becomes

$$\tilde{E}_s = \frac{i\omega\mu_{2r}Z_o\tilde{\rho}}{2\pi r_1 a_2} \qquad (14)$$

and the power losses are given by

$$P = -v^2\int_{-\infty}^{\infty}\tilde{E}_s\tilde{\rho}\,d\omega$$

$$= \frac{I_b^2\bar{R}\ell}{2\pi r_1\sigma_s^{3/2}}\sqrt{\frac{Z_o\mu_{2r}}{2\sigma_2}}\ \Gamma\left(\frac{3}{4}\right) \qquad (15)$$

with I_b = bunch current, $\bar{R}$ = mean radius of the machine, $\Gamma(3/4) = 1.2254..$,

The voltage $\tilde{U}$ drops exponentially with $(r_2-r_1)/d_s$. The longitudinal field E_s is calculated in [1]) and is not discussed here.

b) $r_2-r_1 \ll d_s$

This is the case where the field of the beam either totally or partially penetrates the chamber wall. Eq.(10) becomes

$$\tilde{E}_s = \frac{i\omega}{2\pi}\,\frac{Z_o S\tilde{\rho}}{1-i\omega\mu_o\sigma_2 r_1 S(r_2-r_1)} \qquad (16)$$

A Fourier transformation of Eq.(16) gives the total longitudinal electric field in region 1) and also in the coating.

$$E_s = E_{so}\left(\sqrt{\frac{\pi}{2}}\,V^2 e^{(u+V)^2/2}\ \mathrm{erfc}\left(\frac{u+V}{\sqrt{2}}\right) - V\right)e^{-u^2/2} \qquad (17)$$

with

$$E_{so} = \frac{I_b Z_o\bar{R}S}{\sqrt{2\pi}\,\sigma_s^2}\ , \qquad V = \frac{\sigma_s R'}{r_1 S Z_o}\ , \qquad u = \frac{s-vt}{\sigma_s}$$

and

$$R' = \frac{1}{\sigma_2(r_2-r_1)}$$

R' is the resistance of a square area of the coating. erfc is the complemented error function. Fig.2 shows E_s for different values of V, i.e. for different values of $\sigma_2(r_2-r_1)$.

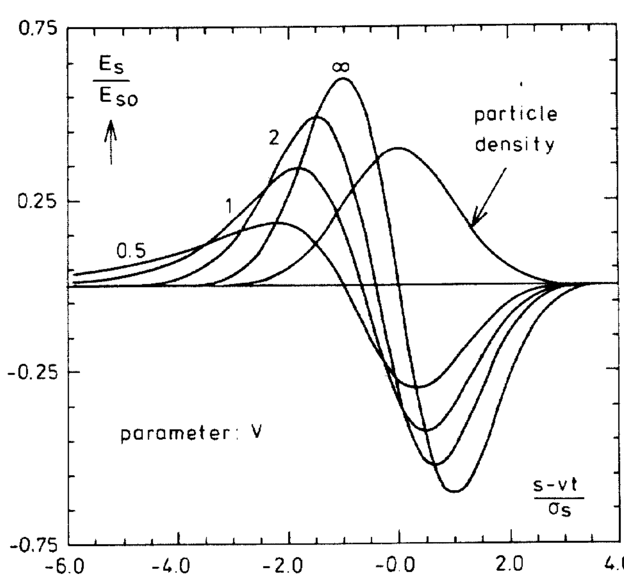


Fig.2 Longitudinal electric field

The power losses are obtained from Eq.(16) and can be written in the form

$$P = \frac{Z_o I_b^2 \ell \bar{R} S}{2 c_s^2} \left(V - \sqrt{\pi}\, V^2 e^{V^2} \operatorname{erfc}(V)\right) \tag{18}$$

Fig.3 shows the power losses for the following PETRA parameters:

1 bunch with 20 mA, $\bar{R}$ = 367 m, r_2 = 37 mm, r_3 = 43 mm, $\varepsilon_{3r} = 9$, $\mu_{4r} = 1$,

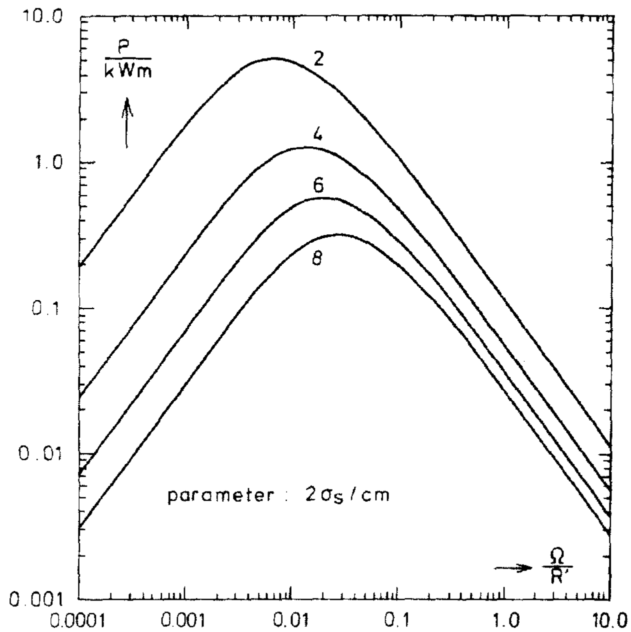


Fig.3 Power losses as a function of $1/R' = (r_2-r_1)\cdot\sigma_2$

On the right hand side the 4 curves asymptotically approach the losses of a metallic vacuum chamber given by Eq.(15).

From Eqs.(12) and (18) follows that the power losses can become larger if there is ferrite in region 4).

The voltage $|\tilde{U}|$ follows from Eq.(13):

$$|\tilde{U}| = \frac{\mu_{4r} \left|\sin(\pi\ell/\lambda)\tilde{\rho}\right|}{\pi\varepsilon_o \sqrt{1+(\omega\mu_o\sigma_2 r_1 S(r_2-r_1))^2}} \tag{19}$$

It is convenient to consider the factor by which the voltage is reduced due to the metallic coating. It can be written in the form

$$\frac{|\tilde{U}|}{|\tilde{U}_o|} = \frac{1}{\sqrt{1+x^2}} \tag{20}$$

with

$$x = 2\pi \frac{Z_o r_1}{R'\lambda} S = 2 \frac{r_2-r_1}{\mu_{2r} d_s^2} r_1 S$$

If $r_1 S \approx r_3 - r_2$, one obtains with $\mu_{2r} \approx 1$ the simple expression

$$x = 2 \frac{(r_2-r_1)(r_3-r_2)}{d_s^2}$$

Fig.4 shows the reduction of $|\tilde{U}|$ for different values of λ.

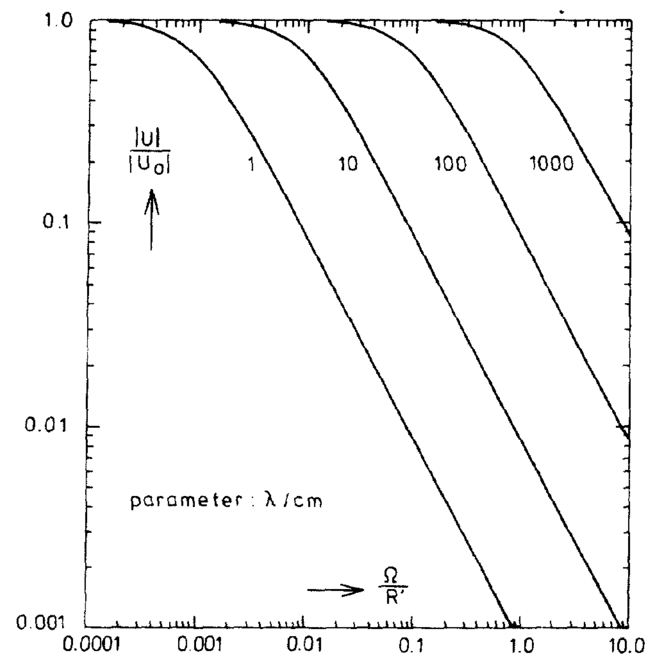


Fig.4 Attenuation of the field as a function of 1/R'

Reference

1) A. Piwinski, DESY 72/72 (1972)