SELECTION OF INJECTOR SYNCHROTRON PARAMETERS TO MINIMIZE COST OF THE 200-BeV ACCELERATOR*

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Summary

In choosing the type of injector system and in optimizing the parameters of any given type of system, we must consider relative cost advantages as well as relative technical advantages eventually entering into the arguments. In this paper we shall discuss a method of determining the cost of accelerator systems and give some examples that arose in the selection of an injector system for the proposed 200-GeV accelerator.[1]

The cost of the injector system cannot be considered apart from the total accelerator cost because the aperture of the main ring depends upon the energy and emittance of the beam transferred from the injector. Since the main ring is relatively much larger and more expensive than the injector system, a change which lowers injector cost can cause a much larger increase in main ring cost.

Calculation of Parameters and Costs

The linac cost is estimated to vary linearly with energy, with a greater cost slope used for energy exceeding 200 MeV. The cost of each synchrotron subsystem is found in terms of accelerator parameters. Insofar as possible, the costs refer to subsystems which have been optimized. The subsystems, together with the accelerator parameters upon which they principally depend, are:

- Magnet - particle momentum, magnetic field strength, aperture.
- Magnet power supply - magnet stored energy, rise time, and the type of supply, whether resonant or alternator-flywheel.
- RF system - particle intensity, injection and extraction energy, momentum spread, average radius, rise time, harmonic number.
- Vacuum system - aperture and average radius.
- Enclosure - average radius.
- Plant and utilities - average radius, power consumption of subsystems.

The costs of accelerator components include 25% for contingencies. Conventional construction such as the enclosure includes 12.5% for architect and engineering services and administration, with 15% added for contingencies. The costs of experimental facilities and of support facilities not directly related to operation were neglected.

The costs quoted in this report are intended for comparative studies and optimization only.

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Although every effort has been made to make the calculations realistic, the primary emphasis has been to make the cost figures internally consistent. Absolute costs will depend upon a number of factors which vary with site and with market conditions at the time of construction. However, the relative accuracy is felt to be adequate for judging significant cost differentials between accelerator systems.

Aperture

In a circular machine the magnet dimensions and stored energy are determined by the beam energy, the magnetic field, and the aperture requirements. The aperture is determined by maximum beam size plus allowances for closed-orbit errors, sagitta, and insurance. The maximum aperture requirements usually occur at injection. If (for the moment) we suppose that space-charge forces can be neglected, the beam size will depend upon the emittance of the preceding accelerator as well as upon the loss of particles and the dilution of phase space that occur in extraction, transfer, and injection processes. The vertical admittance of the booster ring can be written as[2]

$$A_z = \frac{b^2 \pi \nu_z}{F_z R} . \tag{1}$$

F_z is a form factor which depends upon the magnet lattice. See Table I for meaning of other symbols. The subscripts r and z refer to radial and axial (or vertical) directions respectively. Setting the admittance equal to the diluted linac emittance $E_z S_z n_z$, where S_z is the average dilution occurring in the transfer of n_z turns and solving for b, we get

$$b = \left(\frac{F_z R E_z S_z n_z}{\pi \nu_z} \right)^{\frac{1}{2}} . \tag{2}$$

The radial betatron amplitude is given by a similar equation. In most cases a magnet lattice has not been worked out for the given parameters so that ν_z is not known, but a value can be estimated satisfactorily by scaling from known structures.

The total number of injected turns is given by $n = n_r n_z$. Values used for n_r and n_z depend upon the method of injection assumed. For radial multiturn injection, n_z equals 1, and n_r is an independent variable. If n is a very large number, both radial and vertical multiturn injection must be used.

The emittance-determined beam size can be so small for the number of particles contained in the beam that space-charge forces would cause a betatron frequency change, leading to beam instability. The transverse incoherent space-charge limit is computed by means of Laslett's formula which includes image forces[3]; this formula gives a minimum average beam size b_{sc} which will be stable. If this size is larger than the emittance-determined beam size, some dilution must be introduced so that the beam will be stable. If we suppose that only the vertical emittance is increased (as this causes less dilution), we have $b = \sqrt{F_z}\, b_{sc}$ as the new vertical beam size, which now no longer depends upon linac emittance.

In calculating the main ring beam size at injection, we have employed similar considerations. The emittance of the beam from the booster then includes all of the dilution that has occurred both at injection into the booster and at extraction and transfer to the main ring. The emittance has also been reduced by the adiabatic damping occurring during the booster accelerator cycle.

The Fast Cycling Injector

The fast booster requires several cycles to fill the main ring while the main ring guide field is held constant. Important injector parameters are T_L, T_B, n_c, and t_1. Radial multiturn injection from linac to booster is assumed with a linac current of 50 mA. Single-turn, fast extraction from the booster is assumed. The ratio of main ring to booster circumference is equal to n_c.

Figure 1 shows curves of accelerator costs as a function of T_B. A different booster cost curve is obtained for each value of n_c. For $n_c = 15$, the booster aperture is determined by linac emittance, whereas, for lower n_c values it is space-charged determined. For low values of T_B the main ring aperture is determined by space charge, making both its costs and the total cost strongly decreasing functions of T_B. For higher booster energies, at which the main ring aperture is emittance determined, the main ring cost decreases more slowly with increasing booster energy, the decrease being due to greater damping of beam emittance in the booster. Under these same conditions, the booster cost increases with T_B, and the resulting total cost curve is nearly flat for a given n_c. The penalty for increasing T_B is that a decreasing amount of magnet-free circumference becomes available. When this amount becomes insufficient to allow adequate space for injection, ejection, RF cavities, and special-purpose magnets, n_c must be changed to a lower value. On the booster cost curves a short vertical bar indicates the point at which average radius is double the magnetic radius.

Cost versus linac energy is shown in Fig. 2 for $n_c = 10$, with $T_B = 6.6$ BeV. The total cost has a minimum near the point at which the injected linac beam size exactly satisfies the space-charge requirement for the booster at $T_L = 195$ MeV. Below this, dilution occurring at injection into the booster causes both booster and main ring cost to rise more steeply. Above the minimum, the increase in total cost with T_L is due to linac cost.

In Fig. 3 variation in filling time is considered, again for $n_c = 10$. The overall main ring cycle is held constant, so that as t_1 is increased, with a resulting slower repetition rate to the booster, the main ring rise and fall times must be shortened. Thus an increase in t_1 results primarily in decreased booster RF costs and in increased main ring RF and magnet-power supply costs. This gives a minimum near $t_1 = 0.4$ sec.

The Slow Cycling Injector

The injector synchrotron beam contains all of the charge required for one main ring pulse and injects it in a few microseconds. Thus no appreciable injection time need be allowed during the main ring cycle and the main ring cycle time is shortened from 2.6 to 2.25 sec. The main ring charge per pulse has been reduced by 13% so that the slow and fast booster cases will have the same time-averaged particle intensity.

The important parameters are linac and booster energy and the methods of injection and extraction used for the booster. Since with a total charge per pulse of 2.9×10^{13} and a linac current of 50 mA, the number of turns is very large, multiturn injection both radially and vertically must be employed. This method of injection permits some optimization of the beam shape. Let α_B be the ratio of beam width (including synchrotron oscillation amplitude) to beam height in the booster. At extraction from the booster, the circulating beam must be peeled off by some multiturn process. Again, for purposes of optimization we can suppose that the ratio of beam width to height in the main ring (α_{MR}) is a variable. Figure 4 shows accelerator costs as a function of these two variables, and for $T_1 = 200$ MeV, $T_B = 7.4$ GeV. It is seen that $\alpha_B = 1$ is strongly favored, as is $\alpha_{MR} > 3$. (The calculations did not include $\alpha_B < 1$, which does not seem to be interesting from a practical point of view.) To understand this result, consider first the case for which α_B = const. Varying α_{MR} does not change the booster emittance; consequently the product ab in the main ring is a constant. As the cost of an increment in beam height is about four times as expensive as an equal increment of beam width, a > b is favored; this condition corresponds to α_{MR} large. The main ring aperture is emittance-determined, and space-charge effects do not enter.

When α_B is varied the situation is different. First let us suppose that the booster aperture is space-charge determined, which is true for the machines of Fig. 4. Then we find that $b_{sc} \approx (\text{const})/\sqrt{1+\alpha_B}$. On the other hand, if F_p is neglected, the emittance-determined beam size can be expressed as $b_E = (\text{const})/\sqrt{\alpha_B}$. The increase in beam size because of space-charge dilution is given by

$$\frac{b_{sc}}{b_E} = (\text{const})\left(\frac{\alpha_B}{1+\alpha_B}\right)^{\frac{1}{2}}. \qquad (3)$$

The function $[\alpha/(1+\alpha)]^{\frac{1}{2}}$ equals $1/\sqrt{2}$ for $\alpha = 1$, approaching 1 as $\alpha \to \infty$. Thus in the domain of space-charge-determined aperture, $\alpha_B > 1$ leads to more dilution, which causes increased costs both in the booster and in the main ring. If the booster aperture is emittance determined, there is no dilution and the cost becomes nearly independent of α_B. This can happen if the main ring intensity per pulse is substantially reduced.

Figure 5 shows the variation of costs with T_B, with $\alpha_B = 1.2$ and $\alpha_{MR} = 4.7$. For low values of T_B the main ring aperture is space-charged determined, and its cost is a strong decreasing function of T_B. Above $T_B = 4.35$ GeV, the main ring is no longer space-charged determined and its cost decreases with T_B at a lower rate. The booster cost rises almost linearly with T_B. The result is a minimum in the total cost curve at $T_B = 7.4$ GeV. The minimum is very weak. For an increased total cost of less than 3%, T_B can be selected anywhere from 5 to 12 GeV.

Now consider the effect of varying T_L (Fig. 6), with $T_B = 7.4$ GeV. For $T_L < 200$ MeV the booster aperture is space-charge determined, causing dilution, so that both booster and main ring cost decrease with T_L. Above $T_L = 200$ MeV, the booster aperture is determined by linac emittance, and the main ring aperture increases slightly with T_L, because at injection into the booster, an increase in injection particle velocity β requires an increase either in the number of injected turns or in linac beam current to achieve the same beam current in the booster. This causes the main ring beam size to be proportional to $\beta^{1/4}$. Linac cost has little effect in determining the position of the minimum. Above $T_L = 200$ MeV, the sum of booster and main ring costs is essentially constant; thus, since the linac cost is certain to increase with T_L, the minimum cannot be above this point. Below $T_L = 200$ MeV, the sum of booster and main ring costs is a steeper function than the linac cost. The linac cost per MeV would have to be about 50% greater in order to shift the minimum to lower T_L.

Conclusions

The minimum total cost for either of the two cases results when $T_L \approx 200$ MeV and $T_B \approx 7$ GeV. See Table I for a list of parameters. The slow booster scheme is about 2.6% greater in total cost. Allowing for uncertainties in the figures, this difference is not great enough to determine the choice of injector. Note that the slow booster is cheaper than the fast booster, but when the slow booster is used as an injector, the main ring cost is considerably larger due to an increased beam size caused by dilution in the slow booster. Technical arguments seem to favor the fast booster --principally because of the advantage of a smaller beam size in the main ring and the fact that injection and extraction with the fast booster involve only a modest extension of present techniques. With the slow booster both injection and extraction would involve considerable development, and at best it seems impossible to produce a main ring beam that is uniform in both emittance and intensity.

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References

1. 200 BeV Accelerator Design Study, Lawrence Radiation Laboratory Report, UCRL-16000, June 1965.
2. E. D. Courant and H. S. Snyder, Ann. Phys. 3, p. 12, (1958).
3. L. J. Laslett, 1963 Brookhaven Summer Study, BNL-7534, p. 324.

Table I. Parameters near minimum total costs.

Injector parameters		Fast booster	Slow booster
n_c	Pulses per main ring pulse	10	1
T	Kinetic energy (GeV)	6.6	7.4
N	Protons per pulse	3×10^{12}	2.9×10^{13}
t_R	Magnetic field risetime (sec)	0.019	0.4
B	Field at maximum energy (kG)	7.0	12.0
R	Average Radius (m)	69.03	41.45
q	Ratio of R to magnetic radius	1.94	1.80
ν	Betatron oscillation frequency	≈ 7.25	≈ 4.25
a	Horizontal betatron amplitude at injection (cm)	3.6	5.2
b	Vertical betatron amplitude at injection (cm)	1.5	5.2
r_p	Synchrotron amplitude at injection (cm)	0.5	0.8
w	Vacuum chamber half-width (cm)	6.6	8.5
h	Vacuum chamber half-height (cm)	3.3	5.2
g	Half-height of magnet gap (cm)	4.4	5.8
T_L	Linac kinetic energy (MeV)	195	200
I_L	Linac current (mA)	50	50

Injector parameters (cont'd)		Fast booster	Slow booster
E	Linac emittance (mR - cm)	3.0	3.0
n	Total number of injected turns into booster	3.7	60.8
n_r	Radial number of injected turns into booster	3.7	7.8

Main ring parameters		Fast booster	Slow booster
T	Kinetic energy (GeV)	200	200
N	Protons per pulse	3×10^{13}	2.6×10^{-3}
B	Field at maximum energy (kG)	15.07	15.07
R	Average radius (m)	690.25	690.25
ν	Betatron oscillation frequency	16.75	16.75
t_c	Cycle time (with flattop) (sec)	2.60	2.25
	Cycle time (without flattop) (sec)	2.0	1.65
t_R	Magnetic field risetime (sec)	0.79	0.80
t_i	Filling time (sec)	0.35	0.0
a	Horizontal betatron amplitude at injection (cm)	1.8	3.3
b	Vertical betatron amplitude at injection (cm)	0.7	0.8
r_p	Synchrotron amplitude at injection (cm)	0.5	0.5
w	Vacuum chamber half-width (cm)	5.7	7.2
h	Vacuum chamber half-height (cm)	2.6	2.7
g	Half-height of magnet gap (cm)	3.2	3.3

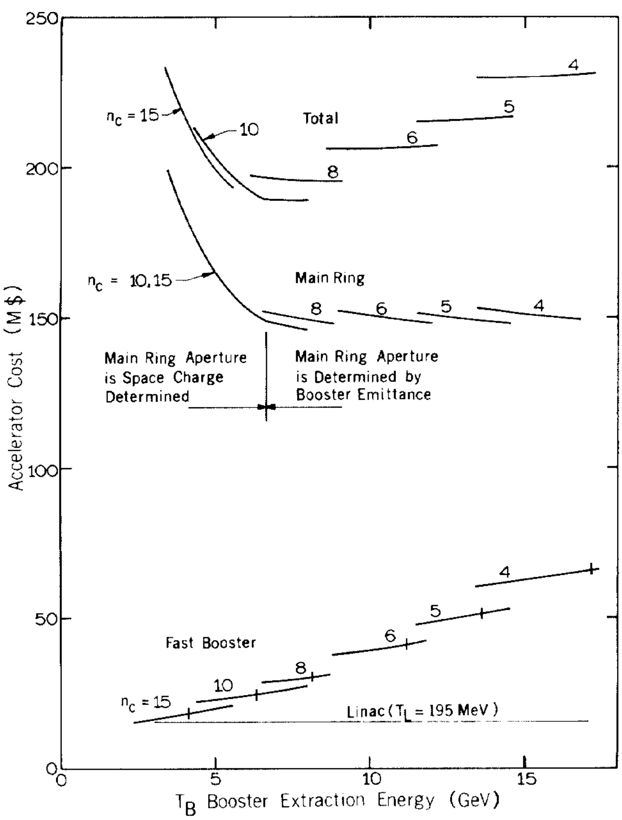

Fig. 1. Cost curves for a 200 GeV main ring with a fast cycling injector, as a function of booster energy.

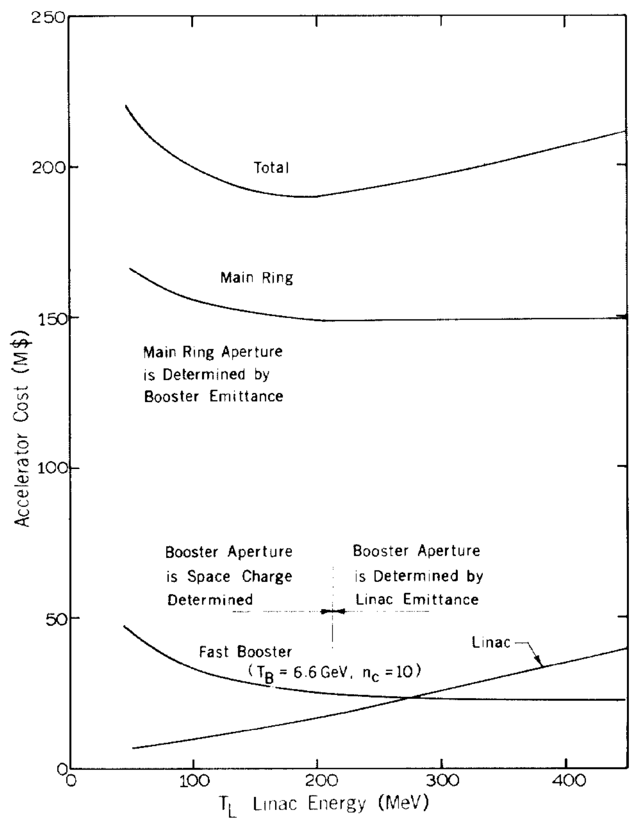

Fig. 2. Cost curves for a 200 GeV main ring with a fast cycling injector, as a function of linac energy.

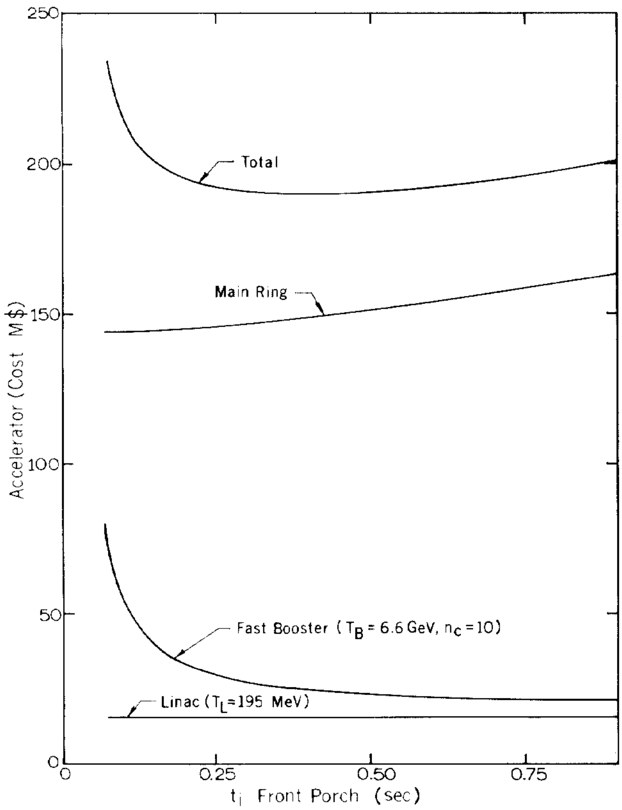

Fig. 3. Cost curves for a 200 GeV main ring with a fast cycling injector, as a function of injection time (front porch).

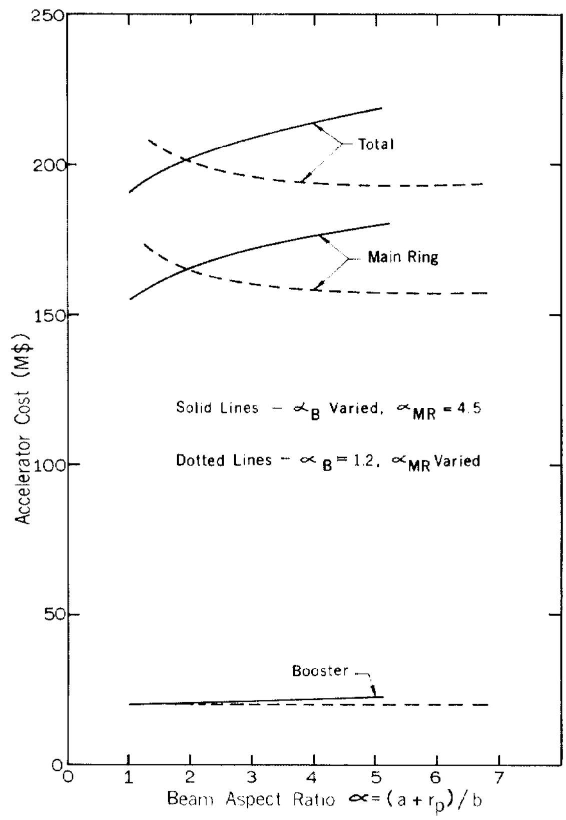

Fig. 4. Cost curves for a 200 GeV main ring with a slow cycling injector, showing effect of varying beam aspect ratio in the main ring and booster.

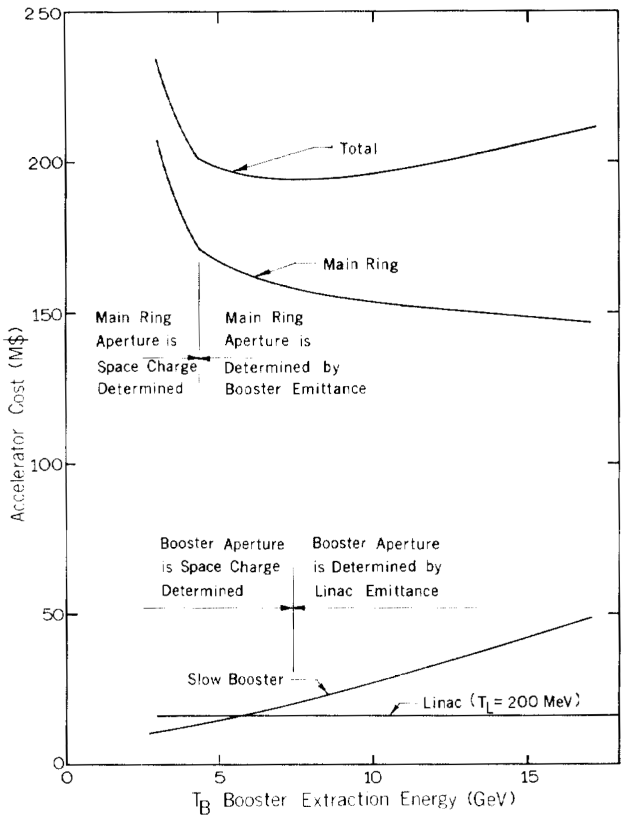

Fig. 5. Cost curves for a 200 GeV main ring with a slow cycling injector, as a function of booster energy.

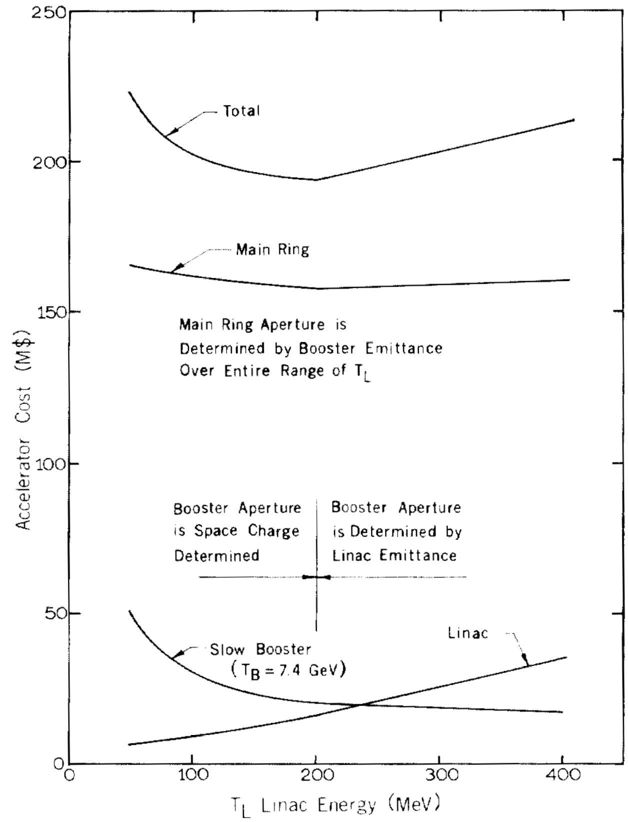

Fig. 6. Cost curves for a 200 GeV main ring with a slow cycling injector, as a function of linac energy.