IEEE Transactions on Nuclear Science, Vol.NS-22, No.3, June 1975

ON BEAM BLOWUP IN A RACETRACK MICROTRON

V. A. Volodin*, A. O. Hanson
University of Illinois, Urbana-Champaign, Illinois U.S.A.

Summary

This paper reports on a preliminary theoretical study of the interaction of electromagnetic fields in the accelerating structure of a racetrack microtron with the transverse motion of electrons circulating through the structure. Basic equations of the transverse beam dynamics and of buildup of the deflecting modes are derived with the help of idealized models of a microtron as well as of the beam-structure interaction process. The dependence of the starting current for beam blowup (BBU) on the parameters of the accelerator has been studied and some features of the Microtron Using a Superconducting Electron Linac (MUSL-2) of the University of Illinois[1] were taken into account to make numerical calculations.

It was shown that the starting current is extremely sensitive to the phase correlation between the returning beams and under some conditions can be much lower than that for a linac. The use of the thin magnetic lenses on each orbit helps to achieve higher values of the starting current.

Introduction

Beam blowup in normal accelerators as well as in superconducting sections have been analyzed in some detail[2-3]. Some of the features of the racetrack microtron, using a linac as an accelerating element, introduce problems associated with the phases and trajectories of the returning beams and by the presence of several beams in the accelerating structure at the same time. It is well known that in many cases BBU sets an upper limit for the beam intensity in the high-current linacs. Although one cannot expect high peak currents in the superconducting accelerator, the BBU effect is still very important because of the high Q-value and of the danger of damaging the superconducting structure.

Since the theoretical investigation of this problem is rather difficult for a linac (which is an essential element of a racetrack microtron) and since BBU has not been considered for a microtron, it seems reasonable to use as simple a model of beam-structure interaction as possible and to pay more attention to those features of the racetrack microtron that can introduce some special BBU properties.

Basic Assumptions and Equations

Consider a racetrack microtron consisting of the following main elements: a linac with the length ℓ, bending magnets with a separation L, and one thin focusing lens in the middle of the free space on each orbit.

If the system of cartesian coordinates is chosen in such a way that z-axis always coincides with the path of the electrons with z=0 at the linacs input, x - the vertical direction and y - the horizontal direction, one can have the following set of recurrence relations for ξ - the transverse beam displacement from the axis (ξ = x or y) and for P_ξ - the transverse momentum at the input of the linac after the n^{th} turn.

$$\begin{aligned}\xi_n(0) &= \left(1-\frac{L_{\xi n}}{2f_n}\right)[\xi_{n-1}(0)+\Delta\xi_{n-1}(\ell)] + \\ &\quad \frac{L_{\xi n}}{P_{zn}}\left(1-\frac{L_{\xi n}}{4f_n}\right)[P_{\xi n-1}(0)+\Delta P_{\xi n-1}(\ell)] \\ P_{\xi n}(0) &= -\frac{P_{zn}}{f_n}[\xi_{n-1}(0) + \Delta\xi_{n-1}(\ell)] + \\ &\quad \left(1-\frac{L_{\xi n}}{2f_n}\right)[P_{\xi n-1}(0) + \Delta P_{\xi n-1}(\ell)]\end{aligned} \tag{1}$$

where P_{zn} - longitudinal momentum; f_n - focal length of thin lens; $L_{\xi n} = \begin{cases} V_e T_n - \ell, & \xi = x \\ 2L - \ell, & \xi = y\end{cases}$; V_e - electron velocity; $T_n = \frac{2\pi}{\omega_o}[q+\Delta q(n-1) + \Delta q_1\delta_{n,1} + \Delta q_2\ \delta_{n,2}]$;

ω_o - driving frequency; q - basic harmonic number; Δq - change of harmonic number per turn; Δq_1, Δq_2 - change of harmonic number for bypasses on the first and second orbits; $\delta_{n,k} = \begin{cases}1, & n=k. \\ 0, & n\neq k\end{cases}$ The changes of the beam position $\Delta\xi$ and of transverse momentum ΔP_ξ within the accelerating structure ($0<z\leq\ell$) are due to the interaction of the beam with some parasitic deflecting modes.

Suppose there is an interaction of the electrons only with TM_{11} - mode in the structure. If, as assumed by P. Wilson[4], the linac is considered to be a series of weak coupled resonant cavities with a specified phase shift from one cavity to the next and if the cavities are assumed to be resonant in the TM_{110} - mode, one has the following expression for the components of the forward propagating fundamental space harmonic at a small distance from the axis.

$$\begin{aligned} E_Z &\simeq gE\frac{k\xi}{2}\ e^{ik_z Z} \\ H_\xi &\simeq -\frac{igE}{2}\ e^{ik_z Z}\end{aligned} \tag{2}$$

Here $k = \frac{\omega_{11}}{c}$, ω_{11}-TM_{11}-mode frequency; c - light velocity, $k_z = \omega_{11}/V_{ph}$, V_{ph} - phase velocity; i = $(-1)^{1/2}$; g - the Fourier coefficient relating the fundamental space harmonic to the peak field. The factor $e^{-i\omega_{11}t}$ is usually omitted and Gaussian units are used.

An interaction of the beam with such a field leads to a transverse displacement of the electrons and to a change of the transverse momentum as given by

$$\begin{aligned}\Delta\xi_n(z) &= \frac{P_{\xi n}(0)}{P_{zn}}\ z - \frac{eEg\ell}{2P_{zn}\alpha}e^{i\phi_n}\ [z-\frac{i\ell}{\alpha}(1-e^{i\alpha z/\ell})] \\ \Delta P_{\xi n}(z) &= -\ \frac{eEg\ell}{2c\alpha}\ e^{i\phi_n}(1-e^{i\alpha z/\ell})\end{aligned} \tag{3}$$

where $\alpha = k_z\ell(1-V_{ph}/V_e)$; $\phi_n = 2\pi\frac{\omega_{11}}{\omega_o}\ n(q+\Delta q_1\delta_{n,1} + \Delta q_2\delta_{n,2} + \Delta q\frac{n-1}{2})$; e - electron charge. The connection between the beam transverse motion and exitation of the TM_{11} - mode can be found by using the energy balance equation

*Visitor from the Moscow Engineering Physics Institute, U.S.S.R.

$$\frac{\partial U}{\partial t} + \frac{\omega_{11}}{Q}U = -\frac{1}{2}\,\mathrm{Re}\int_v \sum_{n=o}^{N} \vec{J}_n\,\vec{E}_n^{\star}\,dv \tag{4}$$

Here U - electromagnetic energy stored in the structure, $\vec{J}_n$ - vector of beam current density; v - volume of the cavity.

After introducing the shunt impedance $r_{sh} = \frac{E^2 \ell Q}{4\omega_{11}U}$ and using (1), (3) one can obtain the following equation for the buildup of the field in the accelerating structure of a racetrack microtron.

$$\frac{\partial E}{\partial t} + \frac{\omega_{11}}{2Q}E = \frac{eIg^2 k\omega_{11} r_{sh} E\ell^2}{2\alpha^3 QP_{zo}} G_{N\xi} \tag{5}$$

where the coherence function $G_{N\xi}$ is given by

$$G_{N\xi} = G_o\Big(1+\sum_{n=1}^{N}\Big(1+\frac{\Delta\gamma_n}{\gamma_o}\Big)^{-1}\Big(1+\frac{2c}{\alpha G_o eEg\ell}\,\mathrm{Im}$$

$$\Big(e^{-i\phi_n}\Big(\frac{P_{zn}}{\ell}\,\xi_n(0)(1-e^{-i\alpha}) - \tag{6}$$

$$P_{\xi n}(0)\ \Big(e^{-i\alpha} + i\frac{1-e^{-i\alpha}}{\alpha}\Big)\Big)\Big)\Big)\Big)$$

$G_o = 1/\alpha\ \Big(\frac{\sin\alpha/2}{\alpha/2}\Big)^2\ \Big(1-\frac{\alpha}{2}\ \mathrm{Ctg}\ \alpha/2\Big)$, I - output beam current; $\Delta\gamma/\gamma_o$ - relative energy gain per turn; N - number of orbits. The BBU starting current I_s corresponds to the condition that $\partial E/\partial t = 0$. Under the assumption that the initial injected beam has no transverse displacement and no transverse momentum the starting current is given by

$$I_s = \frac{2P_{zo}}{er_{sh}kg^2\ell^2 G_{N\xi}} \tag{7}$$

and the time dependence of the field amplitude follows as

$$E(t) = E(0)\ \exp\Big(\frac{er_{sh}\omega_{11}kg^2\ell^2 G_{N\xi}}{4QP_{zo}}(I-I_s)t\Big) \tag{8}$$

These results apply to a linac if N = 0. In this case $G_{N\xi} = G_o$ and the expression for the starting current corresponds to that of P. Wilson[4]. In the case of the racetrack microtron, the dependence of $G_{N\xi}$ on its basic parameters is not quite clear since it is necessary to find $\xi_n(0)$ and $P_{\xi n}(0)$ by solving (1) together with (3).

On the Possibilities of Analytical Solution

For the analytical analysis it is more convenient to have a differential equation rather than the recurrence relations (1). By introducing derivatives $\partial\xi_n/\partial_n = \xi_{n+1}-\xi_n$, $\partial^2\xi_n/\partial_n{}^2 = \xi_{n+1} - 2\xi_n+\xi_{n-1}$; $\partial P_{\xi n}/\partial_n = P_{\xi n+1} - P_{\xi n}$, $\partial^2 P_{\xi n}/\partial n^2 = P_{\xi n+1} - 2P_{\xi n} + P_{\xi n-1}$ one obtains for no focusing

$$\frac{\partial P_{\xi n}}{\partial n} = -\frac{eEg\ell}{2c\alpha}\ (1-e^{i\alpha})e^{i\phi_n}$$

$$\frac{\partial\xi_n}{\partial n} = \frac{1}{P_{zn}}\Big(\Big(1+\frac{L_{\xi n}}{\ell}\Big)P_{\xi n} + \frac{eEg\ell}{2c\alpha}\cdot \tag{9}$$

$$\Big(\Big(\frac{L_{\xi n}}{\ell}+\frac{i}{\alpha}\Big)\ (1-e^{i\alpha})-1\Big)\Big)$$

and with focusing

$$\frac{\partial^2 P_{\xi n}}{\partial n^2} + 2\beta_n\frac{\partial P_{\xi n}}{\partial n} + \Omega_{on}^2 P_{\xi n} = \sigma_n e^{i\phi_n}$$

$$\xi_n = -\frac{f_n}{P_{zn}}\Big(\frac{\partial P_{\xi n}}{\partial n} + \frac{L_{\xi n}+2\ell}{f_n}P_{\xi n} + \tag{10}$$

$$\frac{eEg\ell}{2\alpha}e^{i\phi_n}\Big(\Big(1-\frac{L_{\xi n}}{2f_n}\Big)\ (1-e^{i\alpha}) +$$

$$\frac{i\ell}{\alpha f_n}\ (1-e^{i\alpha}+i\alpha)\Big)\Big)$$

$$2\beta_n = \frac{L_{\xi n}+\ell}{f_n} - \frac{\partial}{\partial n}\ln\Big(\frac{P_{zn}}{f_n}\Big)$$

$$\Omega_{on}^2 = \frac{L_{\xi n}+\ell}{f_n} + \frac{1}{2}\frac{\partial}{\partial n}\Big(\frac{L_{\xi n}+2\ell}{f_n}\Big) - \frac{L_{\xi n}+2\ell}{2P_{zn}}\ \frac{\partial}{\partial n}\Big(\frac{P_{zn}}{f_n}\Big)$$

$$\sigma_n = -\frac{eEg\ell}{2c\alpha}\Big((1-e^{i\alpha})\ \Big(\Big(1-\frac{L_{\xi n}}{2f_n}\Big)\ \Big(i\frac{\partial\phi_n}{\partial n} -$$

$$\frac{\partial}{\partial n}\ln\frac{P_{zn}}{f_n}\Big) - \frac{L_{\xi n}}{2f_n} - \frac{\partial}{\partial n}\Big(\frac{L_{\xi n}}{2f_n}\Big)\Big)\cdot + \frac{i\ell}{f_n\alpha}$$

$$(1-e^{i\alpha}+i\alpha)\ \Big(1-\frac{\partial}{\partial n}\ln P_{zn} + i\frac{\partial\phi_n}{\partial n}\Big)\Big)$$

Solutions of (9) and (10) can be found for some particular cases. For example in the absence of focusing and for $\phi_n = 2\pi m$ $(m=0,1,2,\ldots)$ one has the same function G_N for vertical and horizontal directions

$$G_N = G_o\ \Big(1+s\Big(1+(n-1)\ \frac{\alpha}{2}\ \mathrm{ctg}\frac{\alpha}{2}\Big)\ \ln\Big(1+\frac{N}{s}\Big)\Big)$$

$$s = (q - 2L\omega_o/2\pi c - \Delta q)/\Delta q \tag{11}$$

The presence of focusing lenses makes the solution of (10) possible only with the help of some asymptotic method such as a WKB approximation. As the result expressions for functions $G_{N\xi}$ are usually so complicated that the only possibility to analyze them is by using numerical methods. It is therefore more practical to use the recurrence relations (1) rather than equations (10).

Some Results of Numerical Analysis

A computer program was written to investigate the dependence of the starting current on the basic parameters of the microtron. Recurrence formulas (1) and relations (3) were used. The basic parameters were chosen to be representative of the arrangement in MUSL-2. It was found that the starting current corresponding to the horizontal direction practically always exceeded that for the vertical direction. One could expect such a result since always $L_x > L_y$. So it is sufficient to analyze further only the results for the vertical direction and subscript "x" is omitted.

The calculations showed that under some conditions G_N as a function of the number of orbits N can grow as fast as N^2. This can be a very serious limit for the beam current accelerated in the racetrack microtron. For example the starting current for the HEPL superconducting section[5] is about 500 μA and can be N^2 times less for the Illinois MUSL-2, where the number of orbits may be as large as 20. Fortunately it is possible to reduce the coherence of the recirculated beams in the microtron by introducing focusing on the return paths. The results of some computations

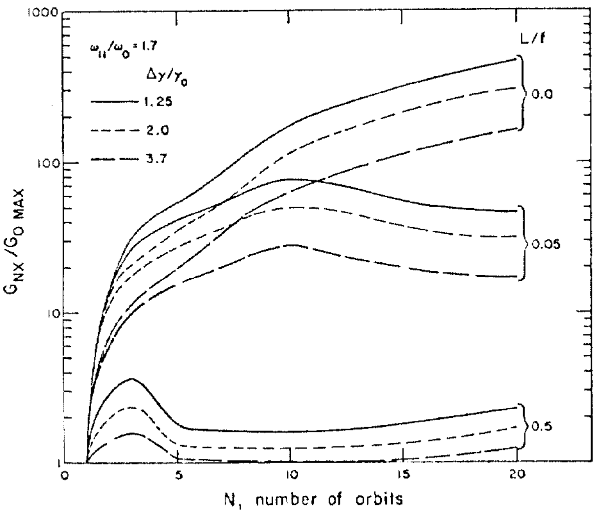

Fig. 1. Dependence of the coherence function on the number of orbits, focusing and energy gain per turn ($L=52\lambda_0$, $\ell=24.5\lambda_0$, $q=110$, $\Delta q=2$, $\Delta q_1=4$, $\Delta q_2=4$, $\omega_{11}/\omega_0=1.7$).

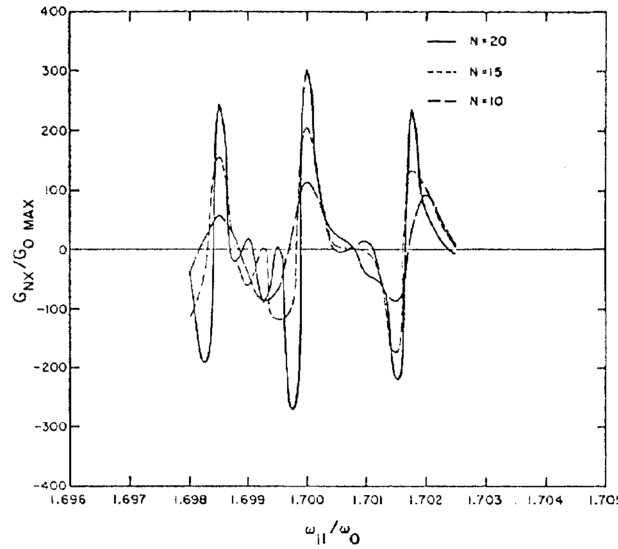

Fig. 2. Coherence function for small changes in the frequency of the deflecting mode ($L=52\lambda_0$, $\ell=24.5\lambda_0$, $q=110$, $\Delta q=2$, $\Delta q_1=4$, $\Delta q_2=4$, $\Delta\gamma/\gamma_0=2.$)

are shown in Fig. 1 where the coherence function G_N is normalized to the maximum value of G_0. This ratio represents the factor by which the starting current is reduced as compared to the linac. It can be seen that focusing reduces the coherence and can make the starting current independent of the number of orbits and can be close to that of the linac. It also shows quite naturally that an increase in the energy gain corresponds to a larger starting current.

Another point of interest is the extreme sensitivity of the coherence function to the phase correlations between the returning beams. Since the basic harmonic number "q" is large (of the order 100) and the value of the return phase ϕ_n increases rapidly from orbit to orbit, even very small deviations of the deflecting mode frequency cause significant changes in the value of the starting current. As can be seen in Fig. 2 the coherence function G_N has an oscillatory character and can have negative values that correspond to the absorption of energy from the deflecting mode by the beam. In principle this fact can be used to suppress BBU by special adjustment of the phases of the returning beams, but the phasing is specified for the acceleration of the particles and the possibilities to change it are rather limited. Since there are many frequencies within TM_{11} band (55 for MUSL-2) that can be excited by the beam it may be practically impossible to reduce the phase coherence for all the frequencies.

Conclusion

The above results illustrate some features of BBU in a racetrack microtron. One can see that recirculation of the beams several times through the same accelerating structure is not quite equivalent to the acceleration of the electrons in a multisection linac even though the output energy and the beam current may be the same. The main difference is the consequence of the fact that in a racetrack microtron the path length of the electrons on each orbit is an integral number of the accelerating mode wavelength λ_0. This is always different from the wavelength of the deflecting mode and their ratio could be an integer only by chance. This fact leads to a rather complicated picture of beam-structure interaction and causes the extreme sensitivity of the starting current to small changes of the parameters of the system which affect the phasing of the returning beams.

At the same time the general arrangement of a racetrack microtron leaves many possibilities for focusing the beams and this can be very effective in increasing the BBU starting current.

In conclusion we should emphasize that these results are preliminary and qualitative and more detailed theoretical and experimental investigation of this problem is necessary.

References

1. P. Axel, A. O. Hanson, J. R. Harlan, R. A. Hoffswell, D. Jamnik, D. C. Sutton and L. M. Young. "Microtron Using a Superconducting Electron Linac." Contributed to this conference.

2. R. H. Helm and G. A. Loew, "Beam Breakup", in Linear Accelerators (P. M. Lapostolle and A. L. Septier, editors) North Holland Publ. Co. (1970). pp. 173-222.

3. P. B. Wilson, "Theory and Design of Superconducting Electron Linear Accelerators", ibid, pp. 1107-1140.

4. P. B. Wilson, "A Study of Beam Blow-up in Electron Linacs", Stanford University, HEPL-297, June, 1963.

5. K. Mittag, M. A. Schwettman and M. D. Schwartz, IEEE Trans. Nucl. Sci., NS-20, No. 3, 86 (1973).