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ON THE GENERATION AND FOCUSING OF INTENSE ION BEAMS FOR PELLET FUSION*

M. Reiser
University of Maryland
College Park, Maryland 20742

Abstract

The current limits of magnetic focusing systems are examined and approximate formulas for long solenoids and periodic channels with short lenses (solenoids and quadrupole doublets) are derived. The characteristics and limitations of conventional sources, pulsed ion diodes, and collective-field accelerators for the generation of intense, energetic ion beams are compared. Use of a collective-ion accelerator with debuncher is suggested as injector into an induction accelerator.

1. Introduction

The desired beam characteristics for pellet fusion with heavy ions and possible accelerator systems have been discussed and documented in the final report of the 1976 ERDA summer workshop on inertial fusion with heavy ions.[1] Typically, for uranium beams, one needs at the target -- depending on pellet radii between 1 and 10 mm -- final energies between 20 and 60 GeV, particle currents from 4 to 10 kA, normalized emittances of 1 to 10 cm-mrad in transverse direction, and 100-500 cm-mrad in longitudinal phase space. For charged particles of lighter mass than uranium, the energy is reduced to give the same range in the target and the current is accordingly higher. In the case of protons, for instance, energies are about 10 MeV and currents are in the range of 10 MA.[2] Pulsed high-voltage ion diodes appear to be the most promising approach to obtain the proton beam powers required, and 1 MeV, 250 kA beams have already been achieved in experiments.[3] Focusing and transport of the beams to the target, repetition rate, and reproducibility are the major problems with ion diodes that need intensive studies in the future. Pellet fusion with high-energy heavy ions, such as uranium, on the other hand, brings into play, and is almost within reach of, the well-developed technology in the particle accelerator field, as was first suggested by Martin[4] and Maschke.[5] While the systems proposed are very large in physical size compared to ion diodes, the electromagnetic energy is distributed over a large volume, and the repetition rate should therefore not be as difficult to achieve.

With regard to heavy-ion fusion, the systems being considered at this time fall into two groups: linac-synchrotron combinations and the induction accelerator proposed by Keefe.[1] Common to both systems is the feature that they start with low currents in long pulses (though the induction accelerator can accept higher currents than a linac) which by various techniques have to be compressed into a few nanoseconds when they hit the target. The focusing problem is therefore most severe at the high-energy end of the accelerator where it was studied by Garren[1] and at low energies near the ion source. In the present paper, we will examine the current limits of solenoidal and quadrupole focusing channels (Section 2) and the possible use of collective-field accelerators as ion source/injector systems for an induction accelerator (Section 3).

2. Beam Focusing and Space Charge Limits in Solenoidal and Quadrupole Channels

The variation of the cross section of a beam propagating along the z-axis in a focusing channel can be determined by equation[6]

$$\frac{d^2x_m}{dz^2} + \kappa_x x_m - \frac{g}{x_m + y_m} - \frac{\varepsilon_x^2}{x_m^3} = 0 \tag{1}$$

for the envelope x_m and a similar equation for y_m.

The function $\kappa_{x(y)}$ represents the applied focusing/defocusing forces, $\pi\varepsilon_{x(y)}$ the beam emittance, and g the space-charge force given by the expression (in MKS units)

$$g = \frac{qI(1-f-\beta^2)}{\pi\varepsilon_o m_o c^3\beta^3\gamma} = 1.29 \times 10^{-7}\,\frac{Z}{A}\,\frac{I(1-f-\beta^2)}{\beta^3\gamma}\ . \tag{2}$$

I is the instantaneous current, $q = Ze$ the total charge of the ion, A the mass number (in terms of the atomic mass unit of 931.48 MeV), β, γ are the usual relativistic velocity and energy parameters, and f represents a fractional charge neutralization due to low-energy electrons in the beam. For ideal magnetic quadrupoles, we have

$$\kappa_{x(y)} = \kappa_q = \pm\frac{qB_o}{m_o c\beta\gamma a} = \pm\,0.322\,\frac{Z}{A}\,\frac{B_o}{\beta\gamma a}\ . \tag{3}$$

Where the sign depends on the field polarity, B_o is the pole-tip field strength and a the semi-aperture.

For an axisymmetric beam propagating through solenoidal magnetic fields, we have $x_m = y_m = r_m$ and obtain a single equation for the radial beam envelope r_m:

$$\frac{d^2r_m}{dz^2} + \kappa_s r_m - \frac{g}{2r_m} - \frac{\varepsilon_r^2}{r_m^3} = 0\ , \tag{4}$$

with

$$\kappa_s = \left(\frac{qB_s}{2m_o c\beta\gamma}\right)^2 = \left(\frac{\omega_L}{\beta c}\right)^2 = 2.59\times10^{-2}\left(\frac{Z}{A}\right)^2\left(\frac{B_s}{\beta\gamma}\right)^2 . \tag{5}$$

$B_s(z)$ is the magnetic field on the axis of the solenoidal channel (in teslas) and ω_L is the Larmor frequency. Eq. (4) applies for a beam launched in a region of zero magnetic field. For a nonzero field at the source, one has to replace ε_r^2 by the term $[\varepsilon_r^2 + (p_\Theta/m_o c\beta\gamma)^2]$; p_Θ is the canonical angular momentum which evidently has the same effect as an increase of the emittance.

We now examine the current-handling capability of a long solenoid and a periodic focusing system with either short solenoid lenses or quadrupole doublets.

(a) Uniformly focused beam in a long solenoid. Let us assume that the beam propagates in a conducting pipe of radius a and that the solenoidal coil windings around the pipe produce a uniform axial magnetic field. The optimum situation of uniform focusing is achieved when the beam is matched, i.e., when $r_m = a =$ constant, and hence, from Eq. (4), when

$$\frac{g}{2} = \kappa_s a^2 - \frac{\varepsilon_r^2}{a^2}\ . \tag{6}$$

With (2) and (5), we can solve for the beam current I and find:

$$I = 2\pi\varepsilon_o m_o c^2 \frac{\beta\omega_L}{1-f-\beta^2} A_N\left[1-\left(\frac{\varepsilon_N}{A_N}\right)^2\right] , \qquad (7)$$

where $\pi\varepsilon_N = \pi\varepsilon_r\beta\gamma$ represents the normalized emittance of the beam and $A_N\pi = \omega_L\gamma a^2\pi/c = 0.16\pi(Z/A)B_s a^2$ is the normalized acceptance of the channel in the limit of zero space charge. From this expression we draw the important conclusion that the maximum current that can be transported in a solenoidal channel (given the particle and channel parameters) decreases with increasing emittance ε_N of the beam. In fact, for high-current transport, the beam emittance should always remain significantly less than the channel acceptance, say, $\varepsilon_N < A_N/3$, in which case it may be neglected. This result differs from the formulae by Maschke and Courant[7] who find that the current increases with the emittance as $\varepsilon_N^{2/3}$.

Substituting numerical values for the constants in (7), we obtain, with $f = 0$, for the maximum current,

$$I_{[A]} = 4.0\times 10^5 \frac{Z}{A}\beta\gamma(B_s a)^2\left[1-\left(\frac{\varepsilon_N}{A_N}\right)^2\right] , \qquad (8)$$

For $f \neq 0$, replace γ by $[\gamma(1-f-\beta^2)]^{-1}$. Neglecting the emittance term, we obtain for the beam power $P = (\gamma-1)(m_o c^2/q)I$ the expression

$$P_{[w]} = 3.75\times 10^{14}\beta\gamma(\gamma-1)(B_s a)^2. \qquad (9)$$

In the nonrelativistic limit, the current is

$$I_{[A]} = 1.865\times 10^4 \frac{Z}{A}\left(\frac{T}{A}\right)^{1/2}_{[MeV]}(B_s a)^2_{[T-m]} , \qquad (10)$$

where T/A is the kinetic energy per nucleon in MeV.

From these results, we conclude that the maximum particle current I/Z transported by the channel is independent of the charge state Z, and the power depends only on the energy per nucleon for given channel parameters ($B_s a$) contrary to reference[7]. As an example, for 40 GeV uranium ($\beta = 0.53$, $\gamma = 1.18$, $B_s = 5$ T, $a = 0.2$m, we get $I/Z \approx 1$ kA, $P \approx 40$ TW. At 1 MeV per nucleon, the uranium particle current in the same channel is $I/Z \approx 80$ A. This charge-state independence is due to the fact that both the space-charge defocusing term g as well as the focusing term κ_s in Eq. (4) are proportional to Z^2.

(b) Periodic focusing with short solenoidal lenses. We now examine a channel consisting of short solenoids of length L separated by a distance s. In such a periodic system, the beam envelope $r_m(z)$ has its minimum at half distance between lenses. We seek an approximate solution by assuming that the lens is short compared to the focal length f (i.e., $L < f$) and by neglecting the space-charge effect within the lens (but not in the drift space between the lenses). The focal length for a short solenoid is given by

$$\frac{1}{f} = \kappa_s L = \left(\frac{qB_s}{2m_o c\beta\gamma}\right)^2 L, \text{ with } \frac{f}{L} << 1. \qquad (11)$$

The beam envelope has its maximum at the center of the lens ($r_m = a$), and periodic focusing requires that at each lens the slope $(dr_m/dz) = r_m'$ changes from r_o' to $-r_o'$, which implies $f^{-1} = r_o'/a$. In drift space between lenses, the envelope can be determined from Eq. (4) with $\kappa_s = 0$; as in case (a), the emittance term can be neglected. The solution of the resulting equation $r_m r_m'' = g/2$ has been discussed by Pierce.[8] With initial conditions $r_m = a$, $r_m' = r_o'$ at $z = 0$ and the substitutions $R = r_m/a$, $\zeta = \sqrt{g}\, z/a$, $R' = dR/d\zeta = r_m'/\sqrt{g}$, one gets the solution

$$\zeta = \int_1^R \frac{dR}{[\ln R + R_o'^2]^{1/2}} = 2e^{-R_o'^2}\int_{R_o'}^{\pm[\ln R+R_o'^2]^{1/2}} e^{u^2}du \qquad (12)$$

where $R_o' = r_o'/\sqrt{g}$. The envelope of the converging beam that leaves a lens ($R = 1$, $z = 0$) decreases to a minimum value given by $\exp(-R_o'^2)$ (negative sign in the upper integral limit) and then increases again (positive sign) until it reaches the maximum $R = 1$ at the next lens ($z = s$). As the initial convergence is increased, one finds that ζ increases at first, reaches a maximum value of $\zeta \approx 2.16$ when $R_o' = r_o'/\sqrt{g} \approx -0.92$, and then decreases again as R_o' goes to larger negative values. For given lens separation s, this means that the space-charge factor g, and hence the current I, reach a maximum defined by the conditions

$$g = 2.16^2\left(\frac{a}{s}\right)^2 , \text{ or } I = 2.16^2\frac{\pi\varepsilon_o m_o c^3\beta^3\gamma}{q(1-f-\beta^2)}\left(\frac{a}{s}\right)^2 \qquad (13)$$

and

$$\frac{1}{f} = \kappa_s L = \frac{3.974}{s} . \qquad (14)$$

The thin-lens approximation implies $f > L$ and therefore $s > 3.974$ L. Combining Eqs. (11), (13), and (14), we obtain for the maximum current that can be transported through the system the result (for $f = 0$)

$$I_{[A]} = 2.36\times 10^5 \frac{L}{s}\frac{Z}{A}(Ba)^2\beta\gamma = 0.59\, I_s\frac{L}{s} , \qquad (15)$$

where I_s is the maximum current in a long solenoid given by (8) (for $\varepsilon_N = 0$) or (10). When $f \neq 0$, replace γ by $[\gamma(1-f-\beta^2)]^{-1}$. In view of the thin-lens condition $s > 4L$, we see that $I < 0.15\, I_s$, i.e., the maximum current in such a periodic channel with short solenoids is considerably less than in a long solenoid, as one would expect. The thin-lens approximation implies $(\kappa_s L)^2 < 1$ which yields the nonrelativistic criterion $(T/A)[\text{MeV}] > 12(L/a)^2(Z/A)^2(B_s a)^2$. If one abandons the thin-lens approximation and places the short solenoids closer together, one can of course obtain currents above the limit (15) up to $I = I_s$ for a continuous solenoidal field. However, in practice, economic reasons and space requirements dictate the use of a periodic system and $s \geq 4L$ is a realistic assumption. A discussion of the stability of the solutions is beyond the scope of this paper. We merely note that right at the limit $\zeta = 2.16$, the flow is unstable and that one should operate at a somewhat lower current. Eq. (15) represents, however, a useful upper limit and scaling law for conceptual design studies.

(c) Periodic focusing with short quadrupole doublets. The above theory can be applied to a periodic channel with short quadrupole doublets when the envelopes in both directions do not differ significantly (i.e., $x_m \approx y_m = r_m$). In the thin-lens approximation, the focal length of a doublet is given by

$$\frac{1}{F} = \kappa_q^2 L_1^2 L_o = \left(\frac{qB_o}{m_o c\beta\gamma a}\right)^2 L_1^2 L_o , \qquad (16)$$

where L_1 is the length of each quadrupole section and L_o the separation distance. In place of (14), we have the condition $s = 3.974$ F. For comparison with case (b), we take $L_o = L_1 = L/2$, i.e., the quadrupole doublet has the same length as the short solenoid. The maximum current in this channel is then (with $f = 0$) found to be

$$I_{[A]} = 9.45\times 10^5\left(\frac{L}{a}\right)^2\frac{L}{s}\frac{Z}{A}(B_o a)^2\beta\gamma = 2.36\, I_s\left(\frac{L}{a}\right)^2\frac{L}{s} , \qquad (17)$$

where $s \geq 4L$. It is larger by a factor $4(L/a)^2$ than with solenoid lenses. (If $L = 2a$ and $s = 4L$, we get

$I = 2.35\ I_s$.) As in case (b), Eq. (17) represents an upper limit for thin lenses. In general, the stability of the flow has to be examined by numerical solution of Eq. (1).

3. Generation of Intense, Energetic Ion Beams

Conventional Ion Sources. The current limitations of focusing systems at low energies requires an ion source or an injector capable of producing heavy ion beams with as high an energy as possible. The typical ion source is basically a diode where the "cathode" is replaced by an ion emitter. If V_o is the diode (or extraction) voltage and d the gap spacing, the current density J is limited according to Child's law

$$J = 1.73\ V_{o[kV]}^{3/2} d_{[cm]}^{-2} \sum (Z_i/A_i)^{1/2} \quad [mA/cm^2]\ , \qquad (18)$$

where Z_i, A_i are the charge states and mass numbers of the various ion species contained in the beam. Conventional sources operate below the electrical breakdown limit $V_{B[kV]} = k\sqrt{d}_{[cm]}$ (where $k \sim 10^2$) which necessitates a compromise between achieving high luminosity (small d) and sufficient focusing (high V_o). In heavy-ion sources, the output current drops rapidly for higher charge states, which tends to offset a decrease in emittance as Z increases.[9] Heavy ion currents from conventional sources are probably limited to 100 mA, with voltages of 100 kV. To get kA of uranium current at the pellet, current multiplication of 4×10^4 would then be required.

Pulsed Ion Diodes. The only alternative for achieving higher currents is to operate the diode above the breakdown limit as is the case in intense relativistic electron beam (IREB) generators and their application to produce intense ion beams. Here, V_o is in the range of megavolts and d in the range of centimeters. Operation in the breakdown regime implies a very short pulse length (30-100 ns) as the diode gap is quickly shorted out (impedance collapse). In such pulsed ion diodes, the emphasis so far has been on the generation of very high proton currents suitable for fusion. Sources of this type could undoubtedly be developed for heavy ions. Large energy spread, poor emittance, and achieving the repetition rate could be a problem, though no systematic measurements have been made so far.

Collective Ion Accelerators. Even with ion diodes, the energies per nucleon for heavy ions, like uranium -- though considerably higher than in conventional sources -- would only be in the 10 keV range. For this reason, collective ion accelerators, where energies many times higher than V_o can be achieved, appear to be the most attractive source for heavy ions. In the method pioneered by Luce and collaborators at Livermore,[10] the electron beam from an IREB generator forms a plasma in the anode from which positive ions are extracted and accelerated by the electron space charge with the use of special electrodes. Luce has observed protons of 45 MeV and heavier ions (C, F) of 7 MeV per nucleon with diode voltages of 1 MV corresponding to energy amplification factors of over 40. Boyer, Kim, and Zorn[11], in similar experiments at our laboratory, confirmed the high energy amplification that can be achieved with this type of collective field accelerator. With electron beams of <1 MV, 30 kA, and 30 ns pulse width, they obtained 12 kA, 4 ns proton bunches with a sharply defined peak of 16 MeV and a radius of less than 0.5 cm when a magnetic field of 1.8 kG was applied. The ion energy appears to be proportional to the peak electron beam power.

Although many aspects of collective accelerators are not yet fully understood, the experimental data indicates that they can produce heavy ion beams with energies of order 1 MeV per nucleon and currents of order 10^2 to 10^3 amperes, i.e., they are suitable as injectors for fusion accelerators. The short pulse length of 5-10 ns requires debunching prior to injection while the ions are still traveling inside the electron beam and a solenoidal magnetic field.

The problem of longitudinal bunching of intense ion beams was studied by L. Smith (in reference 1). We estimate the debunching effect due to the ion space charge only following a slightly different approach. Let $2z_m$ be the bunch length, N the number of ions, a the beam and pipe radius, ϕ_o the potential on axis at the center of the bunch. Then $E_z(z_m) \approx (\phi_o/a)h$, where h is a geometry factor of order unity, and the equation of motion for z_m versus distance s is found to be:

$$\frac{d^2 z_m}{ds^2} = \frac{(Ze)^2 hN}{\gamma^3 m_o 8\pi\varepsilon_o \beta^2 c^2 a z_m} = \frac{K}{z_m}\ , \qquad (19)$$

$K = 0.36 \times 10^{-15} (Z^2/A) hN/(a\sqrt{T/A})$ in the nonrelativistic limit, where a is in meters and T/A in MeV. With $z_o = z_m(0)$, $\zeta = \sqrt{2K}\ s/z_o$, $R = z_m/z_o$, the solution of (19) is formally identical with (12). As an example, take a beam of U^+ with $I = 500$ A, pulse width $\Delta\tau = 10$ ns, $T/A = 1$ MeV, $a = 3$ cm, $z_o' = 0$, where (with $h = 1$) $N = 3 \times 10^{13}$, $K = 1.575 \times 10^{-3}$, and $2z_o = 13.7$ cm. From tables of the integral function in (12), one finds that $\zeta = 8.5$ when $R = 10$, which yields a drift length of 10.4 m for a factor 10 increase of bunch length with these parameters. If one wants a 4 kA, 5 ns beam at full energy, one could stack four 500 A, 10 ns beams after debunching. One then has 50 A in a pulse of 400 ns. To obtain 5 ns at full energy, a compression factor of 80 is required. In view of the charge-state independence of the focusing channel, charge stripping could be employed to reduce the length of the accelerator.

In contrast to pellet fusion with protons, only a very small fraction of the final energy must be provided by the IREB generator when heavy ions are used. The electron-beam power in this case should make the problem of achieving the desired repetition rates more comparable to that of existing induction accelerators.

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