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FEASIBILITY OF ACCELERATOR APPLICATION FOR PULSED MHD POWER GENERATION

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SUMMARY

Prospects for pulsed electrical power output from a MHD self-excited accelerator driven generator is investigated. Feasibility is centered on exploring the magnitude of acceleration and energy conversion ratio in the accelerator channel subject to a spectrum of applied current impulses. Plasma conditions of moderate and high degree of ionizations are studied.

INTRODUCTION [2]

Pulsed electric power output from an MHD generator requires concentrated efforts to improve several performance components, especially its efficiency and means of controlling initial fluid flow and mode of excitation. A set comprising an MHD accelerator-generator combination is one method of adjusting the magnitude of plasma velocity flow and the rate of its increase by controlling the magnitude of the self-excitation impulse at the accelerator inlet.

Self-excitation for an MHD accelerator could be provided by connecting a capacitor bank initially charged to a high level of energy. A direct application for this accelerator-generator set is for a plasma with a moderate degree of ionization with the prospect that it may possess a magnetic Reynolds number less than unity. Another case to be investigated is a highly conductive plasma with a large R_m, where the mode of interaction is not as significant as for a small R_m.

STATEMENT OF THE PROBLEM [4, 5]

1. Refer to Figure 1 showing a self-excited MHD accelerator coupled directly to an MHD generator. Both channels are of rectangular cross-section with a large width to height ratio. Initial excitation for the accelerator can be applied by the charged capacitor in the form of delayed current impulses of variable strengths.

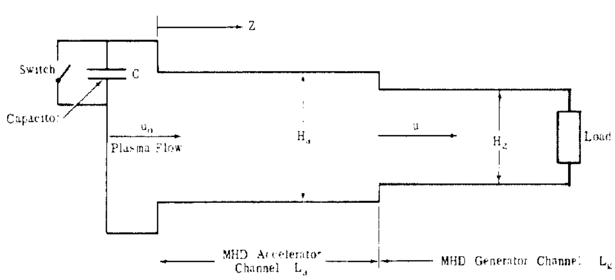

Fig. 1. Self-excited MHD Accelerator Generator Set.

2. Using the principles of electric circuitry and energy balance indicated below:

$$Ri + \frac{d}{dt}(Li) = 0 \tag{1}$$

$$\frac{d}{dt}\left[\frac{1}{2}Li^2 + W\right] + i^2R = 0 \tag{2}$$

L = accelerator inductance at any point Z

$= (L_o - l_a Z)$

l_a = accelerator channel inductance per unit length

R = accelerator channel resistance

W = plasma kinetic energy

From equations 1 and 2, the velocity equation is expressed below:

$$\frac{d^3Z}{dt^3} = \frac{2}{C - Z}\left[\frac{dZ}{dt} - \frac{1}{R_m}\right]\frac{d^2Z}{dt^2} \tag{3}$$

$C = L_o / l_a Z$

R_m = magnetic Reynolds number

3. It is necessary to obtain closed form solutions for the pattern of acceleration and the energy conversion ratio for a spectrum of delayed impulsive current strengths.

   Two cases are to be investigated:

   A = Small values of R_m, implying moderate degree of ionization

   B = Large value for R_m, implying highly conductive plasma

4. Development of a criterion for the feasibility of accelerator utilization to adjust and control plasma flow for pulsed electric power output by an MHD generator.

CLOSED FORM SOLUTION FOR VELOCITY GAIN AND ACCELERATION [1, 2]

Referring to equation 3 above and stated below:

$$\frac{d^3Z}{dt^3} = \frac{2}{C - Z}\left[\frac{dZ}{dt} - \frac{1}{R_m}\right]\frac{d^2Z}{dt^2} \tag{3}$$

With the initial conditions:

$$\begin{aligned} &\text{At } t = 0,\ Z = 0 \\ &\frac{dZ}{dt} = 1 \quad \text{dimensionless} \\ &\frac{d^2Z}{dt^2} = \frac{L_a l_a i_o^2}{2 m u_o^2} \end{aligned} \tag{4}$$

Case A. $R_m < 1$, $\frac{dZ}{dt}$ could be expanded in a double power series with respect to R_m and the magnetic Interaction parameter Q as below:

$$u = u_o + \sum_{n+k=1}^{\infty} R_m^n Q^k u^{(n,k)} \qquad n = 0,1,2,3 \ldots\ldots \quad k = 0,1,2,3 \ldots\ldots \tag{5}$$

or

$$\frac{dZ}{dt} = u_o + R_m\frac{dZ_1}{dt} + Q\frac{dZ_2}{dt} + R_mQ\frac{dZ_3}{dt} + \ldots\ldots \tag{6}$$

where

$u_1 = \frac{dZ_1}{dt}$ is the velocity perturbation due to the influence of the induced magnetic field

$u_2 = \frac{dZ_2}{dt}$ is the velocity perturbation due to the inertia field

$u_3 = \frac{dZ_3}{dt}$ is the velocity perturbation due to the combined effects of the magnetic and inertia flow forces.

Substitution of equations 5 and 6 into equation 3, followed by separation of corresponding terms with respect to R_m and Q resulted in two identical sets as shown below:

$$\frac{d^3 Z_1}{dt^3} = \frac{2}{C - Z_1} \frac{d^2 Z_1}{dt^2} \tag{7}$$

$$\frac{d^3 Z_2}{dt^3} = \frac{2}{C - Z_2} \frac{d^2 Z_2}{dt^2} \tag{8}$$

setting the acceleration $\alpha = \frac{d^2 Z}{dt^2}$ in equations 7 and 8 and using the initial conditions mentioned earlier, closed form solutions have been obtained for u_1 and u_2.

However, the assumption is that the release of energy stored from the capacitor will generate an initial current of an almost impulsive nature, i.e.,

$$i_o = I_o\, \delta(t - \tau) \quad \text{where} \tag{9}$$

τ = time delay for the current I_o actuated after the start of plasma flow in the accelerator channel

$\therefore$ At $t = \tau$

$$\alpha = \frac{L_a I_a [I_o(t - \tau)]^2}{2 m u_o^2} \tag{10}$$

The solutions for u and α are given below:

$$u = 1 \quad [R_m + Q] \left\{ 2t \ln \frac{C}{C - Z} + \frac{L_o [I_o \delta(t - \tau)]^2}{2 C m u_o^2} \right\} \tag{11}$$

and

$$\alpha = 2 \ln \frac{C}{C - Z} + \frac{L_o [I_o \delta(t - \tau)]^2}{2 C m u^2} \tag{12}$$

Case B.

R_m is $\gg$ 1

With R_m large, closed form solutions for the velocity and acceleration are obtained as below:

$$u = 1 + \frac{L_o [I_o \delta(t - \tau)]^2}{m u_o^2 [C - Z]^2} \tag{13}$$

$$\alpha = \frac{L_o\, I_o\, \delta(t - \tau)}{m u_o^2\, [C - Z]^2} \tag{14}$$

ACCELERATOR PERFORMANCE RESULTS[3,5]

1. R_m is small and corresponds to a plasma of moderate degree of ionization injected at the entrance of the MHD accelerator at a relatively high velocity.

   With an infinitisimal time delay between the instant of plasma entrance and the release of energy stored in the charged capacitor, the plasma velocity can be adjusted at the MHD generator channel in the form of repeated pulses.

   Using solutions for flow acceleration and velocity in equations 11 and 12, Table I is obtained for a spectrum of values for C, I_o, u_o and initial time delay.

   Figures 2 through 4 illustrate the accelerator performance for α and u^2. u^2 represents the ratio of total energy conversion into mechanical form (plasma acceleration), with respect to the initially injected electrical energy that was stored in the capacitor.

2. R_m is much larger than unity. This implies that the plasma is of a high electrical conductivity resulting in a state of frozen magnetic force lines associated with great difficulty of penetration into the lumped plasma body.

   Using equations 13 and 14, Table II has been obtained for a spectrum of values for C, I_o, u_o and initial time delay.

   Figures 5 and 6 illustrate various modes of acceleration along the accelerator channel, besides ratios of total mechanical energy (plasma acceleration), with respect to initially injected electrical stored energy.

   Effectiveness of final acceleration at the end of the accelerator channel is shown in Fig. 7 with respect to several strengths of initial current impulses.

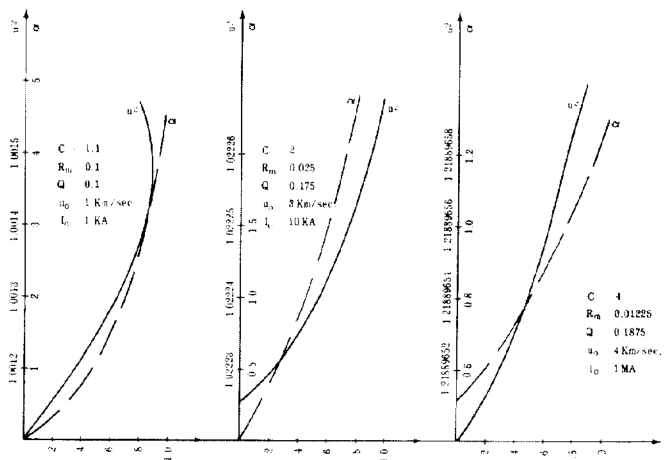

Fig. 2a Fig. 2b Fig. 2c
Ratio of Accelerating to Initial Electrical Energies

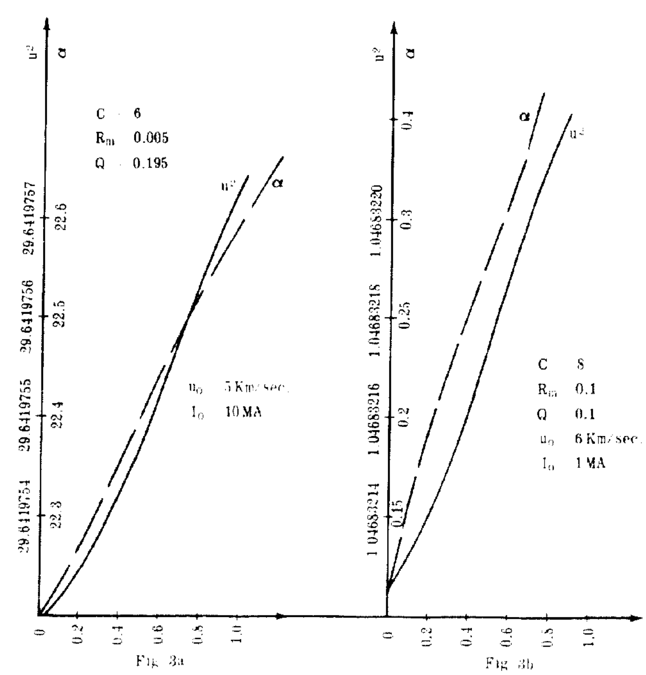

Fig. 3a Fig. 3b
Ratio of Accelerating to Initial Electrical Energies

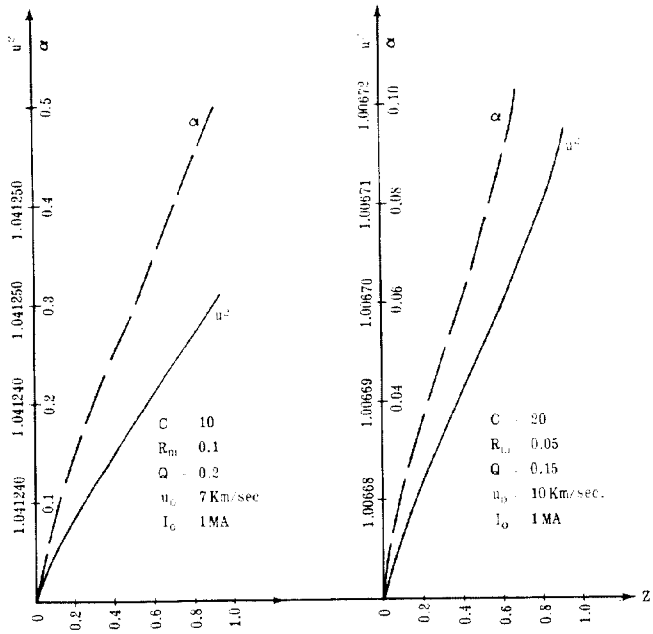

Fig. 4a Fig. 4b

Ratio of Accelerating to Initial Electric Energies

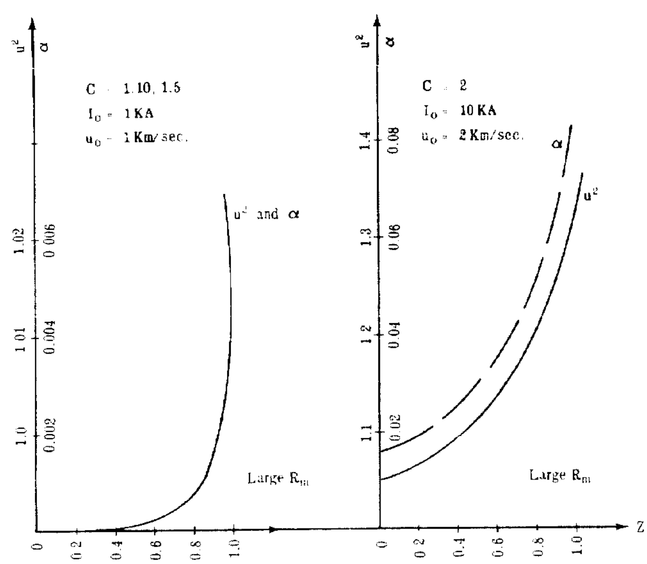

Fig. 5a Fig. 5b

Ratio of Accelerating to Initial Electric Energies with Large R_m

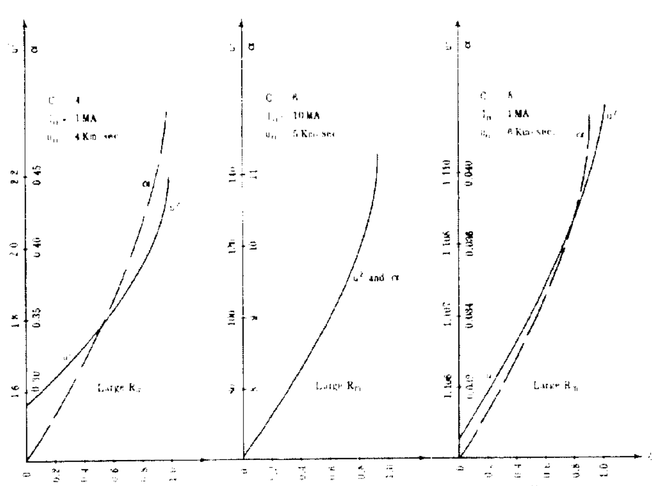

Fig. 6a Fig. 6b Fig. 6c

Ratio of Accelerating to Initial Electric Energies with Large R_m

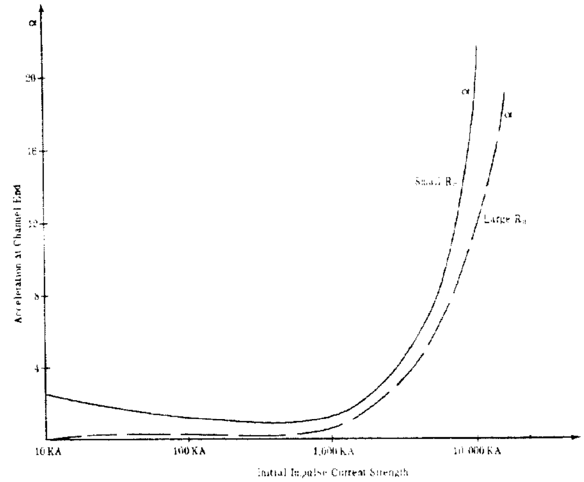

Fig. 7

CONCLUSIONS[3,4,5]

1. Inspection of Figures 2, 3 and 4 demonstrates the pattern of plasma acceleration and ratio of conversion for the initially stored electric energy into mechanical form. Figure 2 indicates the behavior of u^2 and α with increasing parametric values for C, I_o, u_o, Q and with a decreasing level for R_m, where an upsurge in u^2 and a decline in α becomes evident. However, in Figure 3a, where the impulse value of I_o is 10 MA, a sharp increase in u^2 and α is indicated representing a maximum limit for both, especially at the end of the accelerator channel. However, the impulse strengths in u^2 and α with higher values for C, u_o, and Q showed a consistent decline since I_o is being reduced below 10 MA limit, even though R_m is of the same order of magnitude as before.

   The situation demonstrated in Figures 2, 3 and 4 indicates that for a plasma of moderate degree of ionization subjected to an impulse current with a certain time delay beyond the initial plasma flow, one attains an optimum acceleration and energy conversion ratio as the current impulse strength is peaked by adjustment of the self-excitation level.

2. Figures 5 and 6 reveal the pattern of acceleration and energy conversion ratio for a plasma at a high degree of ionization introduced into the accelerator channel at various initial velocities and with increasing current impulses applied after a certain time delay. It is shown that u^2 and α attain their optimum level as I_o is peaked to a limit of 10 MA and inductance ratio of 6, which is the same situation as for a moderately ionized plasma discussed in item 1 above.

3. Commenting on the results discussed in 1 and 2 above, it can be concluded that the rate of energy conversion ratio for a moderately ionized plasma as the initial current impulse increases, accelerates at a slower rate as compared to a highly conductive plasma, while the behavior of acceleration follows a reverse pattern, except at the optimum level of excitation where u^2 and α attained their maximum limits, with the highly conductive plasma attaining larger value for u^2 and lower magnitudes for α. This phenomenon can be attributed to the difficulty of plasma field interaction in the highly ionized case.

4. While the optimum level of plasma acceleration at the end of the MHD channel could be higher for a moderately ionized plasma compared with the highly ionized plasma, the patterns of acceleration at initial current impulses, although fluctuating, are very similar in the cases of moderate and high level of ionization, as demonstrated in Figure 7.

5. It can be concluded that for a set comprising an accelerator-generator, as in Figure 1, control of initial velocity and acceleration at the entrance of the generator channel can be adjusted from the self-excitation system of the accelerator by virtue of an initial current impulse ratio of total inductance to channel induc-

tance (C), degree of ionization and initial plasma velocity at the accelerator channel. Time delay for the actuation of current pulses in an MHD accelerator can be visualized as a repetition rate for plasma injection for the generation of pulsed electric power output from an MHD channel. Although relatively high initial velocities were investigated at the accelerator channel, the feasibility exists for lower velocity values in the order of 1 Km/sec.

TABLE I

R_m is small, x = 1 (End of Channel)

C	I_o	τ μsec.	u_o Km/sec.	R_m	Q	α	u^2
1.1	1KA	0.1	1	0 10000	0.1000	5.00	1.000014
1.5	10KA	0.2	2	0.05000	0.1500	2.50	1.000220
2.0	100KA	0.3	3	0.02500	0.1750	1.50	1.000220
4.0	1MA	0.4	4	0.01225	0.1875	1.20	1.218900
6.0	10MA	0.5	5	0.00500	0.1950	22.60	29.642000
8.0	1MA	0.8	6	0.10000	0.1000	0.40	1.046000
10.0	1MA	100.0	7	0.10000	0.2000	0.30	1.041000
20.0	1MA	500.0	10	0.05000	0.1500	0.12	1.006000

TABLE II

R_m is large, x = 1 (End of Channel)

C	I_o	τ μsec.	u_o Km/sec.	R_m	Q	α	u^2
1.10	1KA	0.1	1	0.10000	0.1000	0.00800	1.020
1.50	10KA	0.2	2	0.05000	0.1500	0.00800	1.020
2.00	100KA	0.3	3	0.02500	0.1750	0.08000	1.200
4.00	1MA	0.4	4	0.01225	0.1875	0.50000	2.200
6.00	10MA	0.5	5	0.00500	0.1950	11.00000	140.000
8.00	1MA	0.8	6	0.10000	0.1000	0.03800	1.080
10.00	1MA	100.0	7	0.10000	0.2000	0.01700	1.034
20.00	1MA	500.0	10	0.05000	0.1500	0.00018	1.000

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