IEEE Transactions on Nuclear Science, Vol.NS-24, No.3, June 1977

MECHANICAL AND THERMAL STRESSES IN SUPERCONDUCTING ACCELERATOR AND BEAM-LINE MAGNETS

S.C. Snowdon
Fermi National Accelerator Laboratory*
P.O. Box 500, Batavia, Illinois 60510

Introduction

A solution to a similar problem has been given previously[1] in which the magnetic field was generated by two sheet currents varying as cosine theta. One sheet current was located at the boundary between the innermost cylinder and the middle cylinder of structural material. The other sheet was located at the boundary between the middle cylinder and the outermost cylinder of structural material. The present note addresses the problem of improving the representation of the magnet excitation by replacing the two current sheets with a thick cosine theta current distribution in the middle structural region.

Equation for Elastic Displacement

If $\vec{u}$ is the displacement vector and the body forces are derived from the Lorentz force then[2]

$$\nabla\times\nabla\times\vec{u}-2\frac{1-\nu}{1-2\nu}\nabla(\nabla\cdot\vec{u})=2\frac{1+\nu}{E}\vec{J}\times\vec{B}\ , \tag{1}$$

where E is Young's Modulus, ν is Poisson's ratio, $\vec{J}$ is the current density and $\vec{B}$ is the magnetic induction.

Generalized Plane Strain Approximation

For simplicity consider only the case for which $u_z = \varepsilon_{zz}z$ with ε_{zz} = constant. The remaining components are considered to be functions of (r,θ) only. This is consistent with an excitation in which J_z is the only component of current density. Hence Eq. (1) becomes

$$\frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru_\theta)-\frac{1}{r}\frac{\partial u_r}{\partial\theta}\right]-\beta\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru_r)+\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}\right]=-2\frac{1+\nu}{E}J_zB_\theta, \tag{2}$$

$$-\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru_\theta)-\frac{1}{r}\frac{\partial u_r}{\partial\theta}\right]-\beta\frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru_r)+\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}\right]=2\frac{1+\nu}{E}J_zB_r, \tag{3}$$

where

$$\beta=2\frac{1-\nu}{1-2\nu}\ . \tag{4}$$

Force on Thick Cosine Theta Conductor

By definition a thick cosine theta conductor carries an axial current between two radii (b,c) with a current density that varies as

$$J_z=J_o\cos\theta.\ \text{(emu)} \tag{5}$$

From this it follows[3] that

$$J_zB_\theta=\frac{\pi}{3}J_o^2\left[4r-3\lambda-b^3r^{-2}\right](1+\cos2\theta),\ \text{(emu)} \tag{6}$$

$$J_zB_r=\frac{\pi}{3}J_o^2\left[2r-3\lambda+b^3r^{-2}\right]\sin2\theta,\qquad \text{(emu)} \tag{7}$$

where for convenience in these and subsequent formulas

$$\lambda=c+\frac{1}{3}(c^3-b^3)r_s^{-2}, \tag{8}$$

the radius of the iron shield being r_s.

Form of Solution

Equations (6) and (7) indicate that the form of the displacement within the conductor may be taken as

$$u_r=P_o(r)+P_2(r)\cos2\theta\qquad u_\theta=Q_2(r)\sin2\theta \tag{9}$$

Substituting into Eqs. (2-3) gives

$$-\beta\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rP_o)\right]=-\mu\left[4r-3\lambda-b^3r^{-2}\right], \tag{10}$$

$$\frac{4}{r^2}P_2-\beta\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rP_2)\right]+\frac{2}{r^2}\frac{d}{dr}(rQ_2)-2\beta\frac{d}{dr}\left(\frac{Q_2}{r}\right)=-\mu\left[4r-3\lambda-b^3r^{-2}\right], \tag{11}$$

$$-2\frac{d}{dr}\left(\frac{P_2}{r}\right)+\frac{2\beta}{r^2}\frac{d}{dr}(rP_2)-\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rQ_2)\right]+\frac{4\beta}{r^2}Q_2=\mu\left[2r-3\lambda+b^3r^{-2}\right], \tag{12}$$

where again for convenience

$$\mu=2\,\frac{1+\nu}{E}\,\frac{\pi}{3}J_o^2\ . \tag{13}$$

Solutions of the Homogeneous Equation

In general these solutions are of the form

$$P_o=Ar^{-1}+Br\qquad P_2=Cr^p\qquad Q_2=Dr^p \tag{14}$$

where p is found by substituting into Eqs. (11-12) to obtain

$$[4-\beta(p^2-1)]C+2[p+1-\beta(p-1)]D=0 \tag{15}$$

$$2[-p+1+\beta(p+1)]C-[p^2-1-4\beta]D=0\ . \tag{16}$$

The determinant of the coefficients is

$$\Delta(p)=\beta(p^2-1)(p^2-9)\ . \tag{17}$$

Setting this equal to zero gives p = ±1, ±3. Hence there are four solutions which must be added together to give

$$P_2=-D_1r-\beta D_2r^{-1}-\frac{2-\beta}{1-2\beta}D_3r^3+D_4r^{-3}, \tag{18}$$

$$Q_2=D_1r+D_2r^{-1}+D_3r^3+D_4r^{-3}\ . \tag{19}$$

Thus the homogeneous solutions are seen to be identical in form with those found previously[1] after it is recognized that ($u_r = \frac{1}{\beta}\,Gr\ln r$, $u_\theta = Gr\theta$) the solution describing pre-tension may also be added.

Particular Solution

Since each term of the RHS of Eq. (10) is of the

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form Br^q, one may take for P_o in Eq. (9): $P_o = Ar^p$. Adding the contributions for q = 1, 0, -2 gives

$$P_o = \frac{\mu}{\beta}[\frac{1}{2}r^3 - \lambda r^2 + b^3]. \tag{20}$$

For the remainder of the solution one may take $\begin{pmatrix} P_2 \\ Q_2 \end{pmatrix} = \begin{pmatrix} C \\ D \end{pmatrix} r^p$ corresponding to $\begin{pmatrix} E \\ F \end{pmatrix} r^q$ as a term on the RHS of Eqs. (11-12). Substituting into Eqs. (11-12) and inverting gives p = q+2 and

$$\begin{pmatrix} C \\ D \end{pmatrix} = \frac{1}{\Delta(p)} \begin{pmatrix} -p^2+1+4\beta & -2[p+1-\beta(p-1)] \\ -2[-p+1+\beta(p+1)] & 4-\beta(p^2-1) \end{pmatrix} \begin{pmatrix} E \\ F \end{pmatrix}. \tag{21}$$

Note that the q-values of interest are q = 1, 0, -2 or p = 3, 2, 0. Equation (21) can only provide solutions for p = 2, 0 since p = 3 gives $\Delta(3) = 0$. In this case one considers

$$P_2 = (A+B\ln r)r^3 \qquad Q_2 = (C+D\ln r)r^3 . \tag{22}$$

Substituting these into Eqs. (11-12) yields terms in r and rlnr. The terms in rlnr vanish in both equations if

$$(1-2\beta)B+(2-\beta)D=0. \tag{23}$$

Equating the remaining coefficients of r then gives two equations which when subtracted yields

$$(1+2\beta)B+(2+\beta)D=\mu. \tag{24}$$

Thus

$$B = \frac{1}{6}\frac{\mu}{\beta}(2-\beta) \qquad D = -\frac{1}{6}\frac{\mu}{\beta}(1-2\beta) \tag{25}$$

Using these values for (B, D) the two equations for (A, C) are identical. Since (A, C) are coefficients of r^3 terms already included in the homogeneous solution one may choose one relation between A and C arbitrarily. Let C = -2A for convenience. Then

$$A = -\frac{1}{36}\frac{\mu}{\beta}(1-9\beta-\beta^2) \quad C = \frac{1}{18}\frac{\mu}{\beta}(1-9\beta-\beta^2). \tag{26}$$

Displacement

The incremental displacements to be added to the homogeneous terms are from Eqs. (20-26)

$$\Delta u_r = \frac{\mu}{\beta}\Big\{\frac{1}{2}r^3 - \lambda r^2 + b^3 + [-\frac{1}{9}(1-2\beta)b^3 - \frac{1}{5}(3+2\beta)\lambda r^2$$
$$-\frac{1}{36}(1-9\beta-\beta^2)r^3 + \frac{1}{6}(2-\beta)r^3\ln r]\cos 2\theta\Big\}, \tag{27}$$

$$\Delta u_\theta = \frac{\mu}{\beta}\Big\{\frac{1}{9}(2-\beta)b^3 + \frac{1}{5}(2+3\beta)\lambda r^2$$
$$+\frac{1}{18}(1-9\beta-\beta^2)r^3 - \frac{1}{6}(1-2\beta)r^3\ln r\Big\}\sin 2\theta . \tag{28}$$

Stress

The incremental strain to be added to the homogeneous solution is found from Eqs. (27-28) using Eqs. (61-63) of Ref. (1). Then using Eqs. (48-50) from Ref. (1) one has after inverting

$$\Delta\sigma_{rr} = \frac{E}{1+\nu}[\frac{1}{2}\beta\Delta\varepsilon_{rr} - \frac{1}{2}(2-\beta)\Delta\varepsilon_{\theta\theta}] \tag{29}$$

$$\Delta\sigma_{\theta\theta} = \frac{E}{1+\nu}[-\frac{1}{2}(2-\beta)\Delta\varepsilon_{rr} + \frac{1}{2}\beta\Delta\varepsilon_{\theta\theta}] \tag{30}$$

$$\Delta\sigma_{r\theta} = \frac{E}{1+\nu}\Delta\varepsilon_{r\theta} . \tag{31}$$

The result is

$$\Delta\sigma_{rr} = \frac{E}{1+\nu}\frac{\mu}{\beta}\Big\{-\frac{1}{2}(1-2\beta)r^2 + \frac{1}{2}(2-3\beta)\lambda r - \frac{1}{2}(2-\beta)b^3r^{-1}$$
$$+[-\frac{1}{6}(2-\beta)b^3r^{-1} - \frac{1}{10}(2+13\beta)\lambda r$$
$$-\frac{1}{12}(1-11\beta)r^2]\cos 2\theta\Big\}, \tag{32}$$

$$\Delta\sigma_{\theta\theta} = \frac{E}{1+\nu}\frac{\mu}{\beta}\Big\{-\frac{1}{2}(3-2\beta)r^2 + \frac{1}{2}(4-3\beta)\lambda r + \frac{1}{2}\beta b^3r^{-1}$$
$$+[\frac{1}{6}b^3r^{-1} + \frac{3}{10}(4+\beta)\lambda r$$
$$-\frac{1}{12}(3+5\beta+2\beta^2)r^2 - (1-\beta)r^2\ln r]\cos 2\theta\Big\}, \tag{33}$$

$$\Delta\sigma_{r\theta} = \frac{E}{1+\nu}\frac{\mu}{\beta}\Big\{-\frac{1}{6}\beta b^3r^{-1} + \frac{1}{10}(8+7\beta)\lambda r$$
$$-\frac{1}{12}\beta(7+\beta)r^2 - \frac{1}{2}(1-\beta)r^2\ln r\Big\}\sin 2\theta . \tag{34}$$

Boundary Conditions

Since the form of the solution of the homogeneous equation is identical with that of Ref. (1) one may utilize Eqs. (55-65) of that reference. To these equations add the particular solutions found here to give the correct expressions for displacement, strain and stress. Apart from the special condition related to pre-tension and discussed in Ref. (1) the boundary conditions are as follows:

At r=a, the innermost radius $\sigma_{rr}^{(+)} = \sigma_{r\theta}^{(+)} = 0$ (35)

At r=b $\quad \sigma_{rr}^{(+)} - \sigma_{rr}^{(-)} = \sigma_{r\theta}^{(+)} - \sigma_{r\theta}^{(-)} = 0$ (36)

$u_r^{(+)} - u_r^{(-)} = u_\theta^{(+)} - u_\theta^{(-)} = 0$ (37)

At r=c $\quad \sigma_{rr}^{(+)} - \sigma_{rr}^{(-)} = \sigma_{r\theta}^{(+)} - \sigma_{r\theta}^{(-)} = 0$ (38)

$u_r^{(+)} - u_r^{(-)} = u_\theta^{(+)} - u_\theta^{(-)} = 0$ (39)

At r=d, the outermost radius $\sigma_{rr}^{(-)} = \sigma_{r\theta}^{(-)} = 0$ (40)

Use of the Virial Theorem

The virial theorem as discussed in Ref. (1) is used to obtain enough conditions to permit the longitudinal strain ε_{zz} to be determined. The only change here is to evaluate the magnetic energy and the Maxwell stress tensor using fields[3] for the thick cosine theta conductor. Hence the virial theorem becomes

$$\iint(\sigma_{rr}+\sigma_{\theta\theta}+\sigma_{zz})\,r\,dr\,d\theta =$$

$$\frac{1}{3}\pi^2 J_o^2\left\{-\frac{1}{2}(c^4-b^4)+[c+(c^3-b^3)r_s^{-2}](c^3-b^3)\right.$$

$$\left.-b^3(c-b)\right\}. \quad \text{(dynes)} \qquad (41)$$

References

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2. R.W. Little, Elasticity, Prentice Hall, Inc., Englewood Cliffs, New Jersey, 1964, p. 77

3. J.P. Blewett, "Iron Shielding for Air Core Magnets" Proc. of 1968 Summer Study on Superconducting Devices and Accelerators, Brookhaven National Laboratory, 1968, p. 1042

ELASTIC STRESS AND STRAIN IN FOUR SHELL BEAM LINE DIPOLE

CENTRAL MAGNETIC FIELD(KG) =	0.0000	INNER BORE TUBE RADIUS(IN) =	3.0000	INNER CONDUCTOR RADIUS(IN)) =	3.5000
OUTER CONDUCTOR RADIUS(IN)) =	5.0000	OUTER RADIUS OF BANDING(IN) =	6.0000	RADIUS OF IRON SHIELD(IN) =	8.0000
BORE YOUNGS MODULUS(LBS/IN/IN) =	10000000.	COND. YOUNGS MODULUS(LBS/IN/IN) =	1000000.	BAND YOUNGS MODULUS(LBS/IN/IN) =	10000000.
BORE POISSONS RATIO =	.3330	COND. POISSONS RATIO =	.1000	BAND POISSONS RATIO =	.3330
BORE THERMAL STRAIN(IN/IN) =	-.0020	COND. THERMAL STRAIN(IN/IN) =	-.0030	BAND THERMAL STRAIN(IN/IN) =	-.0040
BAND DISLOCATION(MILLIRAD) =	5.250	MAGNETIC ENERGY((IN-LBS)/IN) =	0.00	MAGNETIC ENERGY(J/M) =	0.00

TRANSVERSE STRESS(LB/IN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
90.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.
80.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.
70.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.
60.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.
50.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.
40.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.
30.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.
20.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.
10.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.
.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.

TRANSVERSE STRAIN (MUIN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
90.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.
80.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.
70.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.
60.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.
50.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.
40.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.
30.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.
20.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.
10.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.
.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.

LONG. STRESS(LB/IN/IN) AND STRAIN(MUIN/IN); SQRT(3*J2) IN (KLB/IN/IN); TRANSVERSE DISPLACEMENT(IN)

ANGLE (DEG)	ZZAP	ZZBM	ZZBP	ZZCM	ZZCP	ZZDM	EZZ	YAP	YBM	YBP	YCM	YCP	YDM	URA	UTA	URB	UTB	URC	UTC	URD	UTD
90.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0079
80.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0070
70.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0061
60.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0053
50.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0044
40.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0035
30.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0026
20.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0018
10.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0009
.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0000

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CENTRAL MAGNETIC FIELD(KG) =	43.5000	INNER BORE TUBE RADIUS(IN) =	3.0000	INNER CONDUCTOR RADIUS(IN)) =	3.5000
OUTER CONDUCTOR RADIUS(IN)) =	5.0000	OUTER RADIUS OF BANDING(IN) =	6.0000	RADIUS OF IRON SHIELD(IN) =	8.0000
BORE YOUNGS MODULUS(LBS/IN/IN) =	10000000.	COND. YOUNGS MODULUS(LBS/IN/IN) =	1000000.	BAND YOUNGS MODULUS(LBS/IN/IN) =	10000000.
BORE POISSONS RATIO =	.3330	COND. POISSONS RATIO =	.1000	BAND POISSONS RATIO =	.3330
BORE THERMAL STRAIN(IN/IN) =	-.0020	COND. THERMAL STRAIN(IN/IN) =	-.0030	BAND THERMAL STRAIN(IN/IN) =	-.0040
BAND DISLOCATION(MILLIRAD) =	5.250	MAGNETIC ENERGY((IN-LBS)/IN) =	88479.62	MAGNETIC ENERGY(J/M) =	394136.50

TRANSVERSE STRESS(LB/IN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
90.0	-0.	-2651.	-0.	-1772.	-14183.	0.	-1772.	53.	0.	-2122.	-695.	0.	-2122.	17085.	0.	0.	6774.	0.
80.0	-0.	-3722.	0.	-1828.	-14395.	-901.	-1828.	30.	-901.	-2145.	-723.	-160.	-2145.	16838.	-160.	0.	7100.	-0.
70.0	-0.	-6806.	0.	-1991.	-15306.	-1693.	-1991.	-35.	-1693.	-2211.	-791.	-300.	-2211.	16125.	-300.	0.	8041.	-0.
60.0	-0.	-11530.	0.	-2240.	-15943.	-2281.	-2240.	-136.	-2281.	-2313.	-996.	-404.	-2313.	15033.	-404.	0.	9482.	-0.
50.0	-0.	-17326.	0.	-2545.	-17091.	-2594.	-2545.	-259.	-2594.	-2438.	-1025.	-459.	-2438.	13694.	-459.	0.	11249.	-0.
40.0	0.	-23493.	0.	-2870.	-18311.	-2594.	-2870.	-391.	-2594.	-2571.	-1161.	-459.	-2571.	12259.	-459.	0.	13137.	-0.
30.0	0.	-29289.	0.	-3175.	-19452.	-2281.	-3175.	-514.	-2281.	-2696.	-1290.	-404.	-2696.	10929.	-404.	0.	14997.	-0.
20.0	0.	-34013.	0.	-3424.	-20393.	-1693.	-3424.	-615.	-1693.	-2793.	-1395.	-300.	-2799.	9837.	-300.	0.	16338.	-0.
10.0	0.	-37097.	0.	-3597.	-21009.	-901.	-3587.	-680.	-901.	-2864.	-1463.	-160.	-2864.	9124.	-160.	0.	17279.	-0.
.0	0.	-38169.	0.	-3643.	-21222.	-0.	-3643.	-703.	-0.	-2887.	-1487.	-0.	-2887.	8876.	-0.	0.	17605.	-0.

TRANSVERSE STRAIN (MUIN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
90.0	-1411.	-1765.	-0.	-1057.	-2711.	0.	-4719.	-2711.	0.	-4982.	-3417.	0.	-5142.	-2582.	0.	-4495.	-3593.	0.
80.0	-1354.	-1860.	0.	-1052.	-2728.	-120.	-4772.	-2728.	-991.	-5003.	-3438.	-175.	-5133.	-2603.	-21.	-4510.	-3563.	-0.
70.0	-1227.	-2134.	0.	-1040.	-2775.	-226.	-4925.	-2775.	-1862.	-5061.	-3499.	-330.	-5107.	-2663.	-43.	-4552.	-3483.	-0.
60.0	-1017.	-2554.	0.	-1020.	-2847.	-304.	-5161.	-2847.	-2509.	-5150.	-3591.	-444.	-5064.	-2756.	-54.	-4616.	-3352.	-0.
50.0	-750.	-3069.	0.	-996.	-2935.	-345.	-5449.	-2935.	-2854.	-5260.	-3705.	-505.	-5019.	-2869.	-61.	-4694.	-3195.	-0.
40.0	-456.	-3615.	0.	-971.	-3030.	-345.	-5757.	-3030.	-2854.	-5376.	-3825.	-505.	-4969.	-2990.	-61.	-4778.	-3027.	-0.
30.0	-229.	-4133.	0.	-947.	-3118.	-304.	-6045.	-3118.	-2509.	-5486.	-3935.	-444.	-4920.	-3103.	-54.	-4855.	-2873.	-0.
20.0	-19.	-4553.	0.	-928.	-3190.	-226.	-6291.	-3190.	-1863.	-5575.	-4032.	-330.	-4380.	-3195.	-40.	-4920.	-2742.	-0.
10.0	118.	-4827.	0.	-915.	-3238.	-120.	-6434.	-3238.	-991.	-5633.	-4092.	-176.	-4354.	-3256.	-21.	-4962.	-2659.	-0.
.0	165.	-4922.	0.	-911.	-3254.	-0.	-6483.	-3254.	-0.	-5653.	-4113.	-0.	-4345.	-3277.	-0.	-4976.	-2629.	-0.

LONG. STRESS(LB/IN/IN) AND STRAIN(MUIN/IN); SQRT(3*J2) IN (KLB/IN/IN); TRANSVERSE DISPLACEMENT(IN)

ANGLE (DEG)	ZZAP	ZZBM	ZZBP	ZZCM	ZZCP	ZZDM	EZZ	YAP	YBM	YBP	YCM	YCP	YDM	URA	UTA	URB	UTB	URC	UTC	URD	UTD
90.0	-15033.	-19463.	-587.	-697.	10832.	8105.	-3415.	13.9	15.7	1.6	1.4	17.0	7.5	-.0196	.0000	-.0202	.0000	-.0277	.0000	-.0333	.0079
80.0	-15390.	-19553.	-595.	-702.	10742.	8214.	-3415.	13.9	15.9	2.3	1.5	16.5	7.7	-.0191	-.0025	-.0197	-.0019	-.0272	-.0018	-.0328	.0060
70.0	-16417.	-19810.	-618.	-715.	10493.	8527.	-3415.	14.3	16.2	3.4	1.5	16.3	8.3	-.0174	-.0046	-.0180	-.0035	-.0256	-.0034	-.0313	.0042
60.0	-17990.	-20205.	-653.	-736.	10085.	9007.	-3415.	15.8	16.7	4.4	1.7	15.5	9.3	-.0143	-.0062	-.0153	-.0047	-.0233	-.0046	-.0289	.0027
50.0	-19920.	-20685.	-695.	-751.	9593.	9596.	-3415.	18.8	17.2	5.0	1.8	14.5	10.5	-.0117	-.0071	-.0121	-.0053	-.0234	-.0052	-.0260	.0014
40.0	-21974.	-21204.	-741.	-788.	9079.	10222.	-3415.	22.8	17.7	5.1	1.8	13.6	11.9	-.0084	-.0071	-.0087	-.0053	-.0173	-.0052	-.0230	.0006
30.0	-23904.	-21689.	-784.	-814.	8591.	10810.	-3415.	27.0	17.9	4.7	1.8	12.6	13.3	-.0052	-.0062	-.0055	-.0047	-.0144	-.0046	-.0201	.0001
20.0	-25477.	-22083.	-819.	-834.	8194.	11290.	-3415.	30.7	18.1	4.0	1.8	11.9	14.5	-.0027	-.0046	-.0029	-.0035	-.0125	-.0034	-.0177	-.0002
10.0	-26504.	-22341.	-842.	-848.	7934.	11603.	-3415.	33.1	18.2	3.2	1.8	11.4	15.3	-.0010	-.0025	-.0012	-.0018	-.0105	-.0018	-.0162	-.0001
.0	-26860.	-22430.	-850.	-852.	7844.	11712.	-3415.	34.0	18.2	2.9	1.8	11.3	15.5	-.0004	-.0000	-.0006	-.0000	-.0099	-.0000	-.0157	-.0000