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# SEPARATE SECTOR CYCLOTRON BEAM DYNAMICS WITH SPACE CHARGE

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## Introduction

A general description of the GANIL project is given in reference [1] ; essentially the goal is to produce intense beams of ions from Carbon to Uranium with a maximum energy of 100 MeV/amu for Carbon and 8 MeV/amu for Uranium ; the heart of the project consists of 2 separate sector cyclotrons SSC1 and SSC2 working in series. The desired energy spread of the accelerated beam is $4.10^{-4}$ for light ions and $10^{-3}$ for Uranium ; such a requirement, together with requirements on beam transverse emittances made it clear that beam dynamics in these machines have to be thoroughly studied. Although these studies are still under way, it is the purpose of this report to describe first the beam dynamics codes which have been devised for simulating the behaviour of the cyclotrons, and second to discuss some of the results already obtained.

## The computer codes

The main code which is being used extensively is named GOUPIL ; there are 2 different versions of this code, the difference between them being only the way space charge is computed. GOUPIL is a multiparticle code, computing dynamics in a 4 sector cyclotron. Among various inputs the code needs is a map of the magnetic field in a sector ; this map is stored on discs : it is produced by a preliminary code named ORBISO, which works in the following way.

It is assumed that the main magnetic field (without trim coils) has been measured for several values of the current in the main coils. For a given frequency and a given ion, ORBISO computes how the measured field must be modified along orbits starting from a valley with no radial momentum for these orbits are closed and isochronous orbits. This means that trim coils are supposed to be shaped in such a way that they lie along the closed orbits ; this configuration as compared to circular trim coils allows a practical adjustement of trim coils one after the other. The accuracy achieved by ORBISO is such that the residual isochronism error is $5.10^{-5}$. As no measurement specially performed for the GANIL project were available to date, GOUPIL has been run with a map derived from an analytical formulation of a field similar to the field measured in a model at Oak Ridge (The Oak Ridge Laboratory is thanked for allowing us to use these measurements).

The map consists of two arrays the dimensions of which are (126, 91) giving the values of two quantities $V_1$ and $V_3$ at the nodes of a mesh which covers 45 degrees ; the steps are : 0.5 degree for $\theta$ and 2.5 cm for r ; basically an eight-fold symmetry is assumed for the magnetic field. The 3 components of the field are then derived from $V_1$ and $V_3$ by :

$$B_z = V_1 + 3 z^2 V_3$$

$$B_r = z \frac{\partial V_1}{\partial r} + z^3 \frac{\partial V_3}{\partial r}$$

$$B_\theta = \frac{z}{r} \frac{\partial V_1}{\partial \theta} + \frac{z^3}{r} \frac{\partial V_3}{\partial \theta}$$

Other inputs for GOUPIL are : which ion is considered, beam sizes, vertical and radial emittances, energy spread, intensity. With this material the code fills the 6 dimensional phase space with up to 200 particles : the filling is made at random with a probability distribution such that the density is maximum at the center of the bunch and decreases as $1-r^2$ in the transverse directions. Then linear transformations are performed on this distribution ; the over-all result is : one of the particles is exactly at the center of the bunch : it is tagged as the "reference particle" and will be used for checking isochronism. The r.m.s. emittances and sizes are equal to the input values. Two other particles are tagged as $\nu_r$ and $\nu_z$ particles : they perform almost exclusively radial (or vertical) oscillations around the reference particle ; they are used to evaluate the vertical and radial tunes. Finally a pseudo charge suitable for a proper calculation of space charge is allocated to every particle.

GOUPIL integrates step by step the equations of motion :

$$\frac{dp_r}{d\theta} = p_\theta + qer \left( B_z - \frac{p_z}{p_\theta} B_\theta \right)$$

$$\frac{dp_z}{d\theta} = qer \left( \frac{p_r}{p_\theta} B_\theta - B_r \right)$$

$$\frac{dr}{d\theta} = r \frac{p_r}{p_\theta}$$

$$\frac{dz}{d\theta} = r \frac{p_z}{p_\theta}$$

$$\frac{dt}{d\theta} = \frac{mr}{p_\theta}$$

The integration steps are variable : large when the field is almost constant, small in the fringing field. Only part of the $V_1$ and $V_3$ arrays are present in the central memory at a given time ; other parts of the arrays are called from discs when necessary.

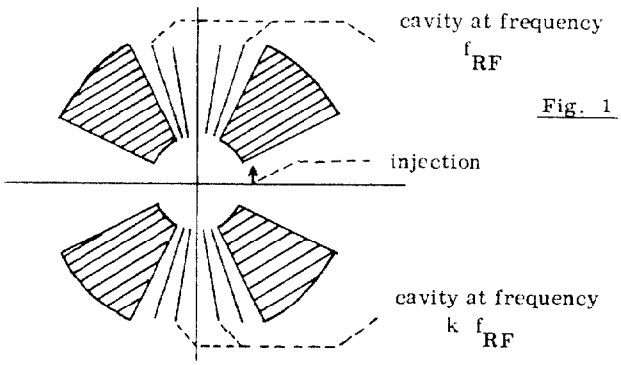



Fig. 1

The cyclotron configuration is shown in fig. 1 : there are 2 double gap cavities at the fundamental frequency $f_{RF} = h f_{rev}$ ($f_{rev}$ is the revolution frequency) and 2 more double gap cavities at frequency $k\ f_{RF}$.

The gaps are treated by the infinitely thin equivalent gap method ; for instance the energy gain is :

$$\Delta W = q e \ V(r) \frac{ch(\alpha z)}{ch(\alpha a)} \cos \varphi$$

where $\begin{cases} a \text{ is the half height of the gap} \\ \alpha = \dfrac{2\pi f_{RF}}{c \beta \gamma} \dfrac{p}{p_0} \end{cases}$

The gap voltage $V(r)$ may be a linear function of the radius ; in this case a vertical magnetic RF field arising from $\dot{B} = - \nabla \wedge E$ perturbs the isochronism[2] ; so a radial kick proportional to $\frac{dV}{dr} \sin \varphi$ is applied to particles to account for possible phase expansion or compression. In addition to that, vertical kicks take care of the vertical focussing properties of the gaps.

Space charge is computed in GOUPIL 1 by the particle to particle interaction method[3], the electrostatic force experienced by each particle is the sum of the forces due to every other particle : each time space charge forces are computed (that is 4 times a turn) $3\frac{N^2}{2}$ components such as

$$\frac{z_2 - z_1}{\left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\right]^{3/2}}$$

are to be calculated if N is the number of particles. So $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$ are only allowed to take discrete values and the calculations are sped up by precomputing tables of the components for these discrete values.

GOUPIL 2 treats space charge by the equivalent continuous distribution methode [4]. The discrete set of particles is replaced for calculating the space charge forces by the continuous distribution :

$$d(x, y, z) = d_0 \exp\left[-\frac{1}{2}\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right)\right]$$

The r.m.s. values of the actual distribution and the continuous distribution are set equal in the 3 directions by a proper choice of a, b, c. The force applied to every particle is then analytically calculated from the continuous distribution.

Both GOUPIL 1 and GOUPIL 2 take into account the neighbouring bunches as well as the image effects.

Magnetic errors can be included in the computations as far as they can be represented by the first few terms of their Fourier expansion as a function of $\theta$. Misalignments of magnets and cavities are also accounted for.

As far as computational precision is concerned, isochronism is the most stringent topic ; round-off errors due to the 36 bits words of the Univac 1110 on which the computations are performed amounts to 0.1 geometrical degree for 300 turns ; integration error with a practical variable integration step is 1 geometrical degree for 300 turns (it can be less with smaller steps).

Computing time without space charge is 7 minutes for 100 particles and 34 turns on Univac 1110. This time is increased to 9 minutes with space charge (with 3 neighbouring bunches on each side).

Some results

Figures 3 to 10 are relative to the acceleration of $C_{12}^{+2}$ in SSC 1 (input energy : 330 KeV/amu ; output energy : 5.3 MeV/amu) : figures 3 to 5 refer to acceleration without flat-topping and figures 7 to 10 with flat-topping ; on figures 3, 6, 9, 10 are shown the bunch occupation in the $\Delta\varphi$, $\Delta W$ phase plane ($\Delta\varphi$ in RF degrees, $\Delta W$ in MeV/amu) ; figures 4 and 7 show how the relative energy spread $\Delta W/W$ varies when the beam is accelerated, (vertical scale must be multiplied by $10^{-3}$) ; figures 5 and 8 show the evolution of the radial dimension of the beam in meter ; abscissa in figures 4, 5, 7, 8 is the turn number.

As can be seen from figures 3 and 4 the required energy spread cannot be achieved without flat-topping (the phase extension of the beam is supposed to be $\pm$ 7.5 degrees).

The phase extension allowed with a flat-topping voltage

$$V_{FT} = \hat{V} \cos(2\pi k f_{RF} t + \varphi_{FT})$$

is a fonction of V, k and $\varphi_{FT}$.

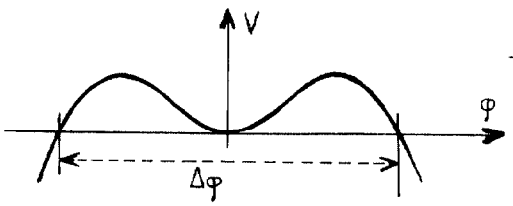


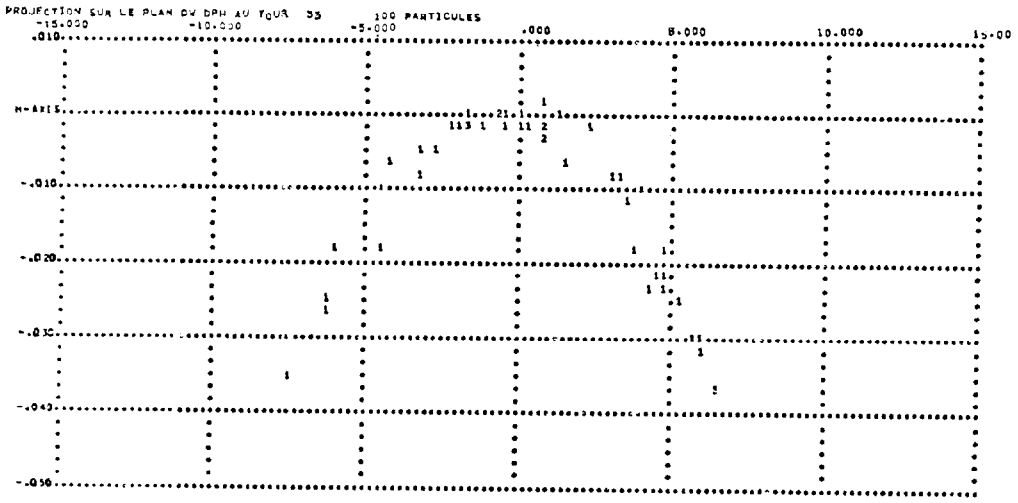

Fig. 2

For k = 2 $\Delta\varphi$ is the largest but also the most sensitive to $\hat{V}$ and $\varphi_{FT}$ fluctuations. If there is no need for a very large $\Delta\varphi$, the use of higher k allows a much easier machine tuning.

In practical cavities the voltage along the dee is not a constant ; as the voltage variations with respect to the radius are different for fundamental and flat-topping cavities, the right value of $\hat{V}$ cannot be maintained everywhere, but only on the average along the dee. Moreover, even if the voltage is correct, the energy gain may be wrong due to the energy gain reduction in the medium plane (term ch ($\alpha$ a) in the above formula. If $\hat{V}$ is locally too far from the right value, the energy spread may increase drastically, and consequently the beam size. Such a situation may arise if one uses large values of h and/or k.

Figures 6 to 8 show an example of a flat-topping working properly with k = 4 ; $\Delta W/W$ is damped almost as 1/W and the radial size of the beam remains constant.

The beam intensities required for the GANIL project are not large enough for space charge being very important in the SSC. For higher intensities the energy spread increases [5] as can be seen in figures 9 and 10. No attempt has been made to compensate for this effect by shifting the RF phase angle of the flat-topping cavities, but this will be tried.




$C_{12}^{+2}$ in SSC 1

h = 8

without flat-topping

i = 0

Fig. 3

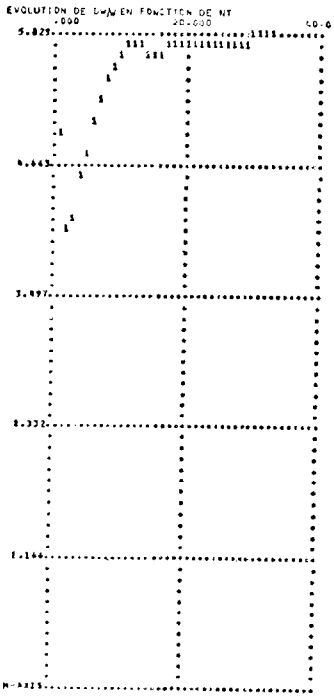



Fig. 4

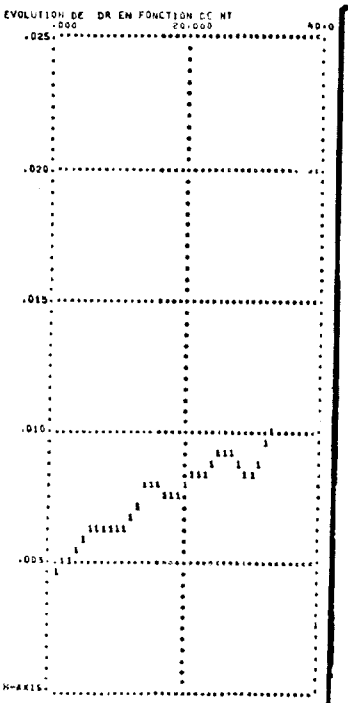



Fig. 5

Fig. 7

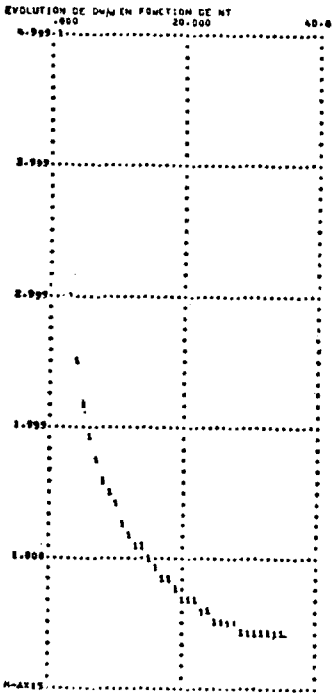
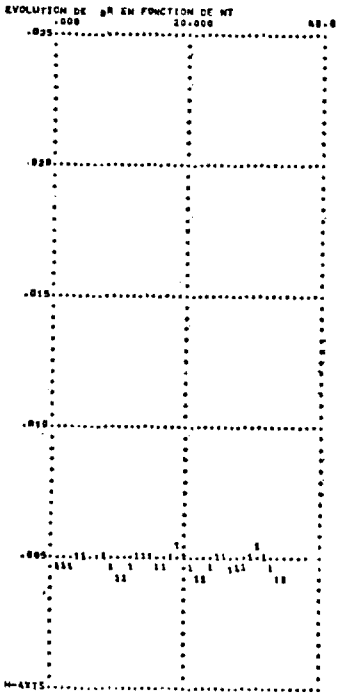



Fig. 8




$C_{12}^{+2}$ in SSC 1

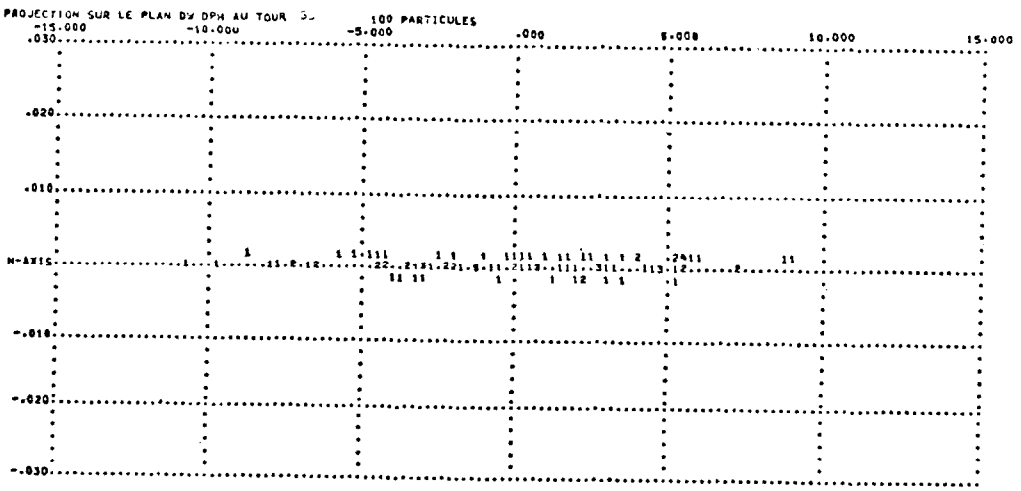
h = 8

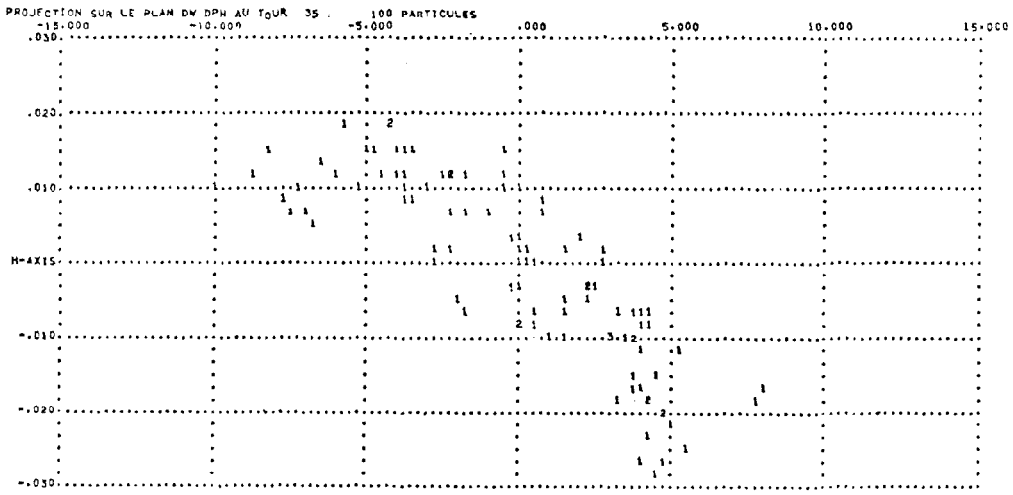
with flat-topping

k = 4

i = 0

Fig. 6



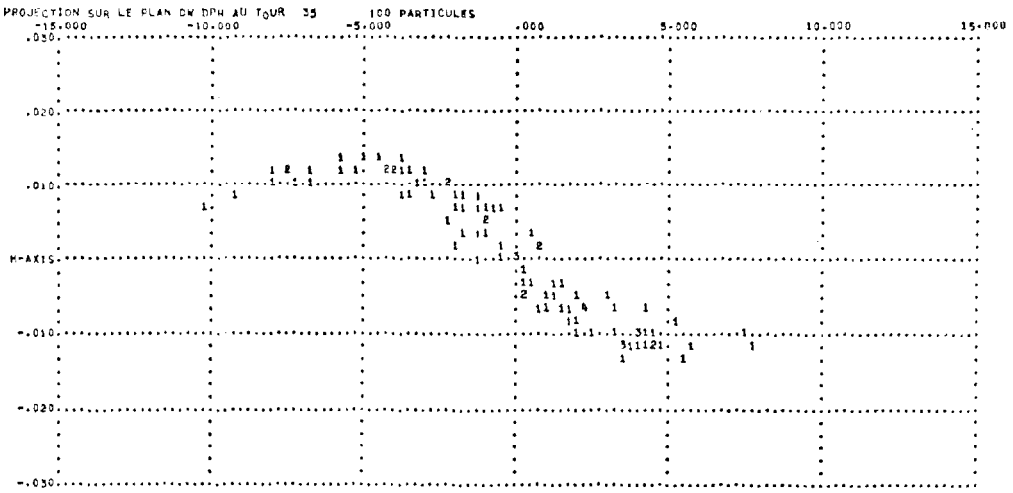



$C_{12}^{+2}$ in SSC 1

GOUPIL 1

i = 200 µA

Fig. 9




$C_{12}^{+2}$ in SSC 1

GOUPIL 2

i = 200 µA

Fig. 10

As far as the computation technique is concerned, results given by GOUPIL 1 (Fig. 9) turned out to be rather noisy ; obviously 100 particles are not enough to simulate space charge fields, especially when the bunch shape departs too much from a sphere.

GOUPIL 2 yields smoother results ; the effect of non linear space charge forces can be easily seen in figure 10 ; the limitations of GOUPIL 2 stem from the mathematical density distribution assumed for the bunch ; such an assumption is probably fair enough for SSC 1, but could not be made for the central region of compact cyclotrons.

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