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RF BEAM LOADING EFFECTS IN EPIC

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Introduction

In the proposed electron-positron storage ring EPIC, very high peak currents in the circulating beams will give rise to transient beam loading effects. These effects have been studied in order to find the extra RF power needed because of them.

We first look at stored energy considerations for the fundamental cavity mode, and find that there is a limit to the RF efficiency. Higher order cavity modes are then introduced, from which we calculate energy loss and wake fields due to single transit effects. Multi-transit effects are then considered, followed by a comparison of calculation models.

Unless otherwise stated, the parameters used in the calculations are:

mean radius of machine = $R_m = 348.8 \text{ m}$ RF frequency = $f_o = 403 \text{ MHz}$ mean circulating current = $i_B = 70 \text{ mA}$ (2 bunches of electrons + 2 bunches of positrons) shunt impedance = $R_S = \frac{R}{Q} \times Qx$ number of cells = 114x37,000x112 = 4.72 10⁸ Ω total length of RF structure = 112x $\lambda/2$ = 41.69 m.

Upper Limit to RF Efficiency

The transient nature of the beam loading arises because the filling time of the RF structure $\tau_{\rm fill}$, is comparable to the time interval between the circulating bunches $\tau_{\rm c}$ (ref.l). It is found that²

$$\frac{U}{W} = \frac{2\tau_o}{\tau_{fill}} \frac{P_B}{P_C}$$
(1)

where U = energy extracted from the structure by the bunch of particles

- W = energy stored in the structure
- $P_{\rm B}$ = power given to the beam
- P_C = power dissibated in the structure (not the average power, but the instantaneous dissipation corresponding to W)

$$\tau_{fill}$$
 = filling time of unloaded structure.

The extra dissipation in the structure due to the ripple component, and the reflected power, may be found³.

$$P_{R} \approx \frac{i_{B}^{2}R_{S}}{6} \left(\frac{\tau_{O}}{\tau_{f111}}\right)^{2} \left(2 + \frac{P_{B}}{P_{C}}\right)$$
(2)

where P_{C} = average power dissipated in the structure neglecting ripple due to beam loading.

Without transient beam loading, since $P_c = V^2/2R_s$, we could increase the structure length indefinitely to improve efficiency. But when equation 2 is taken into account an optimum length of structure will be found.

Higher Order Modes in the RF Structure

In the preceding paragraph we set an approximate limit to the efficiency of the RF system by assuming that all the stored energy in the RF structure was available to a bunch of particles as it traversed the structure. This will in practice not be possible, since the bunch, as well as radiating into the fundamental mode of the structure and cancelling the existing field, will radiate so as to couple to other modes of the structure. The energy left in the structure will therefore be non zero.

Since in EPIC the bunch length will be comparable to the transverse dimensions of the vacuum vessel, the very high frequency components of radiation, $\omega > c/a$, will carry little energy (a is the beam pipe radius in the RF structure). It was therefore vital to find a reasonable model for the lower frequency region where diffraction formulae do not apply.

Keil's⁴ computer programs KN7C and CHIMPS calculate resonant frequencies, shunt impedances and Q values for the simple structure shown below in figure l(b).





Figure 1. Possible EPIC RF structure, and simple model for higher mode calculations.

These programs were used to find mode parameters for an approximation to the EPIC basic cell (a=0.0642 m, b=0.285 m, g=0.244 m, d = 0.375 m). For higher frequencies (ω >2.5 10¹⁰ rad s⁻¹) the Sessler-Vainshteyn diffraction integral was used, (see reference 4).

Energy Loss Calculations

In this section, and the following section on wakefields, we will deal only with single transit effects, so the mode parameter of interest is $\omega R/Q = 1/C = \sqrt{2}/2W$. The shunt impedance R and quality factor Q are of no importance here, since the decay during passage of the bunch is very small. The energy loss is calculated from:

$$U = \frac{1}{2}q^{2} \sum_{\substack{\alpha \in \mathcal{A}^{2} \\ \alpha \neq \alpha \neq \alpha \neq \alpha}} \left[\left(\frac{\omega R}{Q} \right)_{n} e^{-\omega^{2}\sigma_{t}^{2}} \right]$$

+ $C_{0} q^{2} \int_{\omega_{c}}^{\infty} \omega^{2} \frac{\kappa_{i}^{2}}{\kappa_{i}^{2}} \left(\frac{\omega a}{\beta \gamma c} \right) \frac{(M+\sigma)}{[(M+\sigma)^{2}+\sigma^{2}]^{2}} e^{-\omega^{2}\sigma_{t}^{2}} d\omega (3)$

U energy loss (joules) Ne = charge in the bunch (coulombs) q = rms half bunch length (seconds) σt = $(2j^{2}\sigma a^{2})/(\pi^{2}\epsilon_{0}\beta^{3}\gamma^{2}c^{3})$ Co = 2.40482 (first zero of J_0) $\zeta (\frac{1}{2})/\sqrt{\pi} = 0.824$ j, = σ = а ---beam pipe radius (m) d cell spacing (m) particle velocity $(m \ s^{-1})$ velocity of light $(m \ s^{-1})$ βc = с = $E/m_{o}c^{2}$ = relativistic mass ratio γ = upper limit of cavity modes = шc к, modified Bessel function of the first kind

Since q = Ne, the average energy loss per particle (in eV) is given by U/q.

Figure 2 shows the result of power requirement calculations for the EPIC structure as a function of bunch length. It is seen that for very short bunches (σ_{ℓ} < lcm) the power requirements start to climb rapidly, but for the natural bunch length (σ_{ℓ} = 1.75cm), and for lengthened bunches, the extra power required for beam loading is not excessive.



Figure 2. Power requirement v bunch length.





Figure 3. Power requirements v structure length.

Short Range Wake Fields

The response of the structure to a delta function of charge can be written as:

$$V_{\delta}(t) = q \sum_{\omega_{c}} \left[\frac{\omega R}{Q} e^{i\omega t} \right] + 2 C_{0}q \int_{\omega_{c}}^{\infty} \omega^{2} \frac{K_{1}^{2}}{1} \left(\frac{\omega a}{\beta \gamma_{c}} \right) \left[\frac{(M+\sigma)}{(M+\sigma)^{2} + \sigma^{2}} \right]^{2} e^{i\omega t} d\omega \quad (4)$$

By applying the Laplace convolution theorem, the response to a Gaussian charge distribution (truncated at 5 σ) was found. Figure 4 shows the response for a bunch length σ_{g} =2cm. This response is predominantly resistive, whereas very short bunches have a capacitive response, and longer bunches tend to see a more inductive wake-field.



Figure 4. Energy loss v position in the bunch. $(\sigma_{g}{=}2\text{cm.})$

We were worried at first that the shape of the wake-field for a short bunch increases the phase focussing forces, further reducing the bunch length. We are now satisfied that this cannot happen for several reasons.

- (1) No bunch shortening (below the natural bunch length) has ever been observed. The bunch lengthening observed at SPEAR with a high frequency RF system confirms bunch lengthening as a general phenomenon.
- (2) It is now widely believed that Sessler's turbulent modes⁵ contribute more to bunch lengthening than modifications to the potential well⁶.
- (3) Hereward and Germain⁷ using Haïssinski's method⁸ are finding that predominantly resistive wakes (figure 4) lead to small bunch lengthening, while capacitive wakes lead to instability and hence to bunch lengthening through reason (2).
- (4) Resonances elsewhere in the vacuum vessel with frequencies $\omega > 1.3/\sigma_t$ give inductive rather than capacitive wakes, causing bunch lengthening by potential well deformation.

Multiturn Effects

For any resonant mode, we can find the energy lost by a bunch traversing the RF structure, taking into account the resonant build up of RF fields²

$$U = \frac{1}{2}C \delta^2 \frac{(e^{2\alpha \tau} - 1)}{e^{2\alpha \tau} + 1 - 2e^{\alpha \tau} \cos \omega \tau}$$
(5)

where

- $C = (\omega R/Q)^{-1} = capacitance of the equivalent circuit$
- $\delta = q \ \omega R/Q \ exp \ [-\omega^2 \sigma_t^2/2] = voltage \ impulse \ due \ to the passage of a bunch$
- $\alpha = \omega/2Q = \text{decay constant of the circuit}$
- ω = resonant frequency of the circuit
- τ = time interval between bunches.

If equation 5 is averaged over all values of the angle $\omega\tau$ we get

$$U = C\delta^{2}/2 = \frac{1}{2}q^{2}(\omega R/Q)\exp(-\omega^{2}\sigma_{+}^{2})$$
(6)

which is the same result as that obtained for a single non-repetitive transit (equation 3).

Using equation 5 we calculate the power loss in modes other than the fundamental, as a function of τ , for small differences from the nominal value of τ =3.655 usec. The result for one beam only is shown in figure 5.





Note that one mode dominates the plot. This mode is the TM_{011} mode, which in our simple model is resonant at 1.1 GHz. If this mode is excited the power loss per cell rises to 4 times the 811 watts/cell average.

It is argued that due to manufacturing tolerances, the resonant frequencies of the higher cavity modes will be sufficiently staggered to average out the losses. In one block of 7 cells however, the inter-cell coupling will couple together the 7 individual cell modes into 7 structure modes, one of which will be synchronous with the beam. Stagger tuning will therefore only be effective on a block to block basis, but with 16 blocks the staggering should be adequate. As a further precaution, the cavity design should not preclude the damping of undesirable cavity modes by means of selective coupling antennae⁹.

Cavity Models

Since other workers $^{10-12}$ have been obtaining higher losses using a closed pill box cavity as a model, a comparison has been made between 3 models. These are the closed cavity, the cavity in an infinite chain (using KN7C and CHIMPS), and the pill box cavity with end tubes¹³. Figure 6 shows the result of the comparison.



Figure 6. Energy loss for different cavity models.

We believe that the closed pill box cavity model is very pessimistic, because at high frequencies the end tubes are very important in lowering the coupling impedance.

Incidental Cavities

The 'cavity with end tubes' model may be very useful in studying the effects of incidental cavities formed by changes in the cross-section of the vacuum vessel. A start has been made on this study, and it is seen that for long cavities, or for small changes in cross-section, the energy loss becomes independent of cavity length.

Two significant relationships emerge from these studies:

(1) The asymptotic energy loss varies as $1/\sigma_{g}$.



Figure 7. Energy loss v gap length, outer radius of cavity as a parameter.

(2) The loss curves as illustrated in figure 7, peak, and then level off, when the centre of the bunch can receive information about the outer radius of the cavity. The peaks of the curves occur when the path difference $\delta \ell = (A \rightarrow B \rightarrow C) - (A \rightarrow C) \approx 1.3 \sigma_{\ell}$ (figure 8).



Figure 8. Path length difference.

Conclusions

- <u>Calculation models</u>. We believe that the closed pill box model is pessimistic.
- (2) <u>Bunch length</u>. This will certainly not be less than natural.
- (3) <u>Power requirements</u>. For the EPIC design parameters, the extra power required for beamloading will be less than we feared initially.
- (4) <u>Multiturn effects</u>. By stagger tuning blocks of cavities, or by damping some of the higher modes, power loss can be kept to the value calculated for single transit effects.
- (5) Incidental cavities (small variations in cross section). Initial calculations suggest that power loss into these will be much less than that into the RF structure.

- (6) <u>Operating frequency</u>. Since for a given cavity voltage and shunt impedance the stored energy varies as (ω/Q)⁻¹ ∝ ω^{-3/2}, and since the power loss and induced voltage go as ω/Q, a low operating frequency is desirable.
- (7) Length of structure. The length of structure in the design is suitable for very high beam currents. For achieving higher energies at reduced currents, a greater length is more suitable, but this could cause problems for high current operation, particularly at injection energy.

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