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HIGHLY STABILIZED POWER SUPPLY FOR SPECTROMETER*

J. J. Gano
William H. Bates Linear Accelerator
Massachusetts Institute of Technology
Middleton, Massachusetts

Summary

The two 4000A, 150V power supplies for the spectrometer magnet of the Bates Linac have a current regulation and stability of better than .001% for an 8-hour period. Current sensing is accomplished by a 5-volt precision shunt. The output is programmed by a stable dc voltage reference with a 6-digit keyboard. The rate of current increase or decrease for controlling magnet excitation is adjustable up to a rate of 4000A per minute. Power dissipation in the 300 pass transistors of each supply is limited by automatically controlling the emitter to collector voltage with the use of a variable voltage transformer. To avoid possible conducted or radiated interference with experimental signals, no silicon-controlled rectifiers are used. The major portion of this article is devoted to an analysis of the stability.

Introduction - Specifications

An accuracy of .01% was designated as the design goal of the 300-ton spectrometer of the 400 MeV Bates Linear Electron Accelerator. To cooperate in this achievement, the two 4000A, 150V dc power supplies for the magnet were limited to a combined current regulation and stability of less than ±.001%.

The following are the pertinent specifications for the output performance of the supplies:

1) Input voltage: 460 volts, 3 phase, 60 Hz.
2) Output load: .035Ω in series with 0.11 H; or a combination of two such loads in series or in parallel
3) Combined current regulation, stability and ripple at any load between 500A and 4000A not to exceed ±.001% over an 8-hour period for the following disturbances:
   a) AC input voltage variation: ±7%, slow, or ±2%, step.
   b) Output load variation: +15% in resistance.
   c) Ambient temperature variation at the power cabinet location: ±10°C in the range of 5° to 45°C.
   d) Ambient temperature variation at the remote control panel location: ±2° in the range of 21° to 25°C.
   e) Cooling water available: 35°C ±1.5°.
4) Resolution of output current selection: .001%.
5) Output performance and resolution from -40A to 500A: degraded inversely as the magnitude of the current approximately.
6) Remote control panel to be located at 150 feet from the power cabinet.
7) Rate of current increase or decrease: adjustable up to 4000A per minute.

Circuitry and interlocks for the protection of personnel and components were also required. Detection, and both audio and visual indication of partial failures, such as parallel transistors, or rectifier diodes, was included. These failures do not initiate a shutdown.

Circuit Description

Fig. 1 is the circuit schematic for the current regulator and Fig. 2 is the block schematic showing power and signal paths through the various circuits. In order to focus attention on the performance analysis, only those components that enter into the analysis are shown. Conventional circuitry not shown includes:

1) Transformer-rectifier connections.
2) Voltage pre-regulator controls.
3) Relay controls for energization, mode-of-operation selection, interlocks, etc.
4) Auxiliary power supplies.
5) Circuit breakers and fuses, and load-balancing resistors.
6) Protective clamping circuits for transistor over-voltage, reverse voltage, and short circuit.
7) Reverse current source (-40A).

No silicon-controlled rectifiers are used to avoid possible radiated or conducted interference with experimental signals.

Pre-regulator

The circuit of Fig. 1 is a series-regulated, solid-state current supply with a variable transformer for pre-regulation. The output of the variable transformer is automatically controlled to maintain 10-12 volts across the pass transistors Q1 regardless of the magnitude of the output voltage. To limit power dissipation in the transistors, the voltage is just sufficient to meet the required transient demands on the series regulator.

Rectifier-Filter

The transformer assembly operates into a 6-phase bridge rectifier, the output ripple of which is filtered by L1-C2. A peak-to-peak ripple of one volt is considered an economical compromise between cost of the filter and excessive power dissipation in the series transistors.

Series Regulator

The series regulator controls the amount of current supplied to the magnet load R_L-L_L. The regulator consists of 300 transistors Q1 in parallel driven by a 3-stage Darlington circuit, Q2, Q3, Q4, containing the quantity and type of transistors indicated. The 2N 2116 power transistor is rated at 30A, 50V, 250W, with a thermal resistance of 0.45°C/W up to a junction temperature of 175°C, and has a dc current gain of 10.

Output Filter

The output filter C1 acts as energy storage for load transients, reduces ripple and helps stabilize the feedback system.

Current Sensor

Resistor R_S, a precision water-cooled resistor, 1.25mΩ approximately, is connected in series with the load to measure the current and transform it into a voltage V_{fb}, as the feedback signal.

Voltage Reference

The feedback voltage is compared with the reference voltage V_R, and the difference, the error voltage ξ, is applied to the operational amplifier A. The reference voltage which matches the feedback voltage is derived from a stable dc voltage source with a 6-digit keyboard for programming.

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Amplifier

Resistance R4 and the input resistance of the operational amplifier R_A act as a voltage divider for the error signal. The first stage of amplification is performed by a chopper-stabilized operational amplifier having a high dc gain, low input voltage drift, and low input noise.

The output is connected to a transistor amplifier Q5, acting as a signal inverter and buffer between the amplified signal and the first stage of the Darlington circuit Q4.

Both stages of voltage amplification are capacitor coupled back to the input to reduce the gain at higher frequencies and thus achieve feedback stability.

The 15V auxiliary power supplies are voltage regulated to .01%.

Feedback System

The current regulator is presented as a feedback system in Fig. 3 with each block representing a transfer function. Fig. 4 is a hybrid diagram in which part of the system is presented as blocks of a feedback system. The series regulator Q1-3 is shown as an emitter follower circuit. R_S combines with the load resistance R_L to form a divider for the output voltage V_o.

The incremental transfer values of the blocks are as follows:

(1) Voltage divider $\frac{R_A}{R_4+R_A}$	K_D	.75	V/V
(2) Operational amplifier	A	10^8	V/V
(3) Buffer amplifier	K_Q	2	V/V
(4) Series regulator, emitter follower	K_F	1	V/V
(5) Load	$\frac{1}{R_L}$	$\frac{1}{.035}$	A/V
(6) Current sensor	R_S	1.25×10^{-3}	V/A
(7) Feed forward gain, $K_D\,AK_Q\,K_F\,\frac{1}{R_L}$	K_{FF}	$\sim 40\ \times10^8$	A/V
(8) Loop gain, $K_{FF}\,R_S$	K_L	$\sim 5\ \times10^6$	V/V

From feedback system principles, the transfer function of the closed loop system for the incremental effect on output current I_L due to variations in the set point V_R is:

$$\frac{I_L}{V_R} = \frac{\text{feed forward gain}}{\text{1+loop gain}} = \frac{K_{FF}}{1+K_L} \tag{1}$$

$$= \frac{K_D\,AK_Q\,K_F\,\frac{1}{R_L}}{1+K_D\,AK_Q\,K_F\,\frac{1}{R_L}\,R_S} \tag{2}$$

The loop gain is so large that the "1" in the denominator can be neglected and (2) reduces to:

$$I_L = \frac{1}{R_S}\,V_R \tag{3}$$

Equation (3) shows that the output current variation is proportional to the reference voltage variation.

Effect of Drift in Reference Voltage

If the variables in equation (3) are indicated as increments and both sides are divided by I_L:

$$\frac{\Delta I_L}{I_L} = \frac{I}{R_S\,I_L}\,\Delta V_R \tag{4}$$

$R_S\,I_L = V_{fb} = V_R$ essentially

hence

$$\frac{\Delta I_L}{I_L} = \frac{\Delta V_R}{V_R} \tag{5}$$

Thus, the percent change in output current equals the percent change in the reference voltage.

Effect of Variations in Internal Components

At each of the summation points of the feedback diagram Fig. 5, disturbances that may appreciably effect the output current are indicated. The effect on the output, again neglecting the "1" in the denominator, is:

$$\frac{\Delta I_L}{\Delta V_x} = \frac{\text{feed forward gain (from summation point)}}{\text{loop gain}} \tag{6}$$

where ΔV_x is the incremental voltage at the summing point.

If the disturbance enters the loop beyond the amplifier A, the maximum effect is:

$$\frac{\Delta I_L}{\Delta V_x} = \frac{K_Q\,K_F\,\frac{1}{R_L}}{K_L} = 12\mu A/V \tag{7}$$

The stability specification for ΔI_L is 1×10^{-5} or 40ma at full load. Equation (7) indicates that no ΔV_x between the amplifiers A and K_Q will affect the system while it is performing properly.

Drift in Current Sensor

A variation in the value of R_S, which has to dissipate 5kw and is so located in the circuit that the disturbance is multiplied by the loop gain, would appear to have the greatest effect on the output current I_L.

In (3), if the reference voltage V_R is maintained constant and we differentiate I_L with respect to R_S, the result is:

$$\frac{dI_L}{dR_S} = \frac{1}{R_S^{\,2}}\,V_R \tag{8}$$

Essentially: $V_R = V_{fb} = I_L\,R_S$ (9)

Substitution into (8) and replacing the "d" with Δ, yields:

$$\Delta I_L = \frac{I_L}{R_S}\,\Delta R_S \tag{10}$$

The evolution of the equivalent feedback diagram is shown in Fig. 6. Part C of the figure is inserted in Fig. 5.

Normalizing (10) gives:

$$\frac{\Delta I_L}{I_L} = \frac{\Delta R_S}{R_S} \tag{11}$$

This indicates that the output current variation is proportional to the variation in the sensing resistor.

Drift in Voltage Divider at Amplifier Input

Because variations in the value of the resistors in the voltage divider preceding the operational amplifier are multiplied by the enormous gain of

$\frac{K_{FF}}{K_D} = \sim 50\times10^8$, the effect on the output current is investigated.

The transfer function is $K_D = \frac{R_A}{R_4+R_A}$

The output voltage of the divider ξ'

$$= \frac{R_A}{R_4+R_A}\,\xi \tag{12}$$

Differentiating with respect to R_4 while ξ and R_A remain constant, yields the change in the output voltage of the divider, due to variations in R_4:

$$d\xi' = \frac{R_A}{(R_4+R_A)^2}\,dR_4 \tag{13}$$

Substituting K_D and Δ:

$$\Delta\xi' = K_D\,\frac{\xi}{R_4+R_A}\,\Delta R_4 \tag{14}$$

From Fig. 3 and feedback system principles:

$$\xi = V_R - V_{fb} \tag{15}$$

$$V_{fb} = \frac{K_L}{1+K_L} V_R \tag{16}$$

Combining: $$\xi = \frac{1}{1+K_L} V_R \cong \frac{V_R}{K_L} \cong \frac{I_L R_S}{K_L} = \frac{I_L}{K_{FF}} \tag{17}$$

Substituting for ξ in (14):

$$\Delta\xi' = K_D \frac{I_L}{K_{FF}} \frac{1}{R_4+R_A} \Delta R_4 \tag{18}$$

The disturbance ΔR_4 is represented in the feedback diagram of Fig. 5. It first passes through the transfer function $\frac{1}{R_4+R_A}$. The next transfer function $\frac{I_L}{K_{FF}}$ is common to both ΔR_4 and ΔR_A. From Fig. 5, the effect of ΔR_4 on the output current is:

$$\Delta I_L = K_{FF} \frac{I_L}{K_{FF}} \frac{1}{R_4+R_A} \Delta R_4 \tag{19A}$$

$$\frac{\Delta I_L}{I_L} = \frac{\Delta R_4}{R_4+R_A} \tag{19}$$

This relation indicates that the percent change in output current is the change in R_4 represented as a percent of the two resistors in series. It can similarly be shown that a drift ΔR_A has the following effect:

$$\frac{\Delta I_L}{I_L} = \frac{R_1}{R_A} \frac{1}{R_4+R_A} \Delta R_A \tag{20}$$

Drift in Amplifier Input Voltage

The drift at the input voltage of the operational amplifier is inserted directly into the summation point as ΔV_D, Fig. 5. The closed loop transfer function for the output current is:

$$\Delta I_L = \frac{A K_Q K_F \frac{1}{R_L}}{K_L} \Delta V_D = \frac{1}{K_D R_S} \Delta V_D \tag{21}$$

Then, $$\frac{\Delta I_L}{I_L} = \frac{1}{K_D} \frac{\Delta V_D}{V_R} \tag{22}$$

This indicates that the percent variation in output current is the variation in drift voltage represented as a percent of the reference voltage, divided by the voltage divider ratio K_D.

Effect of Variation in Load

In the open loop circuit of Fig. 7, ΔR_L is the change in load. If ΔR_L is shorted and V_L maintained constant, by superposition, the dc load current is:

$$I_L = \frac{V_L+\Delta V_L}{R_L} = \frac{V_L}{R_L} + \frac{I_L \Delta R_L}{R_L} = \frac{1}{R_L}(V_L + I_L \Delta R_L) \tag{23}$$

The summation and transfer function I_L represented by this equation are inserted in Fig. 5.

The closed-loop effect on the output current is:

$$\Delta I_L = \frac{\frac{I_L}{R_L}}{K_L} \Delta R_L \tag{24}$$

Normalizing:

$$\frac{\Delta I_L}{I_L} = \frac{1}{K_L} \frac{\Delta R_1}{R_L} \tag{25}$$

The percent change in output current is equal to the percent change in the output load resistance divided by the loop gain.

Effect of Variation in AC Input Voltage

The equivalent circuit for the series transistor Q1 and the power section is shown in Fig. 8. To facilitate presentation, the rectifier and ripple filter C2-L1 are shown as a Thevenin theorem equivalent. Any dc variation in the rectified voltage V_S affects the open-loop output current as follows:

$$\Delta I_L' = \frac{\Delta V_S}{R_C+R_L} = \frac{R_L}{R_C+R_L} \frac{1}{R_L} \Delta V_S \tag{26}$$

where R_C is the equivalent collector resistance of the transistor bank.

The effect shown by this equation is incorporated into Fig. 5. K_S represents the voltage divider $\frac{R_L}{R_C+R_L}$; K_{LC} represents the filter which is unity for dc.

The closed-loop effect of ΔV_S on the output current is:

$$\Delta I_L = \frac{\frac{1}{R_L}\frac{R_L}{R_C+R_L}}{K_L} \Delta V_S \tag{27}$$

Normalizing:

$$\frac{\Delta I_L}{I_L} = \frac{\frac{R_L}{R_C+R_L}}{K_L} \frac{\Delta V_S}{R_L I_L} = \frac{\frac{R_L}{R_C+R_L}}{K_L} \frac{\Delta V_S}{V_L} \tag{28}$$

Since $\frac{\Delta V_S}{V_S} = \frac{\Delta V_{ac}}{V_{ac}}$, substitution into (28) yields:

$$\frac{\Delta I_L}{I_L} = \frac{V_S}{V_L} \frac{\frac{R_L}{R_C+R_L}}{K_L} \frac{\Delta V_{ac}}{V_{ac}} \tag{29}$$

This equation indicates that the percentage change in output current is proportional to the change in the ac line voltage multiplied by the effect of the higher rectification voltage V_S/V_L and the load resistance-collector resistance divider, with this product then divided by the loop gain. Because of the variable transformer preregulator, the series regulator must provide for only ±1 volt of supply voltage, which in terms of the ac line voltage is ±.5%.

Summary of Effect of Disturbances

Each of the foregoing disturbances that have been investigated have a normalized effect $\frac{\Delta I_L}{I_L}$ on the output current as indicated in Column 1.

	Col. 1	Col. 2
1) Reference-voltage drift	$\frac{\Delta V_R}{V_R}$	$\pm 1\times10^{-6}/^{o}C$
2) Current-sensor drift	$\frac{\Delta R_S}{R_S}$	$\pm 25\times10^{-6}/^{o}C$
3) Voltage-divider drift	$\frac{\Delta R_4}{R_4+R_A}$	$\pm 1.0\times10^{-6}/^{o}C$
4) Op-amp input voltage drift	$\frac{1}{K_D}\frac{\Delta V_D}{V_R}$	$\pm .06\times10^{-6}/^{o}C$ $\pm .2\times10^{-6}$/day
5) Load variation	$\frac{1}{K_L}\frac{\Delta R_L}{R_L}$	$\pm .015\times10^{-6}$
6) AC input voltage variation	$\frac{1}{K_L}\frac{V_S}{V_L}\frac{R_L}{R_L+R_C}\frac{\Delta V_{ac}}{V_{ac}}$	$\pm .06\times10^{-6}$

The following are the maximum values for constants which have not already been given:

ΔV_R	$1\times10^{-6}/^{o}C$
ΔR_S	$25\times10^{-6}/^{o}C$
ΔR_4	$3\times10^{-6}/^{o}C$
ΔV_D	$0.25\times10^{-6}V/^{o}C$ and 1×10^{-6}V/day
ΔR_L	$.15\ R_L$
VS/VL	1.15

When the constants are inserted into the symbols of Column 1, the results are those shown in Column 2. It is seen that the only appreciable effects are those created by drifts.

Excluding the effect of the current-sensor drift, should all the disturbances occur unidirectionally and simultaneously, the specification of $\pm 1\times10^{-5}$ would be satisfied by a factor of two. The temperature coefficient of 25 ppm for R_S is the maximum value guaranteed by the manufacturer. Most times it is appreciably less. A value of 10 ppm would meet the specifications. Should the material have a high temperature coefficient or the water temperature vary up to about 10°C, a voltage divider, as shown in Fig. 9, may be used. A copper resistance R_{CU}, 1Ω, is placed in the water well with R_S. Resistance R_V has a coefficient of 3 ppm. With a constant output current, R_V is adjusted to maintain V_{fb} nearly constant as the water temperature is varied.

Ripple Reduction

The ripple after rectification is reduced by a combination of the L1-C2 filter, the R_C-R_L-C1 output filter, the inductance L_L in the magnet, and the gain in the loop at 60 Hz and 360 Hz. Theoretically, there should be no 60 Hz ripple, but it inevitably appears due to unbalances in the line voltage, the transformer, and the rectification diode voltage drops. A value of 1% rms is a reasonable value to assign.

The following table presents calculated and measured values of ripple and ripple reduction factors:

	360 Hz Factor (Calc)	360 Hz Meas (p-p)	60 Hz Factor (Calc)	60 Hz Meas (p-p)
1) Rectification	$\frac{1}{.057}$	18V	0	.5V
2) L1-C2 filter	60	.3V	1	.5V
3) R_C-R_L-C1 filter	2	-	2	-
4) Ripple at V_L	-	100mv	-	25mv
5) Item 2 x Item 3 or $\frac{\text{Item 1}}{\text{Item 4}}$	120	180	2	20
6) Loop gain= $\frac{\text{(5) meas.}}{\text{(5) calc.}}$	1.5		10	
7) Current ripple in magnet	$.1\times10^{-6}$pp		$.15\times10^{-6}$pp	

$$\frac{\text{Item 4}}{2\pi fL}\cdot\frac{1}{4000\text{A}}=\frac{\Delta I_L}{I_L}$$

Item 7 indicates that the ripple current is well within specifications.

Stability Tests

Both power supplies were tested with a water load at the manufacturer's facility by the manufacturer. One supply was additionally witness-tested. At 4000A, the stability as recorded on a strip chart over an 8-hour period was about 1×10^{-6}, well within specifications. At 500A the stability was 1.2×10^{-5}.

Tests at the accelerator facility with only the coils of the magnet as the load indicated a similar value of 1×10^{-6} at 4000A. The current was measured every half hour. A test at 1000A, using an NMR instrument for magnetic field measurement, showed a variation of 1×10^{-5}.

With such a large power unit, no easy way of controlling the multitude of variables was available. Hence, none were intentionally varied.

Conclusion

The foregoing analysis and tests indicate that a large current power supply can be constructed with a stability in the range of 1×10^{-5} to 1×10^{-6}. The most difficult component to develop is a temperature stable current sensor. No attempt has been made to analyze the dynamic characteristics of the feedback system, an even greater task than the dc analysis.

Acknowledgment

The author is deeply grateful to Ron Rumrill, the designer of the circuit and manager of the project at Alpha Scientific Company. Our many discussions on the practical aspects of the design, both circuit wise and physically, indicate that there is a great deal more to the construction of a power supply than a dc analysis. It was a pleasure working with Dr. S.B. Kowalski, a user of the spectrometer, and performance advisor in drawing the specifications.

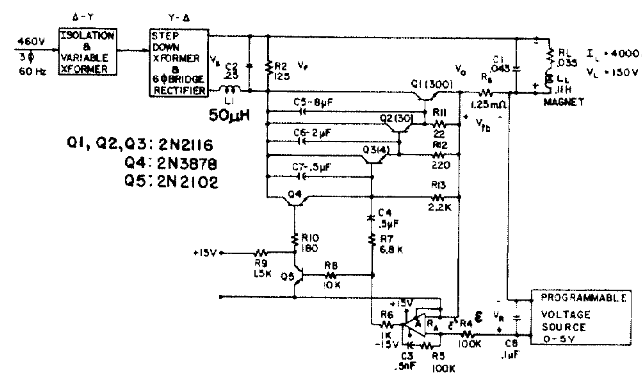

Fig. 1. Circuit Schematic Current Regulated Power Supply.

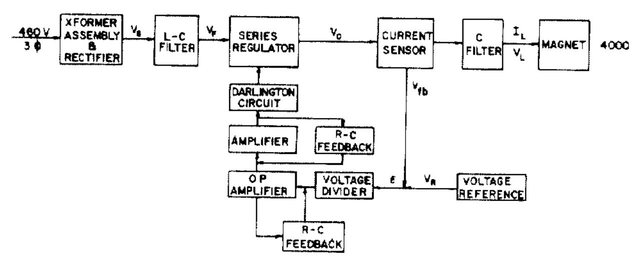

Fig. 2. Block Schematic.

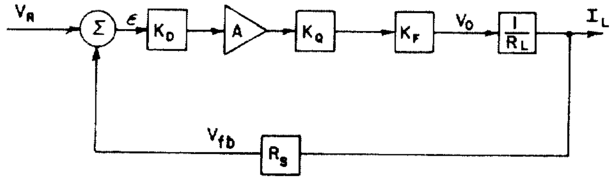

Fig. 3. Simplified Block Diagram Feedback System (dc).

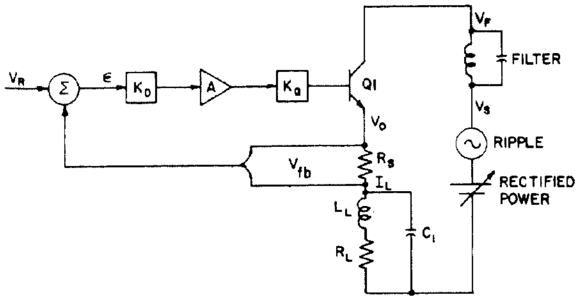

Fig. 4. Hybrid Block Diagram
Power Supply as an Emitter Follower.

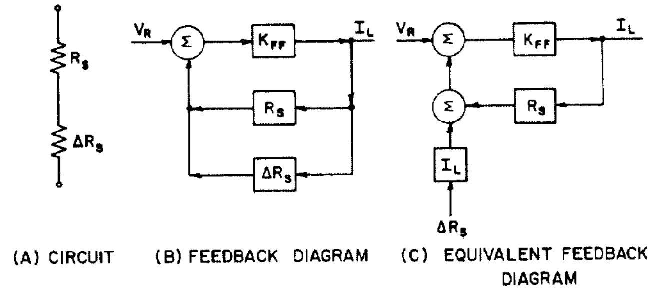

Fig. 6. Effect of Variation in Current Sensor.

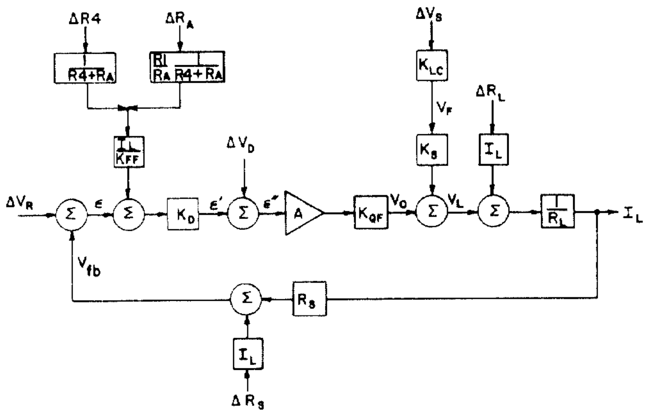

Fig. 5. Feedback System Distrubances on Output Current.

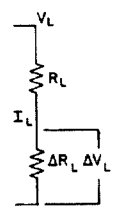

Fig. 7. Variation in Load.

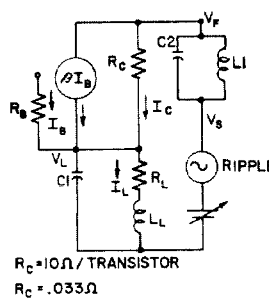

Fig. 8. Equivalent Circuit Power Section.

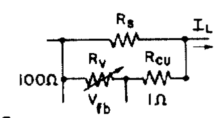

Fig. 9. Temperature Free Feedback Circuit.