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MEASUREMENT OF THE LINEAR COUPLING IN THE BROOKHAVEN AGS*

E.C. Raka
Brookhaven National Laboratory
Upton, New York 11973

Summary

The magnitude and sign of the zeroth harmonic skew quadrupole component of the magnetic field at 28.5 BeV are determined by exciting the normal mode frequencies in part of the debunched beam present during a flat top extraction cycle. Simple rf excitation of the (9-Q) mode is employed. Filtered difference signals from pick-up electrodes are used to measure the frequencies and relative phases of the H and V oscillations. During acceleration when the beam is bunched it is kicked horizontally and the radial position adjusted until the coupled vertical motion in the (9-Q) mode reaches a maximum. Correction quadrupoles are then powered to minimize the observed amplitude. The magnitude of the coupling roughly tracks with the beam momentum. Saturation effects at high fields plus the powering of back-leg bumps and tuning quadrupoles on the SEB flat top are possible sources of the somewhat larger coupling observed under these conditions.

Introduction

During normal operation of the slow extracted beam in the Brookhaven AGS the proton beam is debunched in such a manner as to produce a momentum spread of ≈ 0.8%. This results in a beam whose radial spread due to momentum is ≈ 1 cm at a β_{min}. When a transverse rf field is applied to this beam at frequencies (9-Q) ω_0 where ω_0 is the rotation frequency and Q varies over the expected range of betatron tune, the amplitude of the observed coherent betatron oscillations exhibits a sharp peak at two distinct frequencies. These frequencies are the same for vertical or horizontal excitation and coherent oscillations are observed on pick-up plates sensitive to both horizontal and vertical motion. The H and V motion is in phase at one frequency and 180° out of phase at the other. If these two frequencies are applied simultaneously one to each plane, then the classical interference pattern of coupled linear oscillators is observed.

This behavior can be interpreted with the aid of Fig. 2 which is a plot of the betatron frequencies as a function of position at 28.5 BeV. The beam was bunched when this data was taken and thus Δr is proportional to its momentum,the zero being arbitrary. The essentially straight lines obtained at this energy indicate a negligible octupole component in the magnetic field. If there is linear coupling present, these lines will not intersect at $\Delta r = 0$, but will diverge as shown and the separation will depend upon the strength of the coupling. For the coupling introduced by a zero theta skew quadrupole field it can be shown that the tune vs momentum will have the shape shown when the slopes have opposite signs and are non-zero. There will be two points where the slope is zero and a band of frequencies of the order of the frequency splitting which contain no particles.

On the SEB flat top the debunched beam occupies a region to the left of the extraction radius shown in Fig. 2. Although the relative values of Q_x and Q_y can change somewhat with operating conditions, the beam is always spread across the region where $Q_x = Q_y$ (in the absence of coupling). Hence, if coupling is present some part of the beam will be strongly affected as the negative flat top slope moves it toward the $Q_x = 8\ {}^2/_3$ resonance. The distinct peaks observed with transverse rf excitation of the beam correspond to the frequencies where the slopes are zero in Fig. 2, i.e. points 1 and 2. If one waits until late in the flat top before introducing the excitation, then only the peak at point 1 is seen.

Figure 1 shows a typical response at one of these frequencies. The excitation lasted three milliseconds and was in the vertical plane. The slow decay of the coherent signal can last for many milliseconds since the frequency spread in that part of the beam that is excited is quite small. When excitation is at frequencies above point 1 or below point 2, the response is much smaller because the non-zero slope of the curves results in less protons per unit frequency interval. The decay is very short and the coherence is essentially confined to the plane of excitation. If the excitation is at frequencies between 1 and 2 the response can be an order of magnitude or more smaller depending upon the coupling and the frequency. In general it will not be zero even though there is a hole in the betatron frequency spectrum here.[1]

Coupling Analysis

It can be shown[2] that if the coupling is due to a zero-th harmonic skew quadrupole component the normal mode frequencies will be given by Eq. (1). ($z \equiv y$).

$$Q_X = \left\{\frac{Q_x^2 + Q_z^2}{2} - \sqrt{\left(\frac{Q_z^2 - Q_x^2}{2}\right)^2 + C_x C_z}\right\}^{1/2}$$
$$Q_Z = \left\{\frac{Q_x^2 + Q_z^2}{2} + \sqrt{\left(\frac{Q_z^2 - Q_x^2}{2}\right)^2 + C_x C_z}\right\}^{1/2} \qquad (1)$$

$$\tan\alpha = \frac{C_x}{\dfrac{Q_x^2 - Q_z^2}{2} - \sqrt{\left(\dfrac{Q_x^2 - Q_z^2}{2}\right)^2 + C_x C_z}}$$
$$\tan\beta = \frac{C_z}{\dfrac{Q_x^2 - Q_z^2}{2} - \sqrt{\left[\dfrac{Q_x^2 - Q_z^2}{2}\right]^2 + C_x C_z}} \qquad (2)$$

$$C_{x,z} = -\frac{Q_{x,z}}{2\pi}\oint_C K(s)\sqrt{\beta_x\beta_z}\,ds \qquad (3)$$

$$x_\alpha = A\cos\alpha\cos(2\pi Q_X t + \varphi_1)$$
$$z_\alpha = A\sin\alpha\cos(2\pi Q_X t + \varphi_1) \qquad (4)$$

$$x_\beta = -B\sin\beta\cos(2\pi Q_Z t + \varphi_2)$$
$$z_\beta = B\cos\beta\cos(2\pi Q_Z t + \varphi_2) \qquad (5)$$

Here $Q_z > Q_x$ are the uncoupled frequencies and the expression represents Fig. 2 for $\Delta r > 0$. For $\Delta r < 0$ $Q_x > Q_z$ and one must interchange x and z and X and Z to obtain the other half of the plot. In Eq. (2) α is the angle that the lowest frequency mode makes with the x axis and β the angle that the highest frequency mode

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makes with the z axis. When $Q_x > Q_z$ one must interchange x and z in these expressions also. In Eq. (3) K(s) is the skew gradient and $C_{x,z}$ is the coupling coefficient. For excitation at either of the normal mode frequencies the response in the x and z planes will be given by Eqs. (4) or (5) where A and B depend upon the betatron frequency distribution and the nature of the excitation.[3,4] Here when $Q_x > Q_z$ one must interchange Q_X with Q_Z. Measurement of the relative phase of the motion for a given mode can thus be used in conjunction with Eq. (2) to determine the sign of the coupling.

In order to measure the strength of the coupling with accuracy one must determine Q_Z and Q_X or their difference at known values of Q_x and Q_z. If $Q_x=Q_z=Q$ then only the difference $(Q_Z-Q_X) \cong C/Q$ would be required, and this could be obtained if one kicked a bunched beam in one plane with a pulse whose duration $\leq \tau_o$ the revolution period. Both normal modes would be excited and the beat frequency (Q_Z-Q_X) modulation of the coherent oscillation amplitudes would be observed as the energy is transferred from one plane to the other (Fig. 3). An accurate measurement of this envelope frequency is generally not possible however even if $Q_x = Q_z$ for reasons outlined below.

Returning to Fig. 2 we see that the frequencies at points 1 and 2 are for normal modes at slightly different momenta ($\Delta p/p \approx .84 \times 10^{-3}$). If we take $|C| = \bar{Q}|\delta Q|$ where $\bar{Q}$ is the average of the two measured values and $|\delta Q|$ their difference, it will always be less than the correct value since the separation at $\Delta r = 0$ is always larger as long as the uncoupled slopes are of opposite sign and not equal. In principle C can be determined more precisely only if two or more additional quantities such as the angles α and β or the slopes are known. One can obtain α or β by measuring the ratio of the x and y amplitudes at various positions around the ring when one of the normal modes is excited. The zero-th harmonic of this distribution would give the angle and in principle one could determine the harmonics up to M/2 where M is the number of positions monitored. However, a sufficiently accurate measurement of the angles or the slopes would be extremely difficult. Since the frequencies at points 1 and 2 could be measured quite accurately, the simple expression given above was used to determine the required correction field.

Flat Top Results

Vertical excitation was used early in the flat top at $\approx$ 29.4 BeV/c. Its duration was two milliseconds with a peak strength of .5 microradians per revolution. The driving frequencies were 102.00 kc and 111.20 kc but the beam frequency was measured at the beginning of the decay period with an accuracy of one part in 10^4. For point 1 the value was 101.61 ± .13 kc and for point 2 110.45 ± .11 kc. The short term stability was exceptional when this data was obtained. Thus $Q_1 = 9 - 101.6/371.4 = 9 - .273\underline{6} = 8.726\underline{4}$ and $Q_2 = 9 - 110.4/371.4 = 9 - .297\underline{3} = 8.702\underline{7}$ and $\delta f = 8.84 \pm .17$ kc or $\delta Q = 8.84/371.4 = .0238 \pm .0004$. This gives for $C = \bar{Q}\delta Q = (Q_1+Q_2)\delta Q/2 = 8.714\underline{6}\ \delta Q = .207\underline{4}$.

It was observed that for point 1 the x and z motion were 180^o out of phase while for point 2 they were in phase. Hence C is negative and $\int K(s)$ in Eq. (3) is positive. Using Eq. (3) with $B\rho = 10^8$ gauss/cm, $\sqrt{\beta_x\beta_y} = 15$ meters, and $Q_{x,z} = \bar{Q}$ we obtain for the above value of C an integrated gradient x length of 10^4 gauss as the required skew quadrupole strength to correct the observed coupling.

In the late spring of 1974 four existing skew quadrupoles on the AGS ring were used to obtain the necessary field. When the correction was applied the two peak frequencies moved closer together as expected. However, when the current was within less than 10% of the calculated value the peaks became less distinct and reproducible. Coherence in both planes with a slow decay could still be observed over two regions separated by 1 to 1.5 kc but not consistently. This was true on either side of the value where complete cancellation was expected. There was always some energy coupled to the other plane on most machine cycles in this region. The reason for this is not yet understood (see below). These quadrupoles are now routinely used during slow beam operation to remove an observable tilt in the extracted beam.

Bunched Beam Measurements

Using a horizontal kicker magnet whose pulse duration is $\approx \tau_o$ the coupling was measured at 14.7, 26 and $\approx$ 29.4 BeV/c with a low intensity bunched beam. In each case the beam radius was adjusted until the motion coupled into the vertical plane was a maximum or if a beat pattern of considerable duration was present (Fig. 3) a zero amplitude minimum was the criterion. Then the correction quadrupoles were excited until the motion coupled into the vertical plane was a minimum. Again complete cancellation was not achieved. The required currents were 27, 52 and 58 to 60 amperes, the latter being within the 55-60 A measured on the flat top. Thus the source of the coupling scales with the momentum with a slight increase at the higher field ($\approx$ 11.7 kGauss). The observed strength would correspond to rotating all the 240 AGS magnets by $\approx$ .37 milliradians. It is felt that the net effect could be due to random misalignments[5] but some systematic asymmetry present in the magnets and/or their supports cannot be ruled out.

One characteristic of the signals shown in Fig. 3 indicates a limitation in measuring coupling strength or betatron frequencies with a bunched beam. The period of the beat frequency is increasing with time which means that the $(9-Q)f_o$ component of the motion of the center of charge of the beam changes with time. This effect is due to the tune spread in the beam and has been calculated[6] for essentially the conditions present here, i.e. no tune spread in the unkicked plane, $Q_x \approx Q_z$ and a coupling strength of $\delta Q/2 \approx .012$. Even in the absence of coupling the coherence in the kicked plane would be damped due to the tune spread resulting in a variation of observed betatron frequency with time. As the coupling is reduced toward zero, the beat frequency will decrease and if the period becomes an appreciable fraction of a phase oscillation cycle, additional modulation of the coherent signal can be expected.[7] Figure 4 shows the minimized vertical coherence present at 14.7 BeV/c on two different occasions. The peak amplitude is $\approx$ 1/10 of that with no correction. On the same scale the horizontal signal would be essentially zero by the time the vertical reached its peak value. Here the horizontal and vertical slopes are both negative but the latter is small compared to the horizontal. The bottom photo encompasses a little over half a phase oscillation period ($\tau_{\phi}/2 \approx 4.6$ msec) and in it one can see a distinct echo at this time. Usually a horizontal signal is just barely discernible also. The upper photo at .2 msec/div shows a definite beat pattern which however does not occur on every pulse. One can also observe beat patterns in the echo. The presence of the echo is quite sensitive to the correction and radial position and can serve apparently to locate the $Q_x = Q_z$ point. At 29.4 BeV/c the vertical signal is similar but rises somewhat slower to a larger relative amplitude, i.e. 1/5 to 1/7 of the uncorrected value. However at a given energy the peak amplitude varies linearly with the horizontal kick.

Because of the various effect mentioned above it is not clear that the observed amplitudes are a very accurate measure of the coupled energy. The significance of these observations is still under investigation.

References

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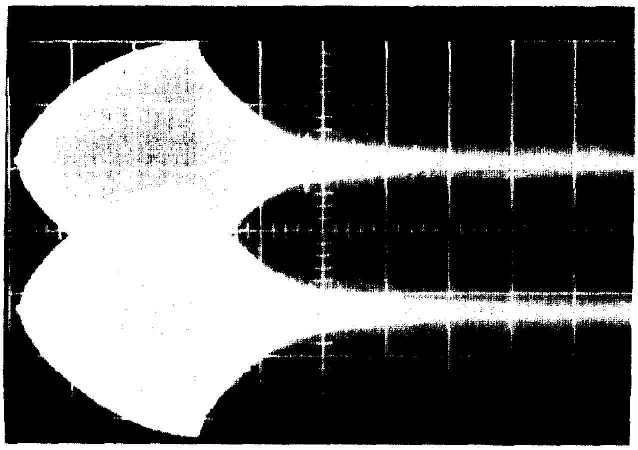

Fig. 1. Filtered H & V difference signals on 28.5 BeV flat top at 1 msec/division. Vertical excitation at $(9-Q)f_o = 83.20$ kc for 3 msec.

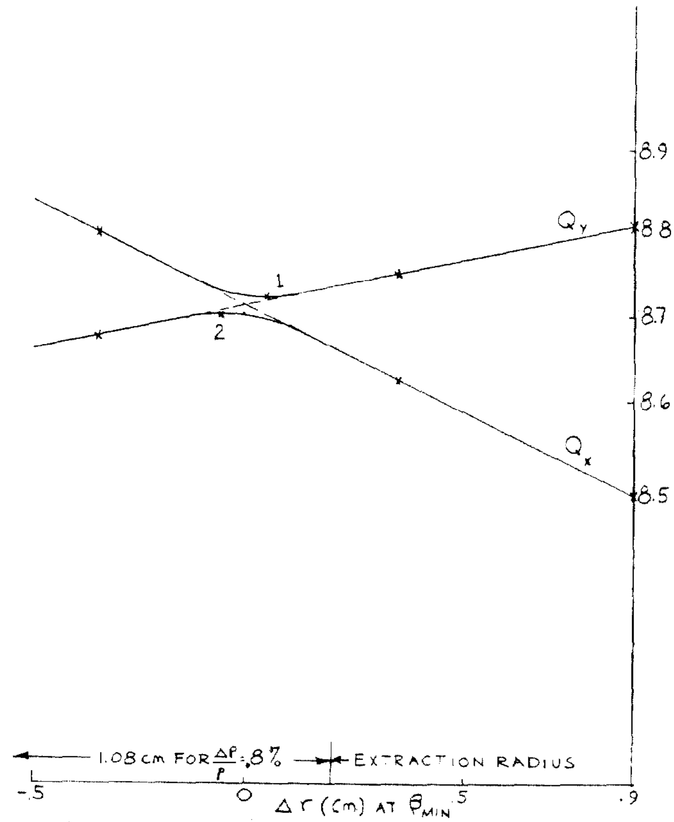

Fig. 2. Q_x, Q_y vs Δr on a 28.5 BeV flat top.

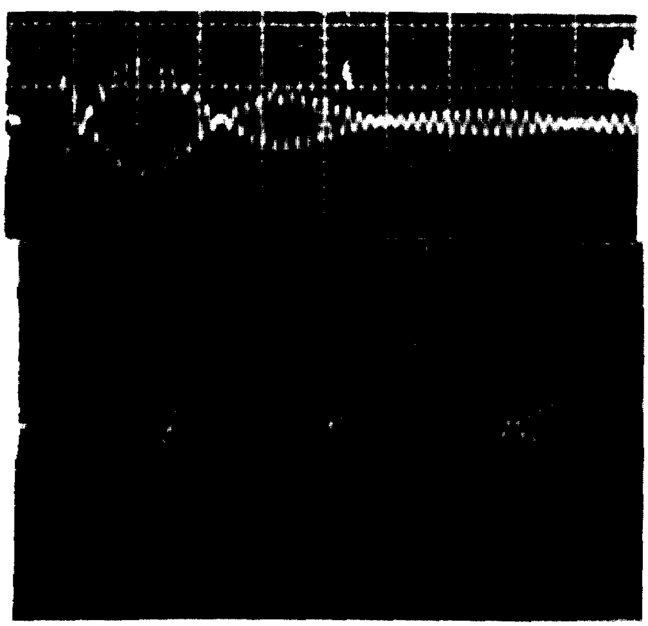

Fig. 3. Filtered H & V difference signal at 26 BeV/c with horizontal kick at start of trace. Vertical scales are not identical but both are 50 μsec/div. No correction current.

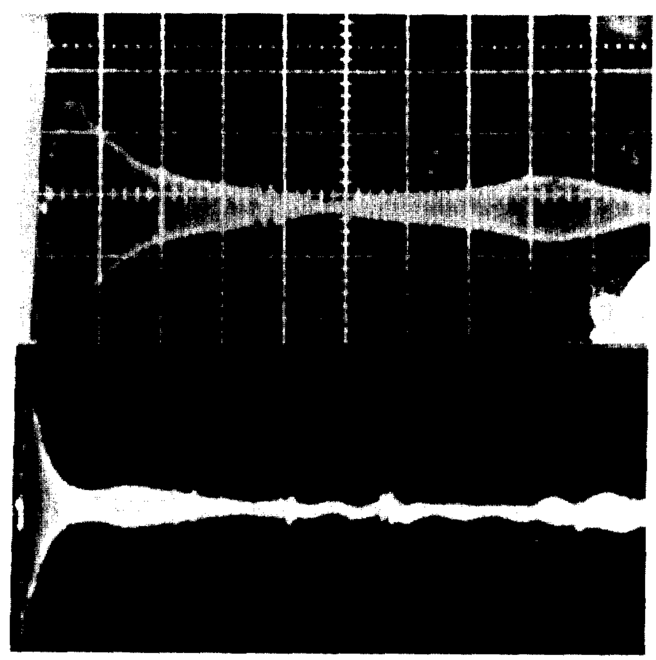

Fig. 4. Filtered vertical difference signal at 14.7 BeV/c. Top at 200 μsec/div. bottom at 500 μsec/div. Not identical vertical scales. Coupling minimized with correction dipoles on both occasions.