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A RESISTIVE THEORY OF BUNCH LENGTHENING*

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SUMMARY

A new theory of bunch lengthening in electron storage rings is proposed. The equilibrium bunch length is that length which stabilizes the bunch against the onset of "fast" resistive instability, caused by the combination of many high frequency resonators such as vacuum flanges. The heat dissipated in these impedance sources follows immediately from the bunch length. It is found that the anomalous bunch length is determined by a scaling parameter $g=(hV\cos\varphi_s)/I$. Data taken in SPEAR I and II, data in which g extends in value by more than three orders of magnitude, can be fit with an appropriate choice of high frequency, large width coupling impedance. The impedance functions for SPEAR I and II are taken to be the same, a reflection of the fact that the high frequency sources are chamber discontinuities rather than structures connected with the rf systems. A parameter search leads to an impedance characterized by a central frequency ~ 5 GHz, a width (FWHM) ~ 1.8 GHz and a peak impedance ~ 0.2 MΩ. The expected and observed higher mode resistance (i.e.heat dissipated) for SPEAR are compared and found to be in agreement. Predictions are given for PEP and PETRA.

I. INTRODUCTION

We give here an overview of a new theory of bunch lengthening in electron storage rings. The method we use to present an account of the theory and its applications is through a sequence of snapshots or figures. These are meant to describe: (1) the line of reasoning that led to the theory, (2) the assumptions used to arrive at relations between observable variables, (3) the capacity of the theory for prediction, (4) tests of the theory from observations and measurements at SPEAR, and (5) extrapolation to the new machines under construction, PEP and PETRA. The paper is divided into sections. In section II, the theory of the "fast" longitudinal instability is given. Comparison of the theoretical predictions with observations at SPEAR I and II of both bunch length and higher mode heating is made in section III. We also make a few brief comments on the impact of the "unstable equilibrium electron state" on the beam quantum lifetime. In section IV considerations related to PEP and PETRA are given.

II. THEORY

Potential well models - predicts lengthening with no energy widening contrary to observation.

Turbulent state model - no specific experimental test.

"Fast" instability model: $\tau_{rev} < \tau_g < \tau_s < \tau_r$ - equilibrium a balance between beam induced high frequency fields and beam frequency spread (Landau damping). Experimental tests of theory: SPEAR I and II identical. Correlation with higher mode heating. Energy widening. Decrease in quantum lifetime. Frequency range of beam induced fields.

Fig. 1. Theories: Potential Well Models,[1-4] Turbulent State Models,[5,6] and "Fast" Longitudinal Instability.[7-10] Time Scales: τ_g (characteristic time for fast instability); τ_s (synchrotron oscillation period); τ_r (radiation damping time); τ_{rev} (revolution period).

Natural Equilibrium[11]:
Balance of radiation damping and quantum fluctuations-long time scale.
Natural bunch length (radians) - $\theta_{nat} = \sigma_o/R$

Unstable Equilibrium[8-10]:
Balance of beam induced fields and Landau damping due to frequency spread.
Short time scale.
Supercedes natural equilibrium.
Equilibrium rms bunch length - $\theta_{rms} = \sigma_{rms}/R$.
Threshold Current: I_{TH}
If $I \le I_{TH}$: θ_{nat} is bunch length
If $I > I_{TH}$: θ_{rms} is bunch length

Fig. 2. General Idea of "Fast" Instability Approach. σ_o is natural bunch length. R is the average machine radius, σ_{rms} is the rms equilibrium bunch length.

Theoretical Procedure[7-10]:

1. Find dispersion relation for oscillation frequency, ω, from Vlasov equation.
2. Take unperturbed solution to be separable in azimuth, θ, and energy, $x = \Delta E/E$. Find Gaussian shape: $\psi_o(\theta,x) = H(\theta)G(x)$, H, G normalized Gaussians.
3. Look for azimuthal coherent modes of the form: $\psi_1(\theta,x,t) = G_1(x)H(\theta)e^{i(n_o\theta-\omega t)}$. Instability is fast — only energy dissipation.
   Important: $G_1(x)$ perturbed form, $H(\theta)$ unperturbed form, n_o azimuthal mode number for single mode.
4. Energy transfer between source impedance and bunch dominant. Neglect smaller and slower energy exchange due to synchrotron motion (except for replacement of mean energy loss due to synchrotron radiation).
5. Revolution frequency spread in bunch implies Landau damping[12,13] and so an instability threshold.[14]
6. Average impedance over circumference (valid if $\tau_{rev} \ll \tau_g$). Induced field can be represented by translation invariant kernel.
7. Average Vlasov equation over azimuth to obtain simple dispersion relation for perturbed frequency.

Fig. 3. General Theoretical Procedure.

Beam Induced Electric Field[7,10,15]

$$\mathcal{E}(\theta,t) = -f_o\int Z(\theta-\theta')\,\lambda_1(\theta',t)d\theta'$$

f_o revolution frequency, Z translation invariant impedance kernel, and λ_1 induced linear charge distribution.

$$\lambda_1(\theta,t) = H(\theta)e^{i(n_o\theta-\omega t)}\tilde{\lambda}_1$$

$$\tilde{\lambda}_1 = (I/c)\int G_1(x)dx$$

Expand $\mathcal{E}$, Z: $\mathcal{E}(\theta,t) = \sum_n \mathcal{E}_n e^{i(n\theta-\omega t)}$

$$Z(\theta) = \sum_n Z_n e^{in\theta}$$

Z_n, usual impedance[14]

Find $\mathcal{E}_n = \tilde{\lambda}_1 f_o Z_n \int H(\theta')e^{i(n_o-n)\theta'}d\theta'$.

Fig. 4. Impedance and Beam Induced Field.

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Dispersion Relation:

Averaging Vlasov equation over θ:

$$1 = ieI/2\pi\eta E(Z_{eff}/n_o)\int[G'(x')/(y-x')]dx'$$

$$\eta = -(p/f_o)(\partial f_o/\partial p) = 1/\gamma_t^2 - 1/\gamma^2 ,$$

p = momentum, E = energy, I = average current, and $y = \omega/(2\pi f_o n_o \eta)$, $G(x) = (1/\sqrt{2\pi}x_{rms})e^{-(x^2/2x_{rms}^2)}$.

Effective impedance:

$$Z_{eff} = \sum_{n=-\infty}^{\infty} Z_n e^{-(n-n_o)^2\theta_{rms}^2} .$$

Many field modes (values of n) contribute to a single coherent beam mode (n_o). The number contributing is limited by the bunch mode spectrum — the exponential cutoff is a result of the Gaussian azimuthal distribution.

Comparison to coasting beam case $Z_{eff} = Z_n \delta_{nn_o}$. A single field mode contributes to a single beam mode.

Fig. 5. Dispersion Relation[15-17] and Effective Impedance.[7-10]

Solution to Dispersion Relation:

Condition for stability: $Im(\omega) < 0$ or $|Z_{eff}/n_o| < (2\pi\eta E/eI)x_{rms}^2$, induced force < frequency spread.

Scaling law: scaling parameter: g, $|Z_{eff}/n_o| < g\theta_{rms}^2$, $g = 2\pi E\nu_s^2/e\eta I = hV\cos\varphi_s k^2(I)/I$, V peak rf voltage, h harmonic number, φ_s stable phase angle, and $k(I)$ is current dependence of particle synchrotron wave number, ν_s.

Include condition that growth rate be "fast": modification of threshold with Z_{eff} real. Equation for equilibrium bunch length θ_{rms}, given Z_{eff}: $Z_{eff}/n_o = g\theta_{rms}^2[1 + (\alpha/\pi n_o\theta_{rms})]$, α = (growth rate)/(synchrotron frequency). Take $\alpha \approx 4$.

Fig. 6. Solution to dispersion relation,[15-17] scaling law[9,10] and equilibrium bunch length.[8-10]

Choice of Impedance:

What is the impedance source? We propose:

- A combination of many closely spaced high frequency resonators
- The sources are small discontinuities in the vacuum chamber (for example, vacuum flanges)
- Addition of resonances leads to a primarily resistive impedance
- Approximate impedance by a Lorentzian shape (a long-tailed function) $Z_n = Z_R[a^2/a^2+(n-n_o)^2]$, Z_R peak impedance at central frequency, f_c central frequency, $f_c = n_o f_o$, and Δf impedance function frequency width-full-width at half-height: $\Delta f = 2af_o$.

Effective impedance determined by 3 parameters, Z_R, f_c, and Δf, together with bunch length θ_{rms}: $Z_{eff} = Z_{eff}(Z_R, f_c, \Delta f, \theta_{rms}) = Z_R \sum_n [a^2/a^2+(n-n_o)^2]e^{-(n-n_o)^2\theta_{rms}^2}$.

Equilibrium bunch length equation: define $G = \{Z_{eff}/n_o\theta_{rms}^2[1+4/(\pi n_o\theta_{rms})]\}$, then, implicit equation for θ_{rms}, $G(Z_R, f_c, \Delta f, \theta_{rms}) = g$.

Fig. 7. Impedance and Implicit Equation for the Equilibrium Bunch Length.[9,10]

As $a \to \infty$ (broad impedance limit)

$Z_{eff} \to Z_R \int_{-\infty}^{\infty} e^{-m^2\theta_{rms}^2} dm = \sqrt{\pi}Z_R/\theta_{rms}$, this leads to $\theta_{rms} \propto I^{1/3}$, roughly what has been known for some time experimentally.

The questionable procedure of using coasting beam theory and replacing ad hoc the average current by the peak current[5] gives the same general result.

Fig. 8. Limit of very broad impedance.

Power dissipation in resistive ring elements: $P = \frac{1}{2} I^2 R_{hm}$.

Higher mode resistance[18]: $R_{hm} = \sum_n Re(Z_n)e^{-n^2\theta_{rms}^2}$.

Suggestion. Heating of ring elements directly correlated with bunch lengthening: both phenomena arise from the same resistive impedance.[9,10]

Implication. $R_{hm} = Z_R\sum_n[a^2/a^2+(n-n_o)^2]e^{-n^2\theta_{rms}^2}$.

Fig. 9. Relation between bunch lengthening and higher mode heating.

III. APPLICATION TO SPEAR

Plot θ_{rms} vs g for SPEAR I and II (Figs. 11 and 12). Fit observations on bunch length. Determine Z_R, f_c and Δf. They should be the same for SPEAR I and II since vacuum chamber unchanged in transition (only rf changed). Since other elements such as ferrite kickers were removed — current dependence of ν_s due to inductive impedance present in SPEAR I, but not in SPEAR II:

$k(I) = [1+25\ I^2]^{-\frac{1}{2}}$ SPEAR I
I in mA

$k(I) = 1$ SPEAR II

Plot R_{hm} vs θ_{rms} (Fig. 13). Use same values for Z_R, f_c and Δf. Compare with measurements on SPEAR II.

Fit to all 3 sets of data obtained with Z_R = 0.2 MΩ, f_c = 5.1 GHz and Δf = 1.8 GHz.

Fig. 10. Application of theory to data from SPEAR I and II.[19-21]

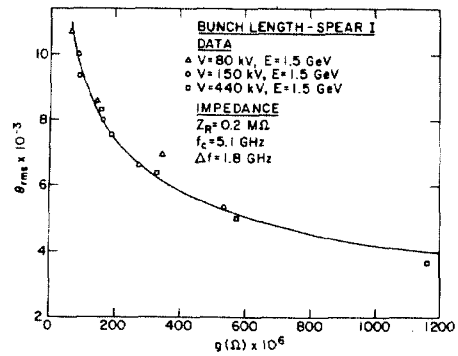

Fig. 11. Bunch Length vs Scaling Parameter, g (SPEAR I).

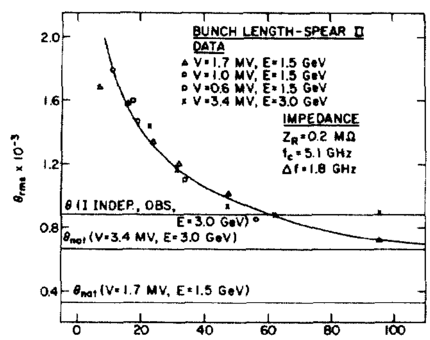

Fig. 12. Bunch Length vs Scaling Parameter, g (SPEAR II).

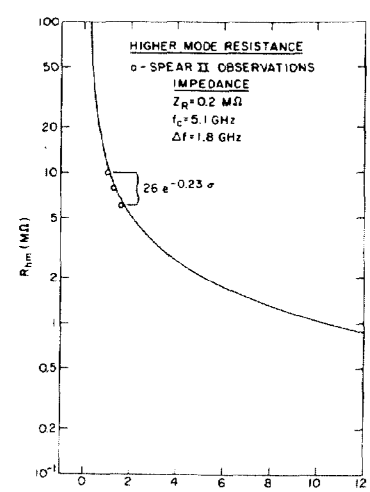

Fig. 13. Higher Mode Resistance vs Bunch Length (SPEAR II).

1. Single mode "fast" instability theory adequately describes the anomalous length of the electron bunch in SPEAR.
2. Scaling law followed over a wide range of the scaling parameter, g – over 3 orders of magnitude. (Note: The scaling is not strictly a consequence of the particular theory presented here, but undoubtedly has a wider significance.)
3. Suggestions that (1) the sources are small chamber discontinuities acting as high frequency resonators and (2) the resulting impedance function is broad and resistive have been shown to be consistent postulates.
4. Suggestion that bunch lengthening and higher mode heating are correlated and due to the same impedance source has been tested and appears to be a correct hypothesis. This is a strong test of the "fast" instability approach.
5. "Fast" instability theory in the class of theories predicting energy widening-consistent with observation.
6. Further tests of theory:
   - Predicts the presence of "small" coherent beam signals in the frequency region 4-6 GHz since the equilibrium is in the nature of an "unstable" state.
   - Effect on quantum lifetime of bunch core increase could be observable. Momentum orbits of core particles (those off the central momentum) are closer to "quantum diffusion sink".

Fig. 14. Discussion of Theoretical Fits[10] to SPEAR Data.

IV. PREDICTIONS FOR PEP AND PETRA

PEP and PETRA paramaters (Fig. 16). Assume f_c and Δf same as SPEAR since vacuum chamber design not too dissimilar.

Plot predicted bunch length (θ_{rms}) vs current (I). For PEP and PETRA (Fig. 17). For 3 values of Z_R: Z_R = 2.0 MΩ (equivalent to SPEAR), Z_R = 1.0 MΩ (2 times better than SPEAR) and Z_R = 0.2 MΩ (10 times better than SPEAR).

Plot predicted higher mode resistance (R_{hm}) vs current (I) for PEP and PETRA (Fig. 18) for Z_R = 2.0 MΩ, 1.0 MΩ and 0.2 MΩ. Use θ_{rms} vs I from previous plots.

Fig. 15. Predictions for PEP[22,23] and PETRA.[23]

PARAMETERS FOR PEP AND PETRA

PARAMETER	PEP Unscaled	PEP Scaled	PETRA Unscaled	PETRA Scaled
Energy, E(GeV)	15	--	15	--
Peak rf voltage, V (MV)	44.0	--	34.3	--
Magnetic Radius of curvature, ρ(m)	169.9	--	192.1	--
Energy loss, U_0 (MeV/turn)	26.4	--	23.3	--
Stable rf phase - cos φ_s	0.8	--	0.749	--
Revolution frequency, f_o (kHz)	138.5	--	130.2	--
Central frequency of impedance, f_c (GHz)	5.1	--	5.4	--
Impedance width, Δf (FWHM, GHz)	1.7	--	1.8	--
Design current, I (mA)	100	--	80	--
Number of bunches, n_B	3	--	3	--
Average radius, R(m)	344.9	115.0	366.7	122.2
Harmonic number, h	2589	863	2304	768
Mode number, n_o	39000	13000	39000	13000

Fig. 16. Table of Parameters for PEP and PETRA. Scaled means with reference to the number of bunches. Formulas apply with $\theta_{rms} = n_B\sigma_{rms}/R$ and both n and n_o should be scaled values. Since n_o is the scaled value, h should also be the scaled value.

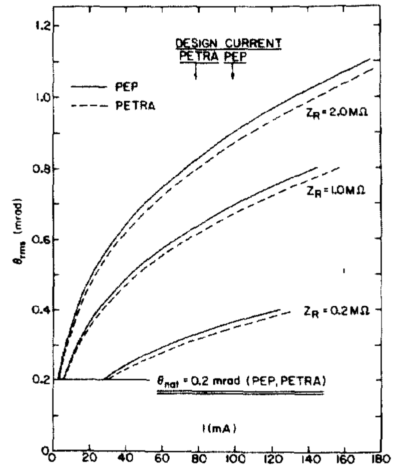

Fig. 17. Predicted Bunch Length vs Current for PEP and PETRA.

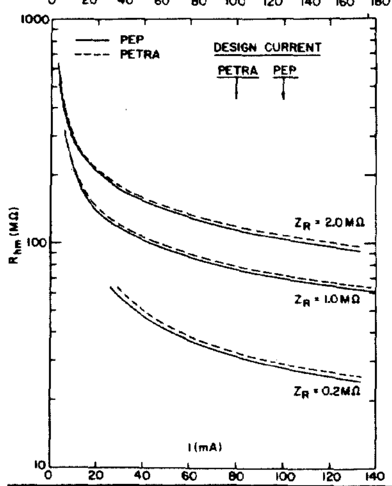

Fig. 18. Predicted Higher Mode Resistance vs Current for PEP and PETRA.

Theoretical Predictions

If impedance same strength as SPEAR ($Z_R \approx 2.0$ MΩ), to reach design currents in PEP and PETRA, bunch length > 4 x natural length, higher mode resistance > 100 MΩ.

If impedance strength 10 x better than SPEAR ($Z_R \approx 0.2$ MΩ), design current reached with bunch length < 2 x natural length, higher mode resistance ≈ 30 MΩ.

Fig. 19. Discussion of predictions for PEP and PETRA.

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