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EFFECTS OF A NON-LINEAR LENS ON THE STORED PROTON BEAM IN THE ISR

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Summary

In order to simulate the beam-beam effect in a proton storage ring, at values of the beam-beam tune-shift much higher than those attainable with proton beams in the ISR, a non-linear lens has been installed. It consists of a pair of bars inside the vacuum chamber, about 1 m long and capable of carrying currents up to 1000 A, corresponding to a linear tune-shift of about 0.1. The current bars, running parallel to the beam at a distance of a few mm above and below it, create a highly non-uniform magnetic field. It has the same symmetry as space charge at a head-on intersection, and excites all non-linear resonances $n_h Q_h + n_v Q_v = p$ for which both n_h and n_v are even. The beam decay rate has been measured as a function of the betatron tunes and the tune shift ΔQ caused by the non-linear lens. When ΔQ changes between 0 and 0.05 the decay rate changes by about 3 orders of magnitude, and depends strongly on the betatron tunes. A computer simulation shows that at $\Delta Q \approx 0.05$ the stochasticity limit is reached.

1. Introduction

It is well known from electron storage rings that their luminosity stops increasing with the circulating current when the linear tune shift ΔQ caused by the beam-beam collisions reaches a certain limit which typically is about $\Delta Q = 0.05$.[1] The linear tune shift is taken as a measure for the strength of the non-linear forces of one beam on the other one which are believed to be responsible for this limitation.

For proton storage rings, it has been suggested that there might be an even lower limit on ΔQ[2,3] arising from slow mechanisms for beam amplitude growth which are not observed in electron storage rings because of radiation damping. An upper limit of $\Delta Q = 0.005$ has been adopted for the design of several machines. Since the luminosity of storage rings is approximately proportional to ΔQ,[4] it is important to find its maximum permissible value also for proton machines in order to have a solid basis for the design of future machines.

A beam-beam tune shift of about $\Delta Q = 5 \times 10^{-4}$ is reached in routine operation in the ISR. It is believed that this machine is not beam-beam limited; in particular, there is no observable increase in the beam decay rate which can be attributed to the beam-beam effect. In order to obtain a higher figure for the permissible tune shift ΔQ, an experiment[5] was done colliding a weak beam at an energy of 2 GeV with a strong beam at 26 GeV. This experiment was rather inconclusive because of the strong effect of intra-beam scattering[6] on the lifetime of the weak beam. The only safe statement which can be derived from the observations is that there is no noticeable effect of a tune shift of $\Delta Q = 0.005$ on the lifetime within about half an hour.

In order to obtain data at significantly higher values of the tune shift it was suggested[7] to simulate the strong beam by an external field acting upon the weak beam. It is obvious that an external field cannot simulate the field of another beam in all its details. However, it should be possible to study the dominant effects of such a field. Following this idea, a non-linear lens (NLL) was built and installed in the ISR. Below, we discuss its construction, the way in which experiments on its effect on the beam were performed, and their results.

2. Mechanical Construction

In order to excite as many non-linear field components as possible within the symmetry of the arrangement, the design of the NLL is completely different from that of the usual non-linear lenses. It consists of two identical current bars which are parallel to the beam, one running above it, the other one below. Cross-sections of the arrangement are shown in Fig. 1.

In order to concentrate as much field as possible in the vicinity of the beam, the current-carrying copper bar is surrounded by a U-shaped piece of high-permeability alloy.[8] As has been verified by field computations, the field generated in this manner closely

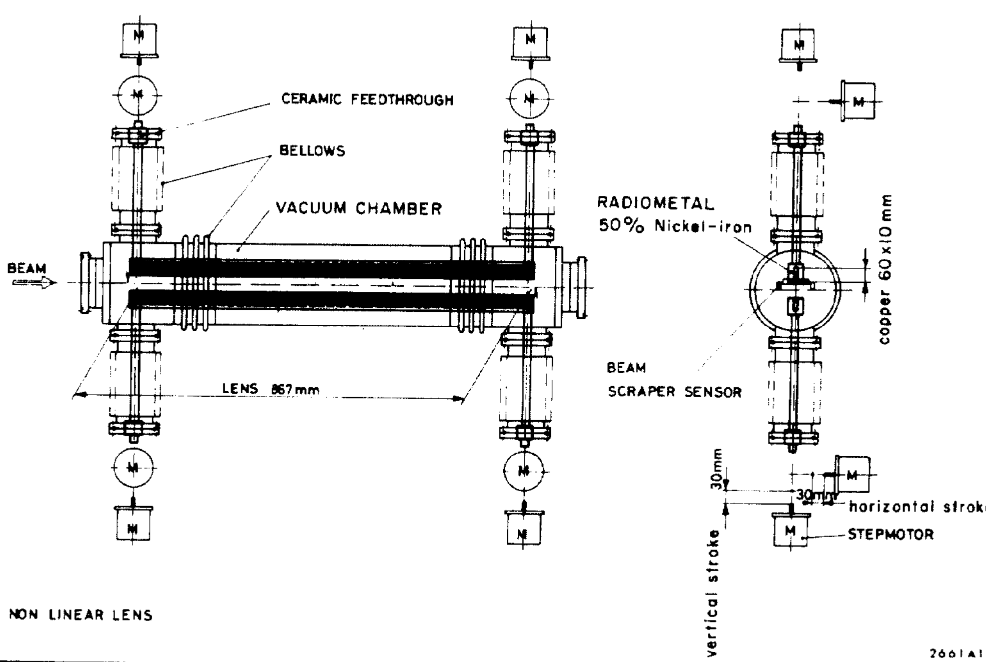

Fig. 1--Schematic cross-sections of the non-linear lens

*Present address: SLAC, P.O. Box 4349, Stanford, California 94305, USA approximates the field obtained from an infinitely thin current sheet of width 2d = 10 mm a distance h away from the beam, as schematically shown in Fig. 2. In this manner, the height of the copper bar can be chosen such that the current density and hence the power dissipated in it cause no particular problems.

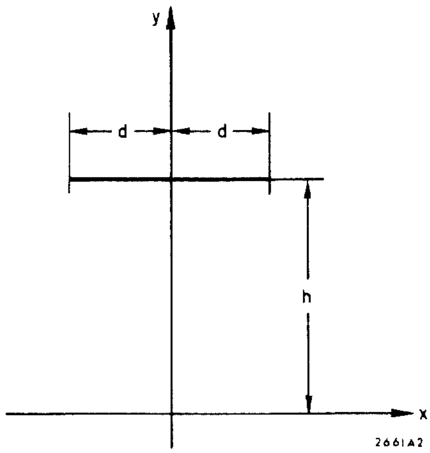

Fig. 2--Co-ordinate system and dimensions of the current sheet

In order to bring the current bars as close as possible to the beam, they are installed inside a vacuum tank. This imposes tight limits on the materials used to avoid deteriorating the ISR vacuum. In addition, the radial and vertical positions of the two current bars and hence their distance h to the centre of the beam are adjustable by stepping motors. The position control and read-out are done by the ISR control computer. Secondary-emission foils are used to detect when the current bars move into the halo of the circulating beam, and to stop their motion.

3. The NLL Field

The magnetic field generated by a current sheet can be given in closed form; its components are:

$$H_x = -\frac{I}{4\pi d}\left[\arctan\frac{d+x}{y-h} + \arctan\frac{d-x}{y-h}\right]$$
$$H_y = \frac{I}{8\pi d}\,\ln\frac{(y-h)^2+(d+x)^2}{(y-h)^2+(d-x)^2} \quad . \tag{1}$$

In the actual lens, the fields generated by the second bar have to be added to those given above. The direction of the current is the same in both bars and hence H_x vanishes in the median plane between them. This condition was imposed on the NLL design in order to avoid a vertical closed orbit distortion which otherwise would have had to be compensated by other means.

Linearizing the field equations above near $x = y = 0$, yields for the gradients of the NLL field (for two bars):

$$\frac{\partial H_x}{\partial y} = -\frac{\partial H_y}{\partial x} = \frac{I}{\pi(h^2+d^2)} \quad . \tag{2}$$

In order to obtain the maximum tune shift for a given strength of the NLL, it was installed at a position where the amplitude function β_y has its maximum value β_y = 53 m, while β_x = 18 m. Using the usual expression for the linear tune shift

$$\Delta Q = \frac{\beta}{4\pi}\,\frac{L}{B\rho}\,\frac{\partial B_x}{\partial y} \tag{3}$$

and assuming the full ISR energy, $B\rho$ = 100 Tm, a bar spacing of 2h = 20 mm, and a length L = 0.867 m, shows that a current of I = 1000 A will yield a tune shift ΔQ = 0.14, well above the beam-beam limit in electron storage rings.

If the fields due to the current sheet at y = -h are added to (1), it may be seen that H_x is odd in y, and that H_y is odd in x. It follows that the potential from which H_x and H_y can be derived is even in x and y. Because of this, the NLL drives only those non-linear resonances, given by

$$n_x Q_x + n_y Q_y = p \quad , \tag{4}$$

where both n_x and n_y are even. This implies that also the orders $n = |n_x| + |n_y|$ of the resonances are even. The head-on collision between two particle beams has the same kind of symmetry.

4. Experimental Procedure

The effect of the NLL on the proton beam was determined by measuring the lifetime of an aperture-limited beam which can be done with good precision. The NLL itself was the aperture limitation. This technique includes two different phenomena which might be responsible for the luminosity half-life in a proton machine:

- a general diffusion-like process which causes a slow growth of the betatron oscillation amplitudes of all protons and hence of the beam size;
- processes which quickly remove protons from the beam but affect only a small fraction of them at a time, such that beam decay rate is enhanced without a visible increase in the beam size.

The detailed procedure is as follows: A proton beam is accumulated while the NLL is withdrawn from the aperture and not excited. The accumulated current ranges from single CPS pulses to small stacks of about 1 A. The choice of the current has to be a compromise between the low currents wanted to avoid the collective phenomena described below and high currents useful to obtain accurate decay rate readings in a short time. The maximum betatron amplitudes of the protons are defined by running horizontal and vertical scrapers into the beam, causing a loss of about 10% of the circulating current. The scrapers are then withdrawn. The NLL is brought into the aperture until the secondary emission foils hit the beam, and then retracted by a few millimetres. The current in the NLL is then increased slowly to the desired value. During the early testing it turned out that the beam current was severely reduced in this operation, presumably due to the beam sweeping across strong non-linear resonances. It was therefore decided to avoid this sweeping across resonances, at least approximately, by changing the setting of the rest of the machine during the NLL current rise such that the small-amplitude tunes of the whole machine including the NLL remain constant. When this technique is adopted, most of the circulating beam survives the excitation of the NLL. At a given current in the NLL the beam lifetime is measured as a function of the tunes by making small changes in Q_x and Q_y.

In addition to the non-linear resonances excited by the NLL, there are other mechanisms which cause a finite beam decay rate in the ISR, such as nuclear scattering and multiple scattering on the residual gas, and intra-beam scattering of the protons in the beam on each other. If these decay rates are not small compared to the effect of the NLL, they must be subtracted from the experimental data. At an average pressure of 10^{-11} Torr, the decay rate due to nuclear scattering is about

$$\dot{I}/I = -10^{-6}/\text{min} \quad . \tag{5}$$

The decay rate due to multiple scattering and due to intra-beam scattering depends on the aperture and on the distribution of the beam within that aperture. An upper limit for the decay rates due to both types of scattering is obtained if we assume that the beam density distribution is less populated near the edge of the aperture than the equilibrium distribution. The latter is proportional to $J_0(j_1 r/h)$ where j_1 is the first zero of J_0 and h is the aperture radius. This assumption is justified since the NLL was brought into the beam and then withdrawn a little. The decay rate and the diffusion constant are related by

$$\dot{I}/I = -(a\ j_1/h)^2 \quad . \tag{6}$$

At a pressure of 10^{-11} Torr and at 26 GeV/c, multiple scattering has the following diffusion constant:[9]

$$a_{MS}^2 = 7.2 \times 10^{-6}\ \text{mm}^2/\text{s} \quad . \tag{7}$$

The diffusion constant for intra-beam scattering[6] is proportional to the circulating current I. For typical running conditions of the NLL it is about:

$$a_{IBS}^2 = 5 \times 10^{-6}\ I\ \text{mm}^2/\text{s} \tag{8}$$

where the current is in A. For typical values of $h \approx 7.5$ mm, both diffusion rates yield decay rates of $\dot{I}/I \lesssim 5 \times 10^{-7}\text{s}^{-1}$.

5. Experimental Results

The tune spread caused by the NLL was calculated by a Monte Carlo method from the fields shown in (1); the vertical tune spread was also measured by Schottky scans.[10] There was not enough signal to measure the horizontal tune spread. The results of the computer simulation are shown in Fig. 3. They agree well with the measurement.

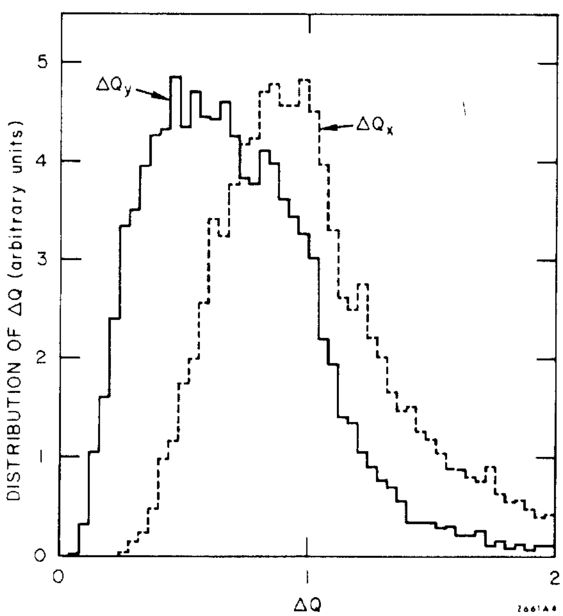

Fig. 3--Histograms of the amplitude variation of the tune shift. The abscissa is in units of the linear tune shift. The rms amplitudes are $\sigma_x = \sigma_y = \frac{1}{2}h$.

The variation of the beam lifetime with the exact values of the tunes is shown in Table I and II; the former giving the results when the NLL is not excited, the latter for an excitation corresponding to a linear tune shift of $\Delta Q_x = -0.054$. At each value to the tune, five readings of the decay were taken. The errors shown in Tables I and II are the standard deviations of these readings. The first table demonstrates that the beam lifetime is rather independent of the tune and compatible with the beam loss phenomena discussed above. The second table shows that the beam decay rate becomes a strong function of the tune once the NLL is excited. It should be noted that tune variations much smaller than the linear tune shift and also the tune spread as calculated above cause a change in the decay rate.

Table I.

Beam decay rates in units of 10^{-8}s^{-1} without excitation of the NLL. $Q_x = 8.654 \pm 0.003$, $Q_y = 8.634 \pm 0.006$, h = 7.4 mm. δQ_x and δQ_y are the applied changes in Q_x and Q_y.

δQ_y	δQ_x		
	-.0025	0	+.0025
.0025	.1 ± 11	22.5 ± 8	1.8 ± 8
0		- .8 ± 9	4.3 ± 9

Table II.

Beam decay rates in units of 10^{-5}s^{-1} with the NLL excited to $\Delta Q_y = -.054$. The parameters are the same as in Table I.

δQ_y	δQ_x		
	-.0025	0	+.0025
.0025	7.8 ± .8	12.8 ± 2.1	17.3 ± 2.6
0	1.5 ± .1	7.0 ± .5, 3.0 ± .2	1.7 ± .2
-.0025	0.6 ± .03	0.5 ± .07	0.6 ± .04

The variation of the beam decay rate with the excitation of the NLL is shown in Fig. 4. The experimental results have been compressed by showing only the beam decay rates obtained with the most favourable and least favourable tunes, as well as their average value for a given excitation of the NLL. The data shown were obtained in two independent runs.

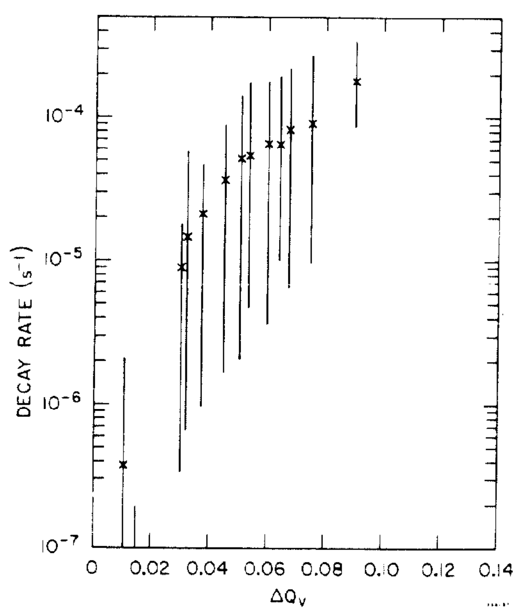

Fig. 4--Beam decay rate. The crosses show the average decay rate; the length of the bars shows the maximum and minimum rate when the machine tune is varied.

6. Computer Simulation

The effect of the NLL was simulated on a computer. The program follows two particles for 10^4 turns, a turn consisting of a linear transformation with phase advance $2\pi Q_x$ and $2\pi Q_y$, respectively, and non-linear kicks derived from (1) with linear strengths ΔQ_x and ΔQ_y. The trajectory starts with $x_0 \neq 0$, $y_0 \neq 0$ and $x_0' = y_0' = 0$. The stochasticity limit is considered to be reached when one of the following two things happens: the y-position becomes bigger than h, i.e., the particle hits the NLL, or local instability occurs.[2] Local instability is believed to be an indication of stochastic motion in systems with several degrees of freedom such as the one being studied. It is detected by observing an exponential growth of the distance in phase space between two particles which are very close together at the beginning. The results of this simulation are shown in Table III.

Table III.

Computed beam-beam limit ΔQ_y for Q_x = .654, Q_y = .634, $\Delta Q_x/\Delta Q_y$ = - 1/3, d/h = 0.3.

y_0/h	x_0/h				
	.1	.2	.3	.4	.5
.1	-.037	-.038	-.041	-.045	-.051
.2	-.035	-.036	-.038	-.043	-.049
.3	-.032	-.033	-.036	-.041	-.047
.4	-.029	-.030	-.034	-.038	-.042
.5	-.025	-.027	-.030	-.034	-.043

7. Discussion

The beam decay rate is a strong function of the excitation of the NLL: changing the NLL current by a factor of 5 changes the decay rate by two orders of magnitude. The decay rate saturates at tune shift values close to the stochasticity limits shown in Table III.

When the NLL is excited, the beam decay rate is very sensitive to the exact value of the tunes. Tune changes which are much smaller than the tune spread present in the beam cause variations in the decay rate by an order of magnitude or more. This observation may be taken as an indication of the order of the resonances involved in the enhanced beam decay. Fig. 5 shows the minimum distance of a working point in (Q_x,Q_y) from resonances up to a given order, averaged over a random sample of working points uniformly distributed in $0 < Q_x,Q_y < 1$. Comparing the tune changes applied in Tables I and II to the resonance spacings in Fig. 5 shows that resonances of orders up to about 10 are involved in the beam decay.

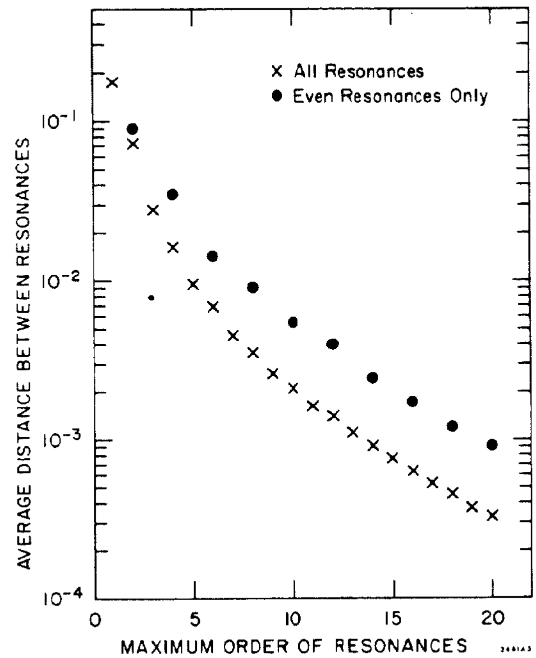

Fig. 5--Average distance between sum resonances of order up to n. The crosses show the distance when all sum resonances are included; the dots show the distance when only even order resonances with even coefficients are included.

8. Acknowledgement

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