SLOW EXTRACTION FROM FFAG ACCELERATORS*

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Summary

The results of earlier analytical calculations on the slow extraction of particles from an alternating gradient accelerator are applied to the special case of a fixed field alternating gradient accelerator. The extraction efficiency and rate of growth of the betatron oscillations are described.

Introduction

In order to continue the photonuclear investigations at Iowa State University, design work has been under way on a fixed field alternating gradient betatron to produce 150 Mev electrons with about a 16% duty cycle and an average current of about 1 ma. To exploit fully the advantages of this accelerator, it is necessary to have an efficient method for the slow extraction of the accelerated beam. A method for slow extraction from a pulsed alternating gradient accelerator was described by Hammer and Laslett[1]. This work was extended by Schoultz and Hammer[2] and by Mead[3] to include radial sector FFAG accelerators. In these papers it was shown that, by adding field perturbations as a function of azimuth to excite a half-integral resonance, the beam could be made to grow at such a rate that a septum placed at the proper position could extract a very large percentage of the circulating beam, and that the extracted beam was of high quality. In the case of a conventional accelerator these field bumps can be time dependent, whereas for an FFAG accelerator they can be time independent bumps which increase with radius. The effects of these bumps as calculated using the linearized equations of motion are used to select the proper perturbations for efficient extraction. The calculations are then checked by calculating the particle orbits using a digital computer and the correct (non-linear) equations of motion. In almost all cases the digital computer results agree well with the analysis and indicate efficient extraction.

The analytical calculation has to be slightly modified in the spiral sector variety of FFAG accelerator since, in contrast to the radial sector machines, the position of maximum magnetic field is not the same as the position of maximum field gradient. In this paper there is first a very brief derivation of these modifications, followed by a description of the important parameters of the proposed accelerator. After this a brief description of the field perturbations and a description of the results of the digital computer calculations are given.

Linearized Equations

In the spiral sector FFAG accelerator the magnetic field is represented by

$$B_z = B_0(1+x)^k\left[1+\sum_j f_j \cos\left(j\theta - \frac{j}{w}\ln|1+x| - \beta_j\right)\right]$$

where

$$x = (r - r_0)/r_0 .$$

The linearized radial equation of motion is,

$$\begin{aligned} x'' = {} & 1 - \left[1+\sum_j f_j \cos(j\theta-\beta_j)\right] \\ & - x\Big[k+1+\sum_j f_j (k+2)\cos(j\theta-\beta_j) \\ & + \sum_j (j f_j/w)\sin(j\theta-\beta_j)\Big] . \end{aligned} \tag{1}$$

From this equation it is apparent (by comparing it with equation (3) of reference 2 that the effects of perturbations on the equilibrium orbit will be shifted in phase with respect to the effects on the oscillations about the equilibrium orbit.

Specifically, the phase angle δ_j which describes the positions of the perturbations appears in Eq. (5) of reference 2 as,

$$W'' + \left\{(k+2)\left[1+\frac{m_\ell}{k+2}H(\theta)+\sum_j \varsigma_j \cos(j\theta+\delta_j)\right]-1\right\}W = 0 \tag{2}$$

The equation for the betatron oscillations as obtained from (1) is,

$$\begin{aligned} & X'' + \Big\{k+1+(k+2)f_N\cos(N\theta-\beta_N)+(f_N/w_N)\sin(N\theta-\beta_N) \\ & + \sum_j\left[(k+2)f_j\cos(j\theta-\beta_j)+(f_j/w_j)\sin(j\theta-\beta_j)\right]\Big\}X = 0 \end{aligned}$$

where N is the number of full sectors. If these two equations are to be identical then,

$$(k+2)\sin\beta_N+(1/W_N)\cos\beta_N = 0 \tag{3a}$$

and

$$\xi_j\cos(j\theta+\delta_j) = (k+2)f_j\cos(j\theta-\beta_j) + (f_j/W_j)\sin(j\theta-\beta_j) \; . \tag{3b}$$

These equations have as a solution

$$f_j = \xi_j\left[(k+2)^2+(1/W_j)^2\right]^{-\frac{1}{2}} ,$$

$$\beta_j = \beta_N - \delta_j ,$$

and

$$\sin\beta_N = (-1/W_j)\left[(k+2)^2+(1/W_j)^2\right]^{-\frac{1}{2}} . \tag{4}$$

Taking these modifications into account, one may use the results of reference 2 directly.

Design Parameters

The approximate parameters of the accelerator being designed at Ames are shown in Table I.

TABLE I

DESIGN PARAMETERS

E_{max}	150 Mev	$(1/W)$	$\simeq$ 60
E_{inj}	1 Mev	ν_x	$\simeq$ 3.3
N	16	ν_y	$\simeq$ 2.8
k	$\simeq$ 9		

Since ν_x is close to 3.5 the ν_x = 3.5 resonance can be used to extract the beam. This requires raising the linear tune to near 3.5 by increasing k slightly, and using a 7th harmonic perturbation to open up the stop band at the 1/2 integral resonance. In order to extract the beam at a definite azimuth, it is necessary to add other harmonics to distort the orbits so that their maximum excursion occurs at one particular azimuthal angle.

The unperturbed field to a good approximation is,

$$B_z = B_o(1+x)^{9.8}\{1+\cos[16\theta-(16)(3.8)\ln|1+x| + 1.38]\} .$$

This gives a tune of almost exactly 3.5.

As indicated in reference 2, harmonics of angular variation 3.5 $\pm$ .5 are especially useful since they give large amplitudes for small perturbations. Also, harmonics of (2)x(3.5) $\pm$ 1 can be added to accentuate the motion at the desired azimuth. So that both the oscillations and the equilibrium orbit give a maximum at one azimuth only, the 3rd and 8th harmonics are selected. The total perturbation field is therefore,

$$B = 0.02\,B_o\left(1+\tanh\frac{x-x_o}{0.02}\right)(1+x)^{9.8} \times \{-\cos[3\theta-(3)(3.8)\ln|1+x| - 2.33] + \tfrac{1}{2}\cos[7\theta-(7)(3.8)\ln|1+x| - 1.2] + \cos[8\theta-(8)(3.8)\ln|1+x| - 2.72]\} .$$

Results of Digital Computations

The results of the digital computations are exactly as one would expect from the results reported in reference 2. Large growth rates at one particular azimuthal position of the accelerator are obtained for relatively small perturbations. For example, for a 7th harmonic perturbation of only 0.15% of the main guide field, a growth rate of 0.2 in. per revolution occurs. As previously reported appreciable vertical growth occurs only after excessively large radial growth. Consequently, the coupling between the vertical and radial oscillations will not affect the extraction efficiency. A typical result for the total oscillation amplitude, radial betatron oscillations plus equilibrium orbit oscillations, is shown in Figure 1.

The predicted extraction point is at the 12th sector shown in the figure. As expected, the maximum radial excursion occurs at this position regardless of which initial radial phase point is used.

There seems to be no inherent difficulty in extracting greater than 90% of the internal beam, the only limitation being the size of the septum. Because the particles all enter the extraction region at essentially the same angle, but with different radii, the external beam will have a very small emittance. Preliminary results indicate an energy resolution of 0.4% which for a 150 Mev peak energy, means an energy spread of 600 kev. However, the optimum radial position of the septum has yet to be obtained.

References

*Work performed at the Ames Laboratory of the U.S. Atomic Energy Commission

1. C. L. Hammer and L. O. Laslett, RSI 32, 144 (1961).

2. W. W. Shoultz and C. L. Hammer, MURA Report No. 666, January 2, 1963, (unpublished).

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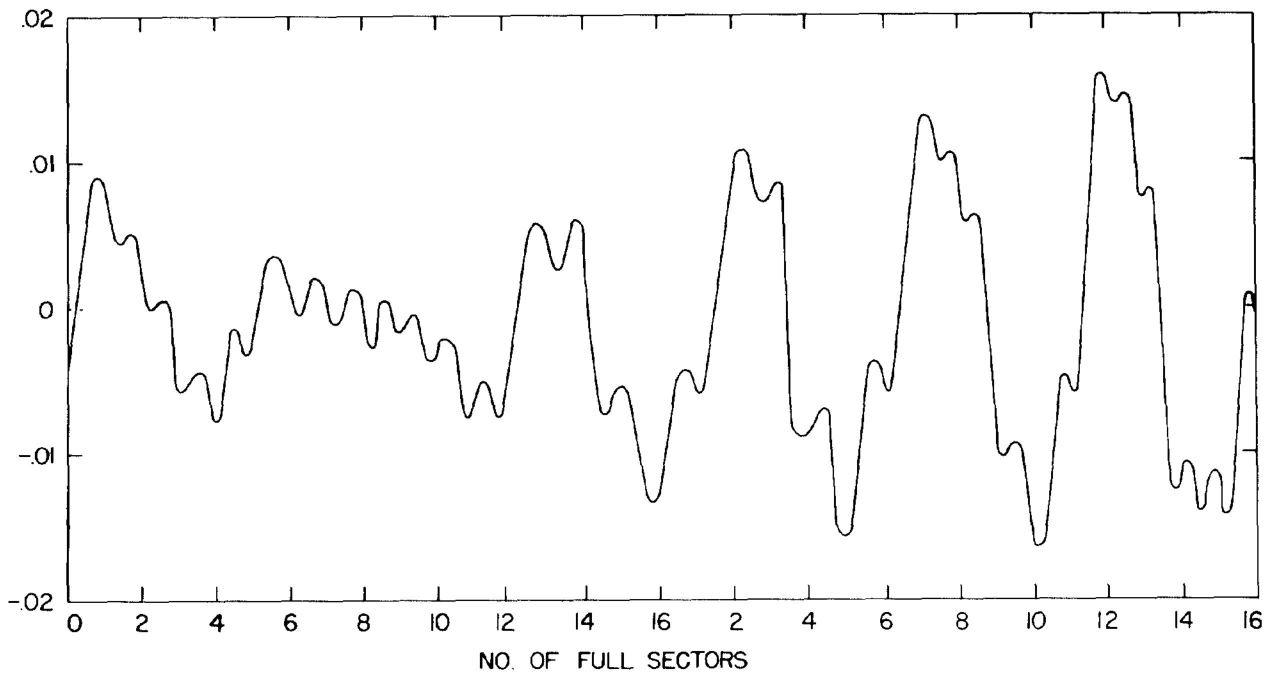

Fig. 1. Total Oscillation Amplitude.