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PROTOTYPE MULTIPOLE MAGNETS FOR THE KEK MAIN RING

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Summary

Two types of multipole magnets were made and tested. They can generate multipole fields up to fourth order, both normal and skew, at the same time by one magnet. One has a twelve pole configuration, that is, a dodecapole magnet, the other is a "cos nθ" magnet. They have an aperture of 160 mm in diameter and a 150 mm long core. The shape and the number of poles, and the number of coils were determined by the demand of the uniformity of multipole fields which was investigated numerically using the computer program LINDA.

Multipole components of the magnets were analyzed by the rotating coils and a spectrum analyzer. The results well agree with numerical calculations.

Introduction

Horizontal and vertical dipole magnets and sextupole magnets are in operation for the correction magnets in the KEK main ring. It is discussed from the results of the initial operation that more uniform distributions of the correcting fields and more kinds of multipole fields may be needed to accelerate much beam stably. For the present, additional sextupole and octupole magnets are installed to estimate the effect on correcting tune spread, and skew quadrupole magnets are also installed for the linear coupling between radial and vertical oscillations.

In consideration of the possibility of the correction up to the fourth order, it is attractive that one magnet can generate all normal and skew multipole fields at the same time, especially for there are no room for one kind of magnets respect to one kind of multipoles. Therefore, so called "combined multipole magnets" were designed and constructed for prototypes of the KEK main ring.

Magnets

A dodecapole magnet was chosen for one of prototypes from the reason that twelve is the least common multiple of numbers smaller than five in consideration that multipole fields up to fourth order can be generated in one magnet. A diagram of its cross section is given in Fig.1. Twelve symmetric poles and twelve spaces for coils are the same size, the core length is 150 mm, the bore radius is 80 mm, the return yoke is the cylinder of which the outside and inside diameters are 290 and 230 mm, and each coil has 400 turns. The poles can be replaced by coils and it is called a "cos nθ" magnet with twenty four coils when all poles are replaced. In this report, the former dodecapole magnet is called Type 1-A, the latter "cos nθ" magnet is called Type 2-A. When these magnets are rotated around their axis by 15° or 7.5°, the new type magnets are obtained and called Type 1-B, Type 2-B respectively, that is, Type 1 has twelve poles and Type A has coils on its medium plane.

For these magnets, current of each coil is weighted by cos nθ for a normal field, or $\cos(n\theta + \frac{\pi}{2})$ for a skew field to obtain the 2n-th-pole field, where θ is an angle of the center of a coil measured from the medium plane.

These configurations are suggested by reference 1, and finally determined by the results of numerical calculations. In this design work, the computer program LINDA was used, and the uniformity of multipole fields are estimated for various number of poles and coils, and for various ratio of the pole width (P) to the coil space width (C). Twelve poles are necessary for a magnet with poles like Type 1 and twelve coils for a "cos nθ" magnet without poles like Type 2 in order that at least there is useful aperture of multipole fields lower than decapole.

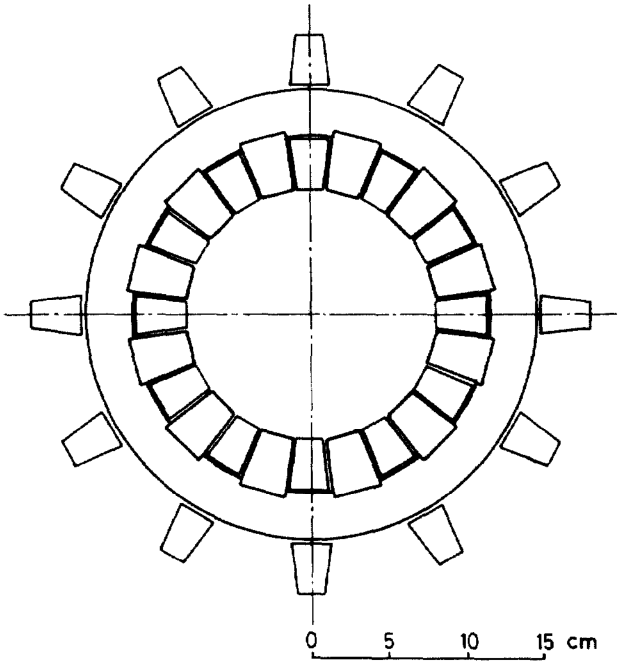


Fig.1 Cross section of the magnet.

Typical results from numerical calculations for dodecapole magnets and cos nθ magnets with 12 or 24 coils are as follows. Increasing the ratio of C/P for magnets like Type 1, there is a little improvement in the uniformity of the field distributions, and the differences between them become negligible and current in coils increases by 10 % at most as the order of the multipole reaches four. Variations in each multipole field of Type-A magnets are equal but opposite to that of Type-B magnets, that is, positive for Type-A and negative for Type-B. Fig.2 shows these variations in the cases of Type-1-A and Type-2-A. Regarding dipole, there is little difference between them when the distance from the center is smaller than 4 cm. As for multipoles higher than dipole, there is obvious difference and Type 2-A must be chosen when variations of a few per cent are allowed in the aperture of about 5 cm. In the case of a cos nθ magnet with 12 coils like Type-2, the region of the uniform distributions of the fields is nearly an average of Type 1 and Type 2 for dipole and larger by about 8 to 4 mm than that of Type 1 for higher multipoles. In comparison with Type 1 and Type 2, Type 2 needs more current by from 20 to 100 % but it has twice as many coils as Type 1, therefore, current density of Type 2 is lower than or equal to that of Type 1.

There are problem in Type 2 that fields directly depend on an accuracy of the construction and position of coils because there are no iron poles of which surface determines the field potential and of which accuracy is rather easily controlled, and that there need so many current sources that there are many coils when several kinds of multipoles are excited at the same time. Finally, the dodecapole magnet with replaceable poles and with an support system which can tilt the magnet by 7.5° and 15° is determined as the prototype.

Field Measurements

The magnet was excited in single multipole mode, that is, in the 2n-pole mode, each coil was electrified by the weight of cos $n\theta$. Multiple fields were analyzed by the rotating coils and a spectrum analyzer for each multipole mode operation.

The rotating coils consist of two coils, one is called the short coil, and the other is called the long coil. The former is 10 mm long and has twelve turns with radius of 37.5 mm, and the latter is an one-turn coil and its length and radius are 1000 and 41.5 mm. They have the same rotating axis and are wound upon a pipe of bakelite along the direction of the axis and their returns and readout are in the axle and their maximum distance from the axis are 1 mm. The coils are protected by the stainless pipe with low permeability, and this stainless pipe is constructed with such accuracy that it is enough to adjust on the basis of its surface. Signals from the rotating coils are obtained through slip rings which consist of carbon brushes and silver rings. This apparatus generates signals which are synchronized with the rotation and obtained by an additional arm traversing a photointerrupter.

If the magnetic potential is expanded around the axis of the rotating coil as

$$\phi = -\sum_{n=1}^{\infty} A_n \gamma^n \sin(n\theta + X_n) \qquad (1)$$

then

$$-\partial\theta/\partial\gamma = B_r \text{ or } \int B_r d\theta \text{ and } -\frac{1\partial\phi}{\gamma\partial\theta} = B_\theta \text{ or } \int B_\theta ds$$

and signals from the rotating coil are discribed as

$$E = \sum_{n=1}^{\infty} E_n \sin[2\pi fn(t - t_0) + X_n],$$

$$E_n = 2\pi fNL\cdot n\cdot A_n\cdot\gamma_0^n$$

where f is the rotating frequency and, N, L and γ_0 are turns, a length and a radius of the rotating coil, respectively. Then the output of a spectrum analyzer is discrete spectra at intervals of f and each amplitude is $E_n/\sqrt{2}$.

Relative phases between multipoles were determined by measuring delay time t_n for each multipole from the signal of a photointerrupter on the oscilloscope, that is,

$$X_n/n - X_n/n' = -2\pi f\ (t_n - t_n')$$

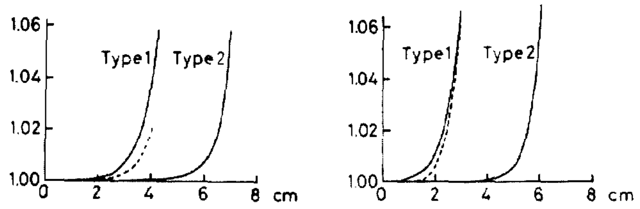
This aim is mainly to determine whether contributions of error fields are constructive or destructive, i.e., constructive when $X_n - X_{n0} \simeq 0°$ and destructive when $X_n - X_{n0} \simeq 180°$ in the $2n_0$-pole mode.

It is the merit in this measurement that the 2n-th multipole component (A_n) does not contribute other component (A_m) higher than A_n i.e. m>n, even if there are position errors of the rotating coil, but A_n is affected by A_m and this error of A_n contributed from A_m (δA_n, m) is given as

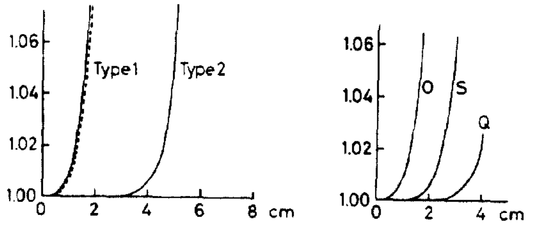
$$\left|\frac{\delta A_{n,m}}{A_n}\right| = \frac{m!}{(m-n)!n!}\,|\delta|^{m-n}\,\frac{|A_m|}{|A_n|} \quad (m>n)$$

and this term is expected small where δ is the distance of the axis of the rotating coil from the magnetic center.

In this measurement, magnitude of signals is about a few mV and analyzing range is 80 dB, for example in the quadrupole mode of Type 1-A, main term was 2.66 mV (5.90 mV), a sextupole term was 4.8 μV (2.5 μV) and higher terms were same order as a sextupole when analyzed for the short(long) coil. The rotating coil was set in the accuracy of ±1 mm respect to the geometrical



(a) Quadrupole (b) Sextupole



(c) Octupole (d) Effective length

Fig.2 Variations of the field distributions.

The solid line shows numerical calculations and the broken line shows the measurements in (a), (b) and (c). Q, S and O show quadrupole, sextupole and octupole fields in (d).

Table 1 Magnitude of multipole fields in each multipole operation

Magnet		Dipole	Quadrupole	Sextupole	Octupole
	B_n	1.08×10^{-2}	1.39×10^{-1}	3.33	1.18×10^{2}
Type 1-A	$\int B_n ds$	3.21×10^{-3}	3.08×10^{-2}	6.78×10^{-1}	2.31×10
	current	249	267	200	267
	B_n	1.12×10^{-2}	1.58×10^{-1}	4.64	1.22×10^{2}
Type 1-B	$\int B_n ds$	3.28×10^{-3}	3.54×10^{-2}	9.48×10^{-1}	2.31×10
	current	267	267	400	267
	B_n		1.42×10^{-1}	2.62	7.24×10
Type 2-A	$\int B_n ds$	not measured	3.46×10^{-2}	5.82×10^{-1}	1.56×10
	current		249	241	267

The maximum current of coils is 400 ampere-turns and B_n is $\partial^{n-1} B_Y/\partial X^{n-1}]_{X=Y=0}$ where n is the order of the multipole. Tabulated values of B_n and $\int B_n ds$ represented in the unit of T/m^{n-1} and T/m^{n-1}, respectively. Current in the table is an average of ampere-turns per coil.

axis of the magnet, therefore errors from higher terms are equal to or smaller than 10^{-3}. Moreover there were inevitable noises of a few μV, that is, order of 10^{-3} respect to the main term. Because of these reasons mentioned above, the position errors of the rotating coil were not considered, and there are rather large ambiquities in the third figure of the results.

As for Type-1, radial field distributions on the medium plane were reduced from measuring the flux at each radial point by using the 50 cm long rotating coil of which area is 1.36 m^2 and verified the results of the multipole analysis.

Results of field measurements

Results of field measurements are summarized in Table 1. The maximum current of coils is 400 AT (ampere-turns) for each multipole mode. Skew fields are not tabulated from the reasons as follows. The skew quadrupole of Type 1-B (Type 1-A) is equal to the normal quadrupole of Type 1-A (Type 1-B) when rotated by 45°. As for the sextupole of Type 1 and also the quadrupole and the sextupole of Type 2, normal and skew fields are the same fields when rotated 30° or 45°. Skew octupole fields are not considered in the KEK main ring at present. It is noticeable that octupole fields of Type 1-A are equal to that of Type 1-B when rotated by 45°. As for Type 2-B, similar results were expected as Type 2-A therefore measurements were not carried out. It is estimated that the resulting accuracy is about ±2 in the third figure in B_n and about ±1 in $\int B_n ds$ because octupole fields of Type 1-A and B are the same, where B_n is $\partial^{n-1} B_Y / x^{n-1}]_{X=Y=0}$.

Radial distributions in the medium plane are calculated from the results and showed in Fig.2 as for Type 1-A and numerical calculation by LINDA are also shown, but dipole fields of Type 1-A and distributions of Type 2-A are not shown because distributions are almost uniform and the variations are smaller than resolutions of measurements. It is clear that central distributions are very similar to numerical calculations and distributions of effective length are similar to the central. It is assumed that permeability of iron is infinite, but numerical calculations well agree with measurements. From these results, magnets which have the cylindrical symmetry are expected to have the same distributions of effective length as central distributions of fields.

It was also measured in the case that only one coil was excited and showed in Table 2. In this case all multipole have the same phase because the symmetric plane of multipoles are the same and angles of these plane are equal to relative phases, that is, from equation (1), field components due to the 2n-pole are as follows,

$$B^n_\gamma(\gamma,\ \theta) = nA_n\ \gamma^{n-1} \sin\ (n\theta + X_n)$$

$$B^n_\theta(\gamma,\ \theta) = nA_n\ \gamma^{n-1} \cos\ (n\theta + X_n)$$

therefore, the symmetric plane is $\theta_n = -X_n/n$, and if θ_n are equal for all n then $X_n = 0$. When the k-th coil of M coils is electrified by $400 \cdot I_k$, the coefficient A_n and phase X_n are described as

$$A_n \cos X_n = A^0_n \sum_{k=1}^{M} I_k \cos n\theta_k$$

$$A_n \sin X_n = A^0_n \sum_{k=1}^{M} I_k \sin n\theta_k$$

where A^0_n is the value in Table 2, and θ_k is the angle between the center of the coil and the medium plane.

In the case of Type 1-A and Type 2-A, if magnets and measurements are free from errors, there are no field components except that satisfy the equation,

$$A_n = p \cdot A^0_n / 2, \qquad X_n = 0$$

therefore allowed terms by the condition of the symmetry are as follows,

mode	Dipole	Quadrupole	Sextupole	Octupole
Type 1-A	1,11,13	2,10,14	3,9,15	4,8,16,
Type 2-A	1,23,	2,22,	3,21,	4,20

Multipoles higher than n=10 cannot be distinguished from noises, and this means that they could be neglected in these magnets.

Radial distributions of multipole fields are clearly understood from the matter mentioned above and the variations are equal quantitatively to values estimated from the Table 2. Therefore it is expected that fields of magnets like these types do not depend on the shape of poles or the fact whether poles are present or absent, and their qualities are determined by the symmetry, and it is similar for the effective length. It is necessary to enlarge the bore radius or increase the number of poles in order to obtain large apertures of fields in the case of magnets with poles like Type 1. As for magnets like Type 2, it is enough in the aperture of the radius of 40 mm, and it would be similar to Type 1 when more large aperture is needed however it is not clear beyond the radius of 40 mm because the measurements are limited in the region of the radius of 40 mm.

The effect of position errors of poles and coils were estimated from the measurements when one pole or one coil was displaced in the quadrupole mode. As for Type-1, if one pole is moved toward the center by 1 mm, the contribution of the error is the same order of the inherent variations due to the symmetry and they could cancel out. As for Type 2, one coil is moved toward the center by 2 mm and in the azimuthal direction by 2 mm but the effect on field distributions is negligible in the quadrupole mode.

Table 2 Coefficients of A^0_n in the expansion of the potential when only one coil is electrified by 400 ampere-turns.

	Type 1		Type 2	
n	central	integral	central	integral
1	1.71×10^{-3}	5.18×10^{-4}	1.22×10^{-3}	3.96×10^{-4}
2	2.20×10^{-2}	4.96×10^{-3}	1.17×10^{-2}	2.80×10^{-3}
3	5.44×10^{-1}	1.16×10^{-1}	2.30×10^{-1}	5.02×10^{-2}
4	1.92×10	3.83	6.22	1.27
5	9.94×10^{2}	1.93×10^{2}	2.36×10^{2}	4.96×10
6	5.64×10^{4}	1.06×10^{4}	1.32×10^{4}	1.99×10^{3}
7	3.90×10^{6}	7.95×10^{5}	1.01×10^{6}.	1.30×10^{5}
8	2.88×10^{8}	6.19×10^{7}		
9	3.92×10^{10}	6.46×10^{9}		
10	———	6.04×10^{11}		

The central and the integral are respect to the two dimensional potential at the center and the effective potential along the axis. They are represented in the same unit as in Table 1.

Reference

1) N. Marks, Multipole Magnets for the Daresbury Synchrotron Radiation Source. Proceedings of Fifth International Conf. on Magnet Technology, 1975, p.22