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SOME IMPROVEMENTS OF CLOSED ORBIT CORRECTION METHODS APPLICATION TO D.C.I.†

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Abstract and Introduction

On the occasion of DCI construction the question was advanced of possible improvements of usual methods concerning closed orbit correction. The calculations we expose here present two advantages :

- The perturbations and corrections are not treated as pointlike dipoles, but inserted in lattice computation by their exactly corresponding transfer matrices.

- The selection is not made among all possible correctors. A set of the statistically most efficient correctors is determined by a computational simulation.

1. Theoretical recollections

Assuming the linear relation between the closed orbit deviations and correcting dipoles :

$$T.\, x = b \qquad (1)$$

with T ($m \times n$) matrix, m : number of beam position monitors, n : number of correctors,

x : correctors vector,

b : vector composed of the orbit deviations at the beam position monitors.

The correcting dipoles are usually approximated by pointlike dipoles[1,2], leading to solve the system (1) by successive approximation.

Taking into account the real configuration of correctors, we obtain an exact expression of the matrix T, allowing to avoid, at least in theory, any iteration.

The closed orbit deviation at the entrance of the i^{th} element of the magnetic lattice is given by the relation :

$$(I - R_i)\, Y^i = V^i \qquad (2)$$

with : I : unit matrix,

R_i : transfer matrix of the whole machine at the entrance of the i^{th} element,

Y^i : orbit deviations vector,

V^i : vector equal to the non homogeneous part of the transfer matrix.

The element T_{ij} is directly obtained by differentiation of the equation (2) :

$$T_{ij} = \frac{\partial Y^i}{\partial A_j} = (I-R_i)^{-1} \frac{\partial V^i}{\partial A_j} \qquad (3)$$

with : A_j dipole strength of the corrector located in the j^{th} element.

It only remains to solve the system (1) by using for instance the Golub-Householder method[3].

2. Calculations procedure

1. Alignment errors are converted to field errors according to Brown formulas[4]. The whole set of dipole perturbations is then simulated by a Monte-Carlo method.

2. For a given set of dipole perturbations, the closed orbit is calculated by solving the equation (2).

3. The elements $\frac{\partial V^i}{\partial A_j}$ are calculated according to the formulas :

$$\frac{\partial V^i}{\partial A_j} = R_{i-1}\, R_N\, R_j^{-1}\, U_j \qquad \text{for } j > i$$

$$\frac{\partial V^i}{\partial A_j} = R_{i-1}\, R_N\, R_i^{-1}\, U_j \qquad j = i \qquad (4)$$

$$\frac{\partial V^i}{\partial A_j} = R_{i-1}\, R_j^{-1}\, U_j \qquad j < i$$

with : N : total number of magnetic elements,

U_j : vector representing the derivative of the non-homogeneous part of the j^{th} element,

and the element T_{ij} are then calculated by expression (3).

4. A set of the statistically most efficient correctors is determined by a computational simulation. The less efficient correctors turn out to correspond to the largest diagonal elements of the matrix $(\tilde{T}\, T)^{-1}$, $\tilde{T}$ being the transpose of T. These results are in agreement with the Arnt-Macgregor theorem[5].

5. A particular set of correctors is then selected to minimize the sum of the squares of the orbit deviations at the observations stations, following the strategy proposed by Autin and Bryant[1].

3. Results

The closed orbit deviations are measured on DCI by 20 beam position monitors regularly spaced on the machine circumference. The accuracy is assumed to be $\pm$ 2 mm.

The correction is realized by a set of dipole fields created by suitable windings on 12 quadrupoles, selected by considering the β functions for the different operating points.

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Results may be summarized as follows :

1. Table 1 shows the statistical results for the different operating points. The orbit deviations turn out to be, depending on the operating point, two or three times more important in the vertical direction than in the horizontal one. This is due to the difference between the β functions in the two directions.

Operating point	1		2		3		4		5		6		7	
	Obs.	Max.	Obs.	Max.	Obs.	Max.	Obs.	Max.	Obs.	Max.	Obs.	Max.	Obs.	Max.
$< X_{max} >$ (mm)	5.4	6.4	6.0	7.0	5.2	6.2	5.9	7.3	5.6	6.7	6.6	8.2	7.7	9.2
$\sigma_{X\ max}$ (mm)	2.5	3.1	2.7	3.3	2.3	2.9	2.55	3.2	2.6	3.2	2.9	3.6	4.9	5.9
max (X_{max})	13.2	15.9	13.9	16.6	11.4	14.2	11.6	14.1	13.1	15.2	16.5	19.5	22.2	26.9
$\beta_{X\ max}$ (m)	8.54	12.51	8.53	12.49	8.23	12.49	7.83	12.49	8.16	12.49	9.79	12.49	9.82	14.52
$< Z_{max} >$ (mm)	12.0	14.9	12.2	15.1	16.4	19.9	16.0	20.2	14.4	17.8	14.7	18.6	11.9	14.8
$\sigma_{Z\ max}$ (mm)	6.4	8.1	6.0	7.4	10.8	13.0	8.6	10.9	7.6	9.7	6.4	8.1	5.1	6.3
max (Z_{max})	31.8	39.9	36.7	45.5	51.6	62.6	46.4	58.6	40.5	51.0	34.0	43.9	25.3	30.8
$\beta_{Z\ max}$ (m)	27.82	43.33	28.37	43.77	43.22	63.14	29.44	47.00	28.72	45.13	29.86	47.22	27.48	41.56

Table 1

According to these results the observed deviations give a good estimation of the maximum deviations.

2. A set of correctors, minimizing the sum of the squares of the closed orbit deviations at the observation stations, is calculated. Figures 1 and 2 show the maximum deviations and the corresponding residue, for the vertical direction of the operating point n° 6, as functions of the number of correctors.

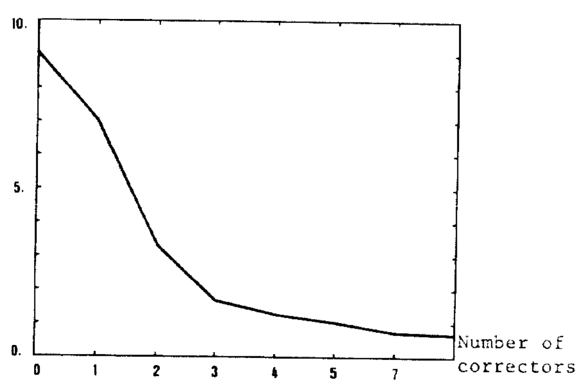

Fig. 1 : Maximum orbit deviations (mm)

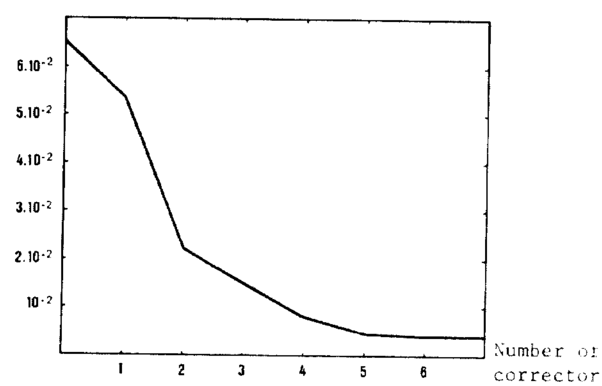

Fig. 2 : Sum of the squares of orbit deviations

3. Figures 3 and 4 show for the operating point n°6 respectively in the X and Z directions, the frequency of utilization of each corrector. The corresponding diagonal elements of $(\tilde{T}\ T)^{-1}$ are given in table 2.

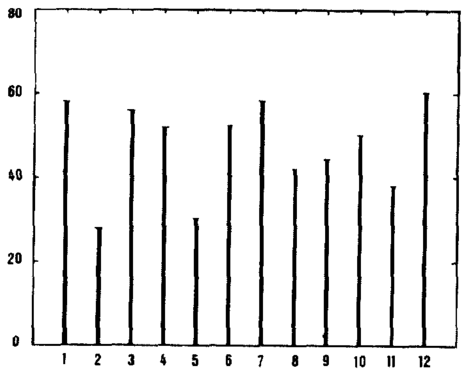

Fig. 3 : Frequency of utilization of correctors (X direction)

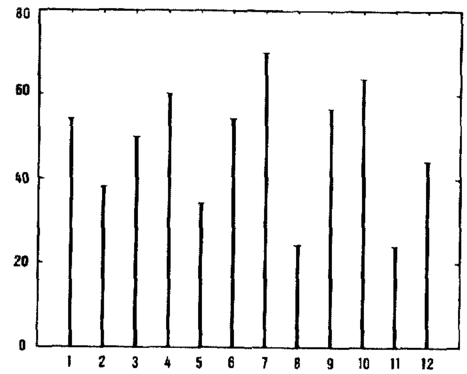

Fig. 4 : Frequency of utilization of correctors (Z direction)

X direction

i	1	2	3	4	5	6	7	8	9	10	11	12
$(\tilde{T}\ T)^{-1}_{ii}$	.1405	.7745	.4556	.4556	.7745	.1405	.1405	.7745	.4556	.4556	.7745	.1405

Z direction

i	1	2	3	4	5	6	7	8	9	10	11	12
$(\tilde{T}\ T)^{-1}_{ii}$	.0867	1.9520	.1785	.1785	1.9520	.0867	.0867	1.9520	.1785	.1785	1.9520	.0867

Table 2

4. Based on these results the initial number of correctors is reduced to 8. Table 3 shows that, for a given residue, the same average number of corrector is used for both cases.

Residue	Complete set		Reduced set	
	X	Z	X	Z
0.02	1.7	3.0	1.7	3.0
0.01	3.4	4.7	3.7	4.8
0.005	5.8	6.2	6.3	6.2

Table 3

4. Conclusion

The proposed method allows to reduce both the number of iterations necessary to correctors calculations and the dimension of the initial set of correctors. The resulting reductions, concerning as well the processing time as the memory allocation, should be appreciable in case of automatic closed orbit corrections.

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