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# POSITIONING TOLERANCES OF BEAM LINE MAGNETS*

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## Summary

The positioning tolerances of magnets are an important consideration in the design and installation of a beam line. A misaligned magnet can both shift the beam centroid and affect the beam line transmission characteristics. To first order and without misalignments, the coordinates at any point of a beam particle are given as a transfer matrix times the initial coordinates. To this we add a term linear in the misalignment parameters giving the beam centroid shift, and a term bilinear in the initial particle coordinates and the misalignment parameters giving the effect on the beam focal properties. This formalism has been incorporated into the computer program TRANSPORT[1]. The effect of either a known misalignment or an uncertainty in position may be exhibited. In a single run, up to ten magnets may be separately misaligned, and their separate effects on the beam coordinates printed in a table.

## Need for Misalignment Information

The positioning tolerences of magnets are an important consideration at every stage of beam line design, installation, and operation. The selection of the optical mode, determination of surveying accuracy requirements, and the choice of correcting elements are all dependent on misalignment information.

In designing a beam line the optical mode is invariably selected to satisfy certain experimental needs. However, the design must also entail realistic installation requirements. Positioning tolerances may make one possible choice of optical mode preferable over another. Correcting elements should then be provided in the design to compensate for residual misalignments. Information on the effect of misalignments is needed to determine the number and strength of such correcting elements.

At installation time, surveying accuracy requirements are needed for proper positioning. It is necessary to know the effect at any critical point in the beam, such as a focus, of the misalignment of any magnetic element in any direction.

Finally, with time the beam line elements will shift in position, creating misalignments that may be known through resurveying. One must then know the corrections necessary to compensate for a given set of misalignments.

Two types of misalignment information are thus needed. To assess the general effect of misalignments in the design stage, one needs to know the uncertainty in beam position and change in beam line transmission characteristics due to errors in the position of each magnetic element in each separate coordinate. Secondly, to provide for correcting elements, one needs to know the effect on the beam of specific misalignments.

## Specifying a Misalignment

The position and orientation of a magnet are specified by six quantities: three spatial coordinates and three angles. We choose an origin such that, for a properly oriented magnet, these quantities are all zero. The coordinate system used is that formed by the beam line reference trajectory and the two transverse axes at the entrance face of the misaligned magnet. The six misalignments consist of displacements of the magnet along each of these three coordinate axes and rotations about each of the axes. For small angles, the effect of the rotations can be approximated by linear terms. In that case, the order in which the rotations are imposed is unimportant, and the three rotational misalignments can be considered independent.

A displacement of a magnet from its aligned position will cause a shift in the beam centroid. However, certain rotational misalignments have no such effect. A rotation of a quadrupole about its axis affects the focal properties of a beam line, and causes mixing of horizontal and vertical planes.

Therefore, in calculating misalignment effects, we wish to include terms to represent both the shift in the beam centroid and the change in transmission characteristics. To first order and without misalignments, the transmission characteristics of a beam line may be represented by a transfer matrix

$$\underline{X}^1 = R\underline{X}^0 \tag{1}$$

where R is a six by six matrix acting on a vector $\underline{X} = (x, x', y, y', \ell, \delta{=}\Delta p/p)$, specifying the position and angles of a particle relative to the beam reference trajectory, and the deviation from the central beam momentum. The misalignment of any given magnet may cause both a shift in the reference trajectory and an alteration of the beam line transfer matrix. Including both terms, equation (1) now takes the form

$$x_i^1 = \sum_j R_{ij} x_j^0 + \sum_j F_{ij} m_j + \sum_{jk} G_{ijk} x_j^0 m_k \tag{2}$$

where here the particle coordinate indices are written explicitly. The quantities m represent the extent of misalignment of a given magnet in each of the six coordinates.

## Mathematical Procedure

Let us first consider the effect of a misalignment on a particle trajectory entering a magnet. The three positional shifts of the magnet cause a reverse shift of the particle coordinates relative to the entrance face of the magnet. Rotations of the magnet about the transverse axes cause a shift in the angles of the particle trajectory. A longitudinal displacement of the magnet effectively inserts a small drift space in front of the magnet entrance. Finally a rotation of the magnet about the reference trajectory causes a mixing of the coordinates x and y and of the angles x' and y'.

The same effects occur in reverse at the exit face of the magnet, but in terms of the shifts and rotations of the reference coordinate system at that face. Since the misaligned parameters are given in terms of shifts and rotations at the entrance face of the magnet, we need to determine the exit face effects in terms of the entrance face misalignments.

For such a transformation, two items are needed. The first is the orthogonal coordinate transformation giving the relationship between the reference trajectory coordinates at the entrance face and those at the exit face. The second is the vector which spans the misaligned magnet, i.e. that reaches from the intersection of the reference trajectory with the entrance face to that with the exit face.

The explicit details of the calculation are given elsewhere.[2] The effects can all be represented by the terms on the right side of equation (2).

We now consider the beam as having a spatial distribution given by an initial centroid position $\overline{x_i^0}$, and set of second moments $\overline{x_i^0 x_j^0}$. The half width of the beam in a given coordinate equals the square root of the diagonal term of the matrix of second moments. For an aligned magnet these quantities are transformed by the R matrix giving

$$\overline{x_i^1} = \sum_j R_{ij} \overline{x_j^0} \tag{3}$$

$$\overline{x_i^1 x_j^1} = \sum_{k\ell} R_{ik} R_{j\ell} \overline{x_k^0 x_\ell^0} \tag{4}$$

When the contribution of a known misalignment is added these equations become

$$\overline{x_i^1} = \sum_j R_{ij} \overline{x_j^0} + \sum_j F_{ij} m_j + \sum_{jk} G_{ijk} \overline{x_j^0} m_k \tag{5}$$

$$\begin{aligned} \overline{x_i^1 x_j^1} = {} & \sum_k R_{ik} R_{j\ell} \overline{x_k^0 x_\ell^0} \\ & + \sum_k \left[ F_{ik} R_{j\ell} + F_{jk} R_{i\ell} \right] \overline{x_\ell^0} m_k \\ & + \sum_{k\ell m} \left[ G_{ik\ell} R_{jm} + G_{jk\ell} R_{im} \right] \overline{x_k^0 x_m^0} m_\ell \\ & + \sum_{k\ell} F_{ik} F_{j\ell} m_k m_\ell \\ & + \sum_{k\ell m} \left[ F_{ik} G_{j\ell m} + F_{jk} G_{i\ell m} \right] \overline{x_\ell^0} m_k m_m \\ & + \sum_{k\ell mn} G_{ik\ell} G_{jmn} \overline{x_k^0 x_m^0} m_\ell m_n \end{aligned} \tag{6}$$

If we wish to express the uncertainty in position of the beam due to unknown misalignments, we must average over the misalignment parameters. The terms linear in the m will vanish, while the quadratic terms will be expressed in terms of the covariance matrix $\overline{m_j m_k}$. The beam centroid is then unaffected, while the second moments are altered by

$$\begin{aligned} \overline{x_i^1 x_j^1} = {} & \sum_{k\ell} R_{ik} R_{j\ell} \overline{x_k^0 x_\ell^0} \\ & + \sum_{k\ell} F_{ik} F_{j\ell} \overline{m_k m_\ell} \\ & + \sum_{k\ell m} \left[ F_{ik} G_{j\ell m} + F_{jk} G_{i\ell m} \right] \overline{x_\ell^0} \, \overline{m_k m_m} \\ & + \sum_{k\ell mn} G_{ik\ell} G_{jmn} \overline{x_k^0 x_m^0} \, \overline{m_\ell m_n} \end{aligned} \tag{7}$$

## Computer Program

The formalism described above has been written into the computer program TRANSPORT[1]. The misalignments may be specified with great flexibility and output may be obtained in various forms.

The magnitude of the misalignment in each coordinate is specified by the user. A misalignment may pertain to a single beam element or a section of the beam. Misaligned sections may also be nested. Alternatively, a single data card can be introduced which causes all subsequent bending magnets and/or quadrupoles to be misaligned by the amount specified.

Misalignments may be known or represent an uncertainty in position. With either option the beam distribution is affected as described in the previous section.

The initial beam distribution is specified by the user. It is transformed by the program through drift spaces and magnetic elements and can be printed at any later point. The existence of a misalignment at any point in the beam line will affect the beam distribution at any later point.

The user may request that the effects of the misalignments be stored in the matrix used to represent the beam. The effects of all misalignments in the beam line will then be combined and shown in the beam distribution wherever the user chooses to have it be printed.

Alternatively, the individual effects of the misalignments may be shown in a table. The table may then be printed at any point desired by the user. It shows separately the effect on the beam of the misalignment of each misaligned element in each of the six coordinates. The misalignments of up to ten magnets may be calculated and displayed in a single run.

The results of such a calculation are shown in the illustration. The misalignment calculation was initiated by a single data card which specified the magnitudes of the misalignment parameters and instructed the program to give an uncertain misalignment to all subsequent quadrupoles, up to ten, and display the results in a table.

The first quantities shown are the unperturbed beam distribution parameters. The two columns give respectively the beam centroid position and distribution half width in each of the beam coordinates. The triangle represents the correlations between coordinates. Following is a table giving the effects of the misalignments. The results of the misalignment of each quadrupole are shown in six pairs of columns, corresponding to the six misalignment degrees of freedom. At the head of each pair of columns is a number giving the magnitude of the misalignment in each direction. The effects of the misalignment of each of the quadrupoles are then shown in turn. Each pair of columns of six numbers shows the resulting beam distribution from a misalignment in one of the six degrees of freedom of the single quadrupole whose label is shown. These numbers may be compared directly with those giving the unperturbed beam parameters.

Following is the transfer matrix R, showing a horizontal focus. In this run the misalignment table was printed at all foci and regions of parallel beam.

## References


1) K.L. Brown, F. Rothacker, D.C. Carey, Ch. Iselin, TRANSPORT,NAL-91, SLAC 91, CERN 73-16, 1974.

2) K.L. Brown, D.C. Carey, TRANSPORT appendix, to be published.



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*DRIFT*        3.              285.83960 FT
   548.829 FT
                                                   0.0     0.086 CM
                                                   0.0     0.329 MR     0.035
                                                   0.0     0.168 CM    -0.000  0.000
                                                   0.0     0.338 MR     0.000  0.000 -0.648
                                                   0.0     0.209 CM    -0.010 -1.000 -0.000  0.000
                                                   0.0     0.0   PC     0.0    0.0    0.0    0.0    0.0

*MISALIGNMENT EFFECT TABLE FOR MISALIGNMENTS OF
      0.020 CM           0.100 MR           0.020 CM           0.100 MR           0.100 CM           1.000 MR

*MISALIGNMENT OF Q1  *
   0.0  0.139 CM     0.0  0.087 CM     0.0  0.087 CM     0.0  0.121 CM     0.0  0.087 CM     0.0  0.094 CM
   0.0  0.330 MR     0.0  0.325 MR     0.0  0.329 MR     0.0  0.330 MR     0.0  0.329 MR     0.0  0.325 MR
   0.0  0.167 CM     0.0  0.200 CM     0.0  0.221 CM     0.0  0.167 CM     0.0  0.167 CM     0.0  0.172 CM
   0.0  0.338 MR     0.0  0.338 MR     0.0  0.338 MR     0.0  0.338 MR     0.0  0.338 MR     0.0  0.338 MR
   0.0  0.209 CM     0.0  0.205 CM     0.0  0.205 CM     0.0  0.209 CM     0.0  0.209 CM     0.0  0.205 CM
   0.0  0.0   PC     0.0  0.0   PC     0.0  0.0   PC     0.0  0.0   PC     0.0  0.0   PC     0.0  0.0   PC

*MISALIGNMENT OF Q2  *
   0.0  0.165 CM     0.0  0.086 CM     0.0  0.086 CM     0.0  0.138 CM     0.0  0.086 CM     0.0  0.094 CM
   0.0  0.330 MR     0.0  0.329 MR     0.0  0.329 MR     0.0  0.330 MR     0.0  0.325 MR     0.0  0.329 MR
   0.0  0.168 CM     0.0  0.182 CM     0.0  0.192 CM     0.0  0.168 CM     0.0  0.168 CM     0.0  0.172 CM
   0.0  0.338 MR     0.0  0.338 MR     0.0  0.338 MR     0.0  0.338 MR     0.0  0.338 MR     0.0  0.338 MR
   0.0  0.205 CM     0.0  0.205 CM     0.0  0.209 CM     0.0  0.209 CM     0.0  0.205 CM     0.0  0.205 CM
   0.0  0.0   PC     0.0  0.0   PC     0.0  0.0   PC     0.0  0.0   PC     0.0  0.0   PC     0.0  0.0   PC

*MISALIGNMENT OF Q3  *
   0.0  0.166 CM     0.0  0.086 CM     0.0  0.086 CM     0.0  0.136 CM     0.0  0.086 CM     0.0  0.094 CM
   0.0  0.330 MR     0.0  0.329 MR     0.0  0.329 MR     0.0  0.330 MR     0.0  0.329 MR     0.0  0.325 MR
   0.0  0.168 CM     0.0  0.183 CM     0.0  0.193 CM     0.0  0.168 CM     0.0  0.168 CM     0.0  0.172 CM
   0.0  0.339 MR     0.0  0.338 MR     0.0  0.338 MR     0.0  0.338 MR     0.0  0.338 MR     0.0  0.339 MR
   0.0  0.205 CM     0.0  0.209 CM     0.0  0.205 CM     0.0  0.209 CM     0.0  0.205 CM     0.0  0.209 CM
   0.0  0.0   PC     0.0  0.0   PC     0.0  0.0   PC     0.0  0.0   PC     0.0  0.0   PC     0.0  0.0   PC

*MISALIGNMENT OF Q4  *
   0.0  0.149 CM     0.0  0.086 CM     0.0  0.086 CM     0.0  0.125 CM     0.0  0.086 CM     0.0  0.094 CM
   0.0  0.330 MR     0.0  0.325 MR     0.0  0.329 MR     0.0  0.330 MR     0.0  0.329 MR     0.0  0.325 MR
   0.0  0.168 CM     0.0  0.190 CM     0.0  0.203 CM     0.0  0.168 CM     0.0  0.168 CM     0.0  0.172 CM
   0.0  0.328 MR     0.0  0.338 MR     0.0  0.338 MR     0.0  0.338 MR     0.0  0.338 MR     0.0  0.338 MR
   0.0  0.209 CM     0.0  0.209 CM     0.0  0.209 CM     0.0  0.205 CM     0.0  0.209 CM     0.0  0.209 CM
   0.0  0.0   PC     0.0  0.0   PC     0.0  0.0   PC     0.0  0.0   PC     0.0  0.0   PC     0.0  0.0   PC
*TRANSFORM 1*          "Q3  "
             -1.70690   0.00121  -0.00000  -0.00000   0.0      -6.33738
             -0.27910  -0.58566  -0.00000  -0.00000   0.0      -0.59894
             -0.00000  -0.00000  -2.49643   0.13069   0.0       0.00000
             -0.00000  -0.00000  -0.21783  -0.38916   0.0       0.00000
              0.07464   0.37123  -0.00000  -0.00000   1.00000  -0.02257
              0.0       0.0       0.0       0.0       0.0       1.00000
```

Figure 1. Sample computer output, showing effect of misalignment of each of four quadrupoles.