(J-4*)

COO-1233-44

RELATED TECHNIQUES IN THERMONUCLEAR RESEARCH*

D. W. Kerst
University of Wisconsin, Madison, Wisconsin

SUMMARY

Among the aspects of thermonuclear plasma research and accelerator technology which have common techniques and problems is the problem of injection which in the case of accelerators is guided by limitations imposed by Liouville theorem. This limitation is being more and more fully appreciated in the problem of assembling a thermonuclear plasma. There are various examples of how this limitation on injection comes into play in the case of some plasma devices.

There is an interesting analogy between the topology of phase space for particle accelerators and the topology of coordinate space for lines of force in a plasma containment device such as a toroidal stellarator. There are integral, 1/2 integral, 1/3 integral, and 1/4 integral, etc. resonances which can be stimulated by magnetic field errors in plasma confining devices and which would cause the flux plot topology to break up into rings of islands for a case where the flux plot has shear. The size of the islands depends upon the type of the multipole field forming the flux plot. Although stellarators are made with twisted or helical multipole windings wound around the major circumferential, they can also be made by sectored multipole magnets rotated between focusing and de-focusing positions as in an accelerator. For the quadrupole case, the transfer matrices include only exponentials with real expotents and no sines and cosines such as are found in the accelerator analogy. K. R. Symon has shown that the problem of trajectories of lines of force can be put in Hamiltonian form.

RELATED TECHNIQUES

Many of the techniques in the study of plasmas for controlled thermonuclear reactions have a clear association with the problems of accelerators. Two of these types of problems are the formation of magnetic fields and the injection of particles into these magnetic fields. Other similarities of problems can be less evident and sometimes surprising -- for example, problems of instabilities in beams and in plasmas, and the formal analogy between the topology of phase trajectories in circular accelerators and the topology of the traces of magnetic lines of force in toroidal plasma confining systems such as stellarators. Some of these topics of mutual interest to both fields will be discussed.

The most hopeful development in the formation of magnetic fields is the superconducting coil. For accelerators it is used for space saving, power saving, and for the high fields available with hard superconductors. To contain a realistic thermonuclear reaction by magnetic pressure it is essential to have very high magnetic fields -- above the value easily available with iron magnets. Again power is an immense problem in these coils.

There is, however, immediate application for superconducting coils for our present experiments with plasmas. This is a result of the fact that for the first time we have been able to produce in a toroidal device, a quiet, hardly-fluctuating plasma, apparently not lost quickly by anomalous diffusion.[1,2] This containment system, technically termed a toroidal multipole, has hoops suspended within the plasma for its field-producing conductors. (Figure 1.)

In present experiments these hoops are suspended by their supports which gradually anihilate the plasma; but a superconducting hoop with its persistent magnetic field would need no external connections and in a properly shaped field it would be self-supporting against gravity. The persistent nature of the field will allow plasma injection and plasma building experiments of the type that require some time.

---

* Work supported by U. S. Atomic Energy Commission

There has not been much superconducting effort in controlled thermonuclear work yet, but the pioneer is Richard Post who has been studying the use of cryogenic coils for plasma confinement for several years[3] and who has used a small superconducting quadrupole coil shaped like a baseball seam and made at the Argonne National Laboratory.

The creation of a plasma within a magnetic field has been accomplished in the earliest plasma experiments by means of a discharge. More controllable means were soon used, particularly at Oak Ridge, which made use of plasma building by injection of particles. The problem is old to accelerator builders for whom Liouville's theorem is gospel, but it has not always been the guiding principle for plasma accumulating experiments. One of the difficulties for plasma creation by injection is that for stability the velocity distribution should be as close to isotropic in the center as possible and there should be an energy spread similar to the Maxwellian distribution. This is quite the opposite of the requirement for accelerators. Beams circulating inside plasmas and vacant angles in the velocity distribution of particles both tend to cause plasma instabilities. Consequently for injection of plasmas phase space should be scanned. Instead of having a small phase emittance from the injector, a large emittance would assist in rapid scanning but would sacrifice particle density in phase space.

By injecting a spray from an injector which deposits particles in the middle of a confinement region and which is withdrawn steadily as the particles begin to return to the injector vicinity, a volume can be filled to something like a percent of the Liouville limit. Before the volume is filled, there would be an incomplete velocity distribution and it may be that instabilities will have ejected many of the particles because the isotropic state was never achieved.

Thus we do not have as much possibility of control of the injection process by beams as we need. It is interesting that if the plasma is all instantaneously shot into a toroidal multipole as a cloud, it undergoes a turbulent evolution and forgets its specialized directed distribution becoming a well behaved isotropic plasma with about 10% of the ions remaining trapped. Thus the instabilities acting in this evolution seem to accomplish exactly what is difficult to do by orderly scanning of phase space.

An injection system that seems to escape the limitations of Liouville's theorem is that of changing the charge state of a beam of ions or of neutral atoms within the confinement region. For example, if a neutral atomic beam in a high state of quantum excitation, say n = 8, passes through a strong magnetic field, the Lorentz forces on the charges in the atom split it into a trapped ion and an electron when it encounters the magnetic field. As yet neutral beam injection has not built high density plasmas, although the equivalent of .05 amperes of neutral atoms can be made by passing an ion beam through a magnesium vapor cell for subsequent injection. We can ask how Liouville's theorem might apply to neutral beam injection. If we say that each atom brings in a volume $n\hbar^3$ of phase space, then we find that in the Liouville limit on density would indeed be so high that it is far higher than that needed for thermonuclear reactions. Although the charged particles are born within the magnetic field and hence could create very high density, to avoid instability direction and energy scanning is needed.

Within the last year there has suddenly arisen a great concern about the containment of the magnetic field lines themselves in toroidal stellarator-like plasma containing devices. Since the plasma particles tend to follow lines of force we want to be sure these lines remain within the confinement region. For answers to this problem we can turn directly to the technology of sectored strong focusing accelerators, which treats with the effect of errors and of sectors in driving integral, half integral, third integral, etc. resonances in particle beams in the presence of non-linearities in focusing forces. These techniques, used to test stability limits by computer, can be taken over for stellarator magnetic fields. The pearls or islands at fractional resonances in phase space are found as islands or isolated bundles of flux lines in the coordinate space of a toroidal confining field.[4] The stellarator field is a multipole field transverse to the circumference of a toroidal vacuum tube. In addition, the main field of the stellarator is a solenoidal or circumferential called B_z. The multipole conductors are twisted so that their field pattern rotates about the central circumferential line as a line of force progresses around the tube. Thus the multipole field is periodic with a sector length equal to the distance for the field pattern to repeat. It is also possible to have nontwisted multipoles which are segmented so that successive segments are progressively rotated within a magnet period having the sector length for field pattern repetition just as they do when applied along an accelerator's circumference.

Symon has shown that the problem of trajectories of lines of force can be put into Hamiltonian form,[4] and thus that there is a formal analogy between the topology of phase space for particles in an accelerator and the topology of coordinate space for lines of force in a stellarator.

When quadrupole fields (L = 2) are used on a stellarator the lines of force circulate around in the minor cross section with the same pitch for all distances (amplitudes) from the center (equilibrium orbit) in the minor cross section. What is called the "transform" angle, iota, is thus constant for all lines. If sextupole fields, (L = 3) are used, the pitch, or the iota, changes for lines of different distance from the center. This is called a magnetic field with shear - - it is equivalent to the tune of a betatron oscillation changing with amplitude due to non-linearities of focusing forces.

An example of what a transverse field error does to fields in an L = 3 stellarator is shown in Figure 2. There are seven pearls or islands representing a tube of flux which circulates seven times around the stellarator and then rejoins itself. This isolated tube of flux cannot support a plasma pressure gradient across itself. Such a topology has actually been found by computation to exist for a real stellarator with certain local field trimming. Figure 3 shows a case found in the Wisconsin toroidal octupole with an added circumferential B_z field which causes a stellarator-like helical field pattern. A permanent magnet is brought up close for a field perturbation. The plasma pressure has a plateau across the isolated tube of flux, and the tube is found to assume a potential different from the surrounding plasma. This tube can be followed as it spirals around in the toroid. Figure 4 shows a computer test of an L = 3 stellarator with a local octupole bump. Here we see a one-fourth integral resonance with islands small enough that the lines of force do not leave the aperture.

It is interesting that for the quadrupole (L = 2) with straight multipole segments within each sector, the linear differential equation solved by transfer matrices gives all solutions of growing or decaying exponentials as the matrix elements -- there are no sines or cosines. Nevertheless, "focusing" or containment of lines of force is possible up to the essential resonance at which a half of a period for the line of force occurs in a whole sector. This is the same as the essential limit of the strong focusing accelerator equations. It is necessary to have at least three segments of different orientation per sector to achieve confinement.

FOOTNOTES:

---

* Work supported by the U. S. Atomic Energy Commission.

[1] D. W. Kerst, R. A. Dory, W. E. Wilson, D. M. Meade and C. W. Erickson, Phys. Rev. Letters 15, 396 (1965).

[2] R. A. Dory, D. W. Kerst, D. M. Meade, W. E. Wilson and C. W. Erickson, Phys. Fluids 9. 997 (1966).

[3] R. F. Post, Nucl. Fusion Part 1, 99 (1962).

[4] D. W. Kerst, Jour. Nucl. Energy C 4, 253 (1963).

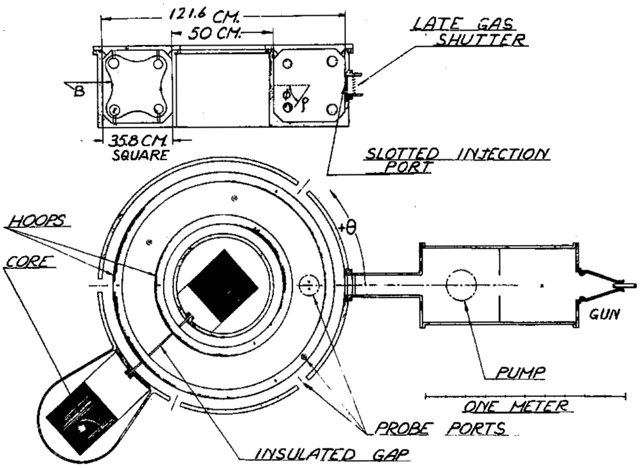

Fig. 1. The toroidal aluminum box wall forms a return conductor for the oppositely-directed currents in the four hoops forming an octupole magnetic field with a deep magnetic well for plasma trapping. These hoops are each supported by three thin berillium copper supports passing through the plasma which surrounds the hoops.

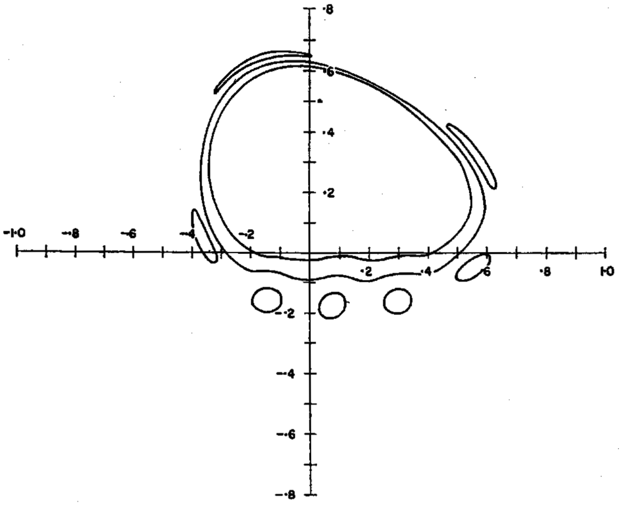

Fig. 2. The transverse field error applied to a 6-pole stellarator causes a separated flux tube which circulates 7 times around the stellarator and connects to itself. A similarity to phase space maps with quadratic nonlinearities is evident.

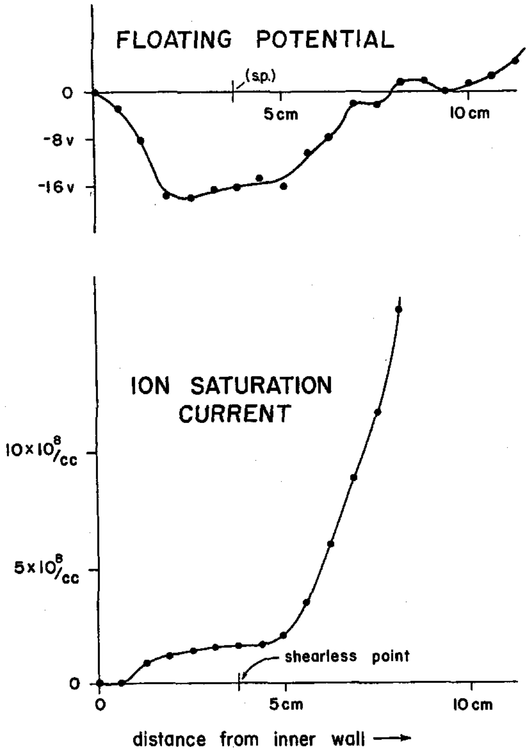

Fig. 3. Particle density in a toroidal octupole to which has been added a circumferential magnetic field causing the spiraling of lines of force. Where the line of force spirals 11 times around the minor axis and then connects with itself, in accelerator language, $\nu = 11$, an isolated flux tube appears with a plateau and particle density showing no pressure gradient across the tube of flux and a potential signature appears which identifies the tube of flux.

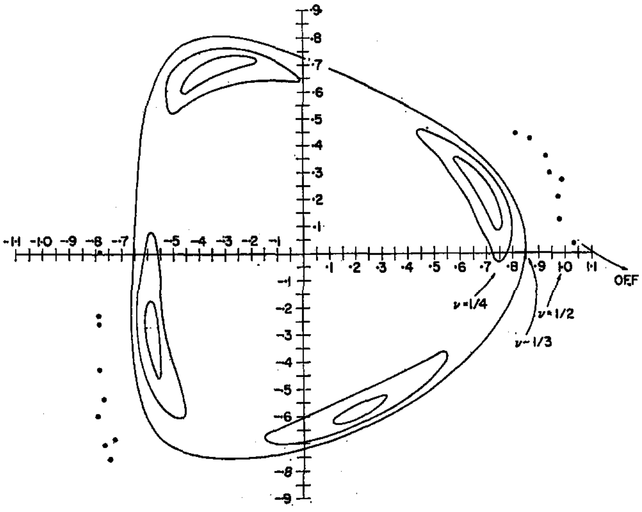

Fig. 4. Figure 4 shows an octupole error applied to a 6-pole stellarator forming islands at the location of the 1/4 integral resonance. The error destroys confinement of lines of force outside the 1/2 integral resonances.