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MULTI-CAVITY PREBUNCHING SYSTEMS

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Resume

The theory of multi-cavity klystrons is briefly reviewed (as being equivalent to the proposed problem). The difficult aspect of the solution is that a broad band solution is antithetical to an arrangement which causes nearly all electrons to arrive at the "catcher cavity" (or final plane) at the same time. A spatial Fourier analysis solution is discussed, which constitutes an acceptable compromise.

Introduction

Shortly after the introduction of the single-gap cavity prebuncher as a component of the injection system of the linear electron accelerator[1] the question naturally arose whether one could, in analogy to klystrons, achieve improved performance by using a cascade of "driver" cavities. The computational attack on the problem proceeded in two directions; some analysts calculated the trajectories of particles through a sequence of gaps and drift spaces using ballistic theory,[2] others analyzed the "driver section" as a product of 4-poles for which the overall transform could be obtained by matrix multiplication, using a plasma wave beam model.[3]

In this case the cavities are characterized by a resonant frequency (ω_0) and figure of merit (Q) only. The techniques of "stagger tuning" is to achieve broad band response of the driver; this case, which is of no importance in accelerators, will not be considered here. We will assume the "synchronously" tuned case. For a drift space we may write[4]

$$\begin{vmatrix} i_b \\ v_b \end{vmatrix} = \begin{vmatrix} \cos\vartheta_{ba} & jg\sin\vartheta_{ba} \\ \frac{j}{g}\sin\vartheta_{ba} & \cos\vartheta_{ba} \end{vmatrix} \begin{vmatrix} i_a \\ v_a \end{vmatrix} e^{-j\psi_{ba}}$$

where $g = -\omega I/\omega_q u_o$
I, DC beam current
ω, driving frequency
u_o, DC beam velocity
$\omega_q = r\omega_p$
$\omega_p = (e/m)\ I/\varepsilon_o u_o$, reduced plasma frequency
$\theta_{ba} = \omega q\ d_{ba}/u_o$, plasma transit angle of drift space
d_{ba} = drift distance
$\psi_{ba} = \omega d_{ba}/u_o$, transit time angle of drift space
r = beam radius

For a gap,

$$\begin{vmatrix} i_b \\ v_b \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ M^2 Z(\omega)\frac{e}{mu} & 0 \end{vmatrix} \begin{vmatrix} i_a \\ v_a \end{vmatrix} e^{-j\psi_g}$$

where $Z(\omega)$, shunt impedance of the gap
M, beam coupling coefficient
ψ_g, transit angle of gap

Clearly, by a successive application of the transforms Eqs. (1) and (2) the overall transform for any structure, consisting of gaps and drift spaces (and a defined beam) can be computed. Suppose we are considering three cavities, Figure 1; then i_4 can be written, neglecting the phase factor,

$$i_4 = -j\sum_1^3 a_{nk} V_k$$

where

$$a_{nk} = M_k Y_o \sin\vartheta_{nk} \qquad Y_o = \frac{e}{mu_o}\left(\frac{\omega I}{\omega_q u_o}\right)$$

and the voltage across the gap of the n-th cavity

$$V_n = M_n Z_n i_n$$

where M_n, Z_n and i_n are the beam coupling coefficient, shunt impedance and exciting current of the n-th cavity, so that, expanding, we have for one, two and three cavities, respectively,

$$\frac{i_2}{V_1} = -jM_1Y_o\left(\sin\vartheta_{21}\right)$$
$$\frac{i_3}{V_1} = -jM_1Y_o\left(\sin\vartheta_{31} - jM_2^2Z_2Y_o\sin\vartheta_{32}\sin\vartheta_{21}\right)$$
$$\frac{i_4}{V_1} = -jM_1Y_o\left(\sin\vartheta_{41} - jM_2^2Z_2Y_o\sin\vartheta_{42}\sin\vartheta_{21} + jM_3^2Z_3Z_2Y_o\sin\vartheta_{43}\sin\vartheta_{32}\sin\vartheta_{31}\right)$$

Evidently, the current at the exit plane cannot be analyzed in analogy to a cascade of amplifying stages; the input to any stage depends upon input from all previous stages.

The procedure outlined above is obviously the space-charge wave theory using the Hahn-Ramo model, which has been shown to be equivalent to the Llewellyn-Peterson equations. It is discouraging to observe that the simple-minded ballistic theory (which completely disregards space-charge forces) often leads to results which are experimentally more accurate than the SCW theory. But the SCW theory is obviously a superior analysis; therefore a considerable amount of work has been done to remedy the shortcomings of this theory.

The foregoing remarks apply to a cascade of cavities in which each cavity (except the first) is excited by the beam current, which generates a voltage across its gap and which, in turn, reacts upon the beam.

Another possibility exists, of interest to accelerator engineering, which is the excitation of two or more cavity gaps with an applied RF signal. We consider here only the case of a two cavity pre-buncher, using the ballistic theory, calculated non-relativistically in the small signal approximation. The arrival time of an electron at the end of the second drift space (t_3) as a function of entry time into the first gap (t_1), neglecting gap transit time and expressed in phase, is

$$\varphi_3 = \varphi_1 + \varphi_o - (X_1 + X_2)\sin\varphi_1 - X_2\sin\left(\varphi_1 + \frac{\omega S_1}{u_o} - X_1\sin\varphi_1 + \psi\right) \tag{1}$$

where $\phi_o = \omega(s_1 + s_2)/u_o$

$X_1 = (\omega s_1/u_o)(V_1/2V_o)$

$X_2 = (\omega s_2/u_o)(V_2/2V_o)$

ψ, excitation phase shift between cavity gaps and, V_1 and V_2 are the gap voltages; V_o and u_o are the DC beam voltage and corresponding velocity. Adjusting the excitation phase so that $\psi = -\omega s_1/u_o$ causes $\phi_3-\phi_0$ to be anti-symmetrical about ϕ_1 and ought to be the best arrangement (as well as simplifying the mathematical relationship). Now, it can be seen that a minimum phase interval between the edges of the double-peaked bunch occurs for values of the second bunching parameter which makes $d\phi_3/d\phi_1 = 0$ when $\phi_1 - X_1 \sin\phi_1 = 0$ because then ϕ_3 is independent of X_2. Differentiating Eq. (1)

$$\frac{d\varphi_3}{d\varphi_1} = 1 - (X_1 + X_2)\cos\varphi_1 - X_2(\varphi_1 - X_1\sin\varphi_1)(1 - X_1\cos\varphi_1)$$

which vanishes when

$$X_2 = \frac{1 - X_1\cos\varphi_1}{1 + \cos\varphi_1 - X_1\cos\varphi_1}$$

This latter applies only to the phase limits of the bunch. From Eq. (1) and the condition $\phi_1 = X_1 \sin\phi_1$ (above) we find, after some algebraic manipulation that $\phi_3-\phi_0 = -\phi_1(s_2/s_1)$ or (rather unexpectedly) that the second drift tube ought to be short compared to the first drift tube because ϕ_1 ought to be as large as possible to compress the largest fraction of current into the bunch. The practical limit of the choice of s_2 is obviously derived from the small signal condition. Examination of trajectories shows that values of $s_2/s_1 = 0.25$, $X_1 = 2$, $X_2 = 1.15$ will compress five radian injected phase into one radian at the output (eighty percent capture).

Literature Cited

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3. G. Branch and T. Mihran, Trans IRE PGED ED-2, 3 (1955) based upon earlier work of W. Hahn, S. Ramo, E. Feeberg and others. A. Bers, ScD Thesis, MIT (1959).

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4. The wrtier is indebted to Dr. S. V. Yadvalli for the ideas in this section, based upon conversation when he (S.V.Y.) was consultant to Litton Industries Tube Division about 1960.

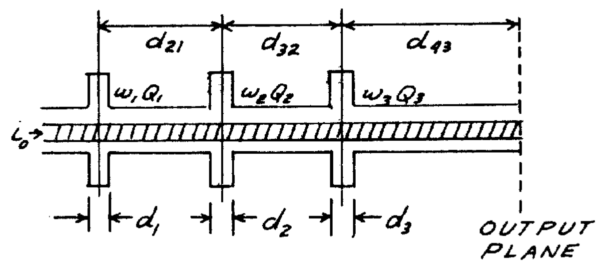

Fig.1. Parameters of 3-cavity structure.