RADIOFREQUENCY STIMULATION OF RESONANT SLOW EXTRACTION SYSTEMS†

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Introduction

Resonant slow extraction systems in synchrotrons are capable of giving tightly bunched output beams. The bunching frequency can, with certain limitations, be chosen at will.

In certain instances, for example for RF particle separators, a tightly bunched primary beam is desirable. This will be especially true for long pulse separators of the future using superconducting cavities. Here, the provision of a bunched beam would make a separator with one external cavity practicable, and so eliminate the necessity for the synchronization of two superconducting cavities. For time-of-flight experiments, bunching is essential.

Bunching can be achieved by the insertion of a deflecting cavity into the ring, and the necessary deflections are almost trivial (typically, 20 μrad peak).

Alternately, one can adiabatically rebunch the beam within the synchrotron, but this is time consuming and also requires relatively high radiofrequency power.[1] Worse still, in slow extraction schemes, the radius of the orbit is usually varied during the extraction period, and consequently the period of rotation of the particles varies. The bunching frequency must be a multiple of the revolution frequency of the particles and therefore cannot be held constant over the extraction period (as pointed out by H. Hahn, private communication).

No such troubles arise when a microwave deflecting cavity is used to stimulate the slow extraction process. A fixed deflecting frequency is sufficient and the emergent bunches are extremely short. In 2/3 or 1/3 integral resonant systems, the bunch length is no greater than 60° or approximately one-sixth of a free space wavelength at the deflecting frequency. In half integral systems their length is 90°.

The Bunching Process

The simplest case to consider is that in which an RF deflection frequency of precisely 2/3 ω_o is superimposed upon a 2/3 integral resonant slow extraction system, ω_o being the particle revolution frequency.

Phase plots for this case are given in Fig. 1. The plots are for particles on the equilibrium orbit, but with different initial azimuths. The initial azimuth is the azimuth of the particle at t = 0, when the RF deflecting force is switched on. Particles with different azimuthal initial conditions encounter the RF deflecting electrode at different RF phases.

Each phase plot is a system of points forming a pattern with threefold symmetry. Each plot shows the history of a single particle in transverse phase space. A point is plotted every time the particle completes a revolution, so that the number of points in a plot equals the total number of revolutions of the particle. Ideally, each point should be labelled with the revolution number appropriate to it. If this were done, it would be seen that the points go from branch to branch in a counterclockwise direction from revolution to revolution.

The general pattern of phase plots repeats itself with a period of 180° in initial azimuth. The only difference between the plots for two particles whose initial azimuths differ by 180° is that their representative points lie on different branches for the same revolution number. This is an important difference in determining the extraction pattern, but one that would not be apparent unless each point were labelled with its revolution number.

It should be noted that the scales are different in the two plots because the plotting routine sets the scale so as to produce plots of approximately the same size.

The initial azimuth was varied in 30° steps. In three cases emission occurred while in three cases the particles were "trapped" on closed contours.

Over the full circumference of the machine, trapping occurs for initial azimuths of -330°, 0°, -30° and also -150°, -180° and -210°.

The regions of initial azimuth over which trapping occurs are shown as shaded zones in Fig. 2. If one now imagines the zone pattern to be rotating at the same angular velocity as the particles, the trapped particles will be contained within the shaded zones for all t > 0, and no emission can occur while a shaded zone is passing the septum.

At first sight, one would expect the emergent beam to consist of bunches and spaces of equal length, but this is not the case. The three branches of the first phase plot of Fig. 1 have been numbered 1, 2 and 3, and the clear zones of Fig. 2, from which emission is possible, have been

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labelled A and B. At some time, while region A is passing the septum, all the particles contained in A will be on branch 1 of the phase plot and emission will be possible. A bunch of length not greater than a quarter of the machine circumference will be emitted.

After emission of this bunch, a shaded zone passes the septum and no emission can occur. When zone B passes the septum, all the particles in it will be found to be on branch 2 of the phase plot and therefore there will be no emission. This is followed by a shaded zone of trapped particles, and when zone A again passes the septum all the particles in it will be found to be on branch 3 of the phase plot. Emission will not occur again until zone B passes the septum. This time, the particles in zone B will be found to be on branch 1 of the phase plot and a bunch will be emitted. Between the two bunches, three shaded zones and two clear ones pass the septum. The gap between bunches will therefore be five times larger than the bunch length. The bunches emerge every $1\frac{1}{2}$ revolutions, or at the deflecting frequency of $2/3\ \omega_o$. In terms of the deflecting frequency, the bunches are 60^o long, with spaces of 300^o between them.

There is no need to restrict oneself to the fundamental deflecting frequency of $2/3\ \omega_o$. One can use any frequency $(N \pm 2/3)\ \omega_o$, where N is an integer. The bunches then emerge at the deflecting frequency and are still 60^o in length referred to the new frequency. The zone pattern of Fig. 2 will now consist of much narrower and more numerous zones. Each zone will be 60^o in length referred to the deflecting frequency and emission will occur from every third clear zone. The necessary deflection is independent of the deflecting frequency.

Releasing the Trapped Particles

A system that leaves half the particles trapped within the machine is of little interest, but fortunately this can be avoided.

If one detunes the deflecting frequency so that it is not quite equal to $2/3\ \omega_o$, slip is introduced between the speed of rotation of the zone pattern and the speed of rotation of the particles. A particle that finds itself in a trapped zone will drift out into a clear zone and be emitted later. This is shown in Fig. 3, where the deflecting frequency has been detuned by $0.005\ \omega_o$ to $0.6717\ \omega_o$. All the particles are released. As before, the frequency of the emitted bunches is equal to the deflecting frequency and their length is 60^o referred to this frequency. With detuning, the phase plots are similar to Fig. 3 for all initial azimuths.

The Mechanism of Trapping

This is best understood by inspecting the forcing terms in the differential equations governing the betatron motion of the particles.

The differential equation governing a 2/3 integral resonant system with RF stimulation is:

$$\frac{d^2x}{d\theta^2} + \nu^2 x = \frac{1}{M}\left[ax^2 \cos 3\,\nu_o\,\theta + B\cos\left[(\nu_o+\Delta)\,\theta - \frac{2}{3}\,\theta_o\,\right]\right] \tag{1}$$

where x = distance of particle from equilibrium orbit, in units of machine radius,
θ = azimuth angle in radians,
ν = ν value of particle,
$\nu_o = 8\ ^2/_3$,
Δ = detuning increment (between deflecting RF and $2/3\ \omega_o$),
ω_o = angular frequency of rotation of particles,
θ_o = initial azimuth of particle at $t = 0$,
a = strength of sextupole,
M = mass of particle,

$$B = \frac{H_{RF\ deflector}}{H_{guide\ field}} \cdot \frac{\Delta L}{L},$$

ΔL = length of RF deflector,
L = circumference of machine.

Consider a particle whose betatron oscillation amplitude is growing under the influence of the RF deflecting force. This will be the only appreciable force at low amplitude, as the sextupole force depends on the square of the amplitude. For growth to occur, the particle motion must be approximately in phase with the RF force, so that the particle motion can be approximated by

$$x \simeq x_o \cos\left[\ (\nu_o + \Delta)\ \theta - \frac{2}{3}\,\theta_o\ \right]$$

where x_o is the peak amplitude of oscillation.

The force due to the sextupoles is then:

$$F = a\,x_o^2 \cos^2\left[(\nu_o + \Delta)\ \theta - \frac{2}{3}\,\theta_o\ \right]\cos 3\,\nu_o\,\theta$$

$$= \frac{ax_o^2}{2}\left[1 + \cos 2\left[(\nu_o+\Delta)\ \theta - \frac{2}{3}\,\theta_o\ \right]\right]\cos 3\,\nu_o\,\theta\ .$$

The fundamental component of this is

$$F_1 = \frac{ax_o^2}{4}\cdot\cos\left[(\nu_o - 2\Delta)\ \theta + \frac{4}{3}\,\theta_o\ \right]. \tag{2}$$

The RF deflecting force is

$$F_{RF} = B\cos\left[(\nu_o + \Delta)\ \theta - \frac{2}{3}\,\theta_o\ \right]. \tag{3}$$

The phase difference between the two forces is

$$\delta\,\varphi = -\,3\Delta\cdot\theta + 2\theta_o = 2\theta_o \quad \text{when } \Delta = 0\ .$$

Therefore, for some values of θ_o, a particle whose amplitude of oscillation is increasing under the influence of the RF deflection forces will meet opposing sextupole forces as the amplitude grows. These particles will be trapped. For other values of θ_0 , the two forces will aid one another and there will be no trapping.

When $\Delta \neq 0$, the phase relationship between the forces is θ dependent, and all particles will be emitted sooner or later.

The Slow Extraction Process

The system under consideration is the same as that described by M.Q. Barton[2] except for the addition of an RF deflecting force. In Barton's scheme, the beam is moved slowly across the vacuum chamber by varying the guide field. The ν value varies across the chamber, and as ν approaches $8\ ^2/_3$ particles become unstable.

Whether a given particle is stable or unstable depends on the proximity of its ν value to $8\ ^2/_3$ and the amplitude and phase of its betatron oscillations. There is a small ν spread among the particles in the beam, and the betatron oscillations are incoherent. This distribution of properties of the individual particles makes the slow spill possible.

The RF deflecting force causes coherent betatron oscillations to build up, so that emission starts earlier than it would have in the absence of such a force. In addition, the coherence of the oscillations causes the emitted beam to be bunched.

For the purpose of this study, the following parameters were taken:

Initial ν values of particles in beam	$8.683 \geq \nu \geq 8.675$
RF deflecting frequency	$0.6717\ \omega_0$
Peak RF deflecting amplitude ..	20 µrad
Initial betatron oscillation amplitude	0

It is necessary to detune the deflecting frequency from $(N \pm 2/3)\ \omega_0$, to avoid trapping. Here it has been detuned by $+\ 0.005\ \omega_0$.

The RF deflecting amplitude was chosen by trial and error to be just sufficient to cause the particles of lowest ν value (8.675) to be emitted after a few hundred revolutions.

To simulate the movement of the beam across the vacuum chamber, the ν value was decreased linearly and very slowly. The rate of change of ν ($- 6\pi \times 10^{-8}$ per revolution), was chosen so that an initial ν value of 8.683 would become 8.675 after 40,000 revolutions. This gives an extraction period of about 100 ms.

When the RF deflecting frequency and the fractional part of the ν value are not the same, the amplitude of betatron oscillations will rise and fall in a series of "beats". The greater the frequency difference, the faster the beats and the lower their amplitude. With the parameters used here, the amplitude of a beat is just sufficient to drive particles with a ν value of 8.675 into the unstable region. If the ν value is greater than this, the beats are of insufficient amplitude to cause emission.

The beat pattern can be explained by means of Cornu Spirals, as shown in previous reports;[3,4] however, in this case, where the RF forces in the neighborhood of the separatrix are small compared with those due to the sextupoles, the Cornu Spiral approach is only of qualitative value.

In practice, it would be a simple matter to determine the optimum RF deflection amplitude. One would simply increase the amplitude from a low value and observe the structure of the slow extracted beam. This would be much simpler than finding it with a computer.

The computer program for all the phase plots used matrix transformations to follow a particle around a "perfect" machine. Four dc sextupoles of alternating sign were inserted at 90° intervals. The sextupole strength was 100 µrad at one inch off axis. At each sextupole an RF deflector was included. The use of four RF deflectors per revolution was a matter of convenience. A single deflector of four times the strength gives the same result, and all deflection amplitudes given here refer to a single deflector.

Figure 4 shows time of emission in revolutions as a function of initial ν value for particles that are initially on their equilibrium orbits. The peak RF deflection amplitude is 20 µrad.

Conclusions

The addition of a low power RF deflector to a slow extraction system will cause the emergent beam to be bunched at the deflecting frequency. In 1/3 or 2/3 integral resonant systems, the bunch length will be 60°, referred to the bunch frequency. In half integral systems the bunches will be 90° long.

The deflecting frequency, and therefore the frequency of the emergent bunches, must be approximately equal to $(N \pm 2/3)\ \omega_0$ in 2/3 integral resonant systems and $(N \pm 1/2)\ \omega_0$ in half integral resonant systems, where N is any integer and ω_0 is the particle revolution frequency.

One must avoid using frequencies of exactly $(N \pm 2/3)\ \omega_0$ or $(N \pm 1/2)\ \omega_0$, or else trapping will occur. On the other hand, the detuning from these values should not be too great. If one detunes so much that the fractional part of the deflecting frequency falls within the range of the fractional parts of the ν values present in the beam, the pattern of beats of increasing amplitude will not occur. Also, excessive deflection amplitude is to be avoided as it shortens the extraction period. Apart from these restrictions, there appear to be no critical parameters. Although beam emittance has not been considered here, it seems reasonable to assume that it will not be affected by a relatively small RF force.

References

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3. K.M. Terwilliger, MURA Internal Report KMT-3, August 15, 1956.

4. D.S. Robertson, "Multiturn Beam Extraction in AG Synchrotrons," BNL Accelerator Dept. Internal Report AADD-85, June 10, 1965.

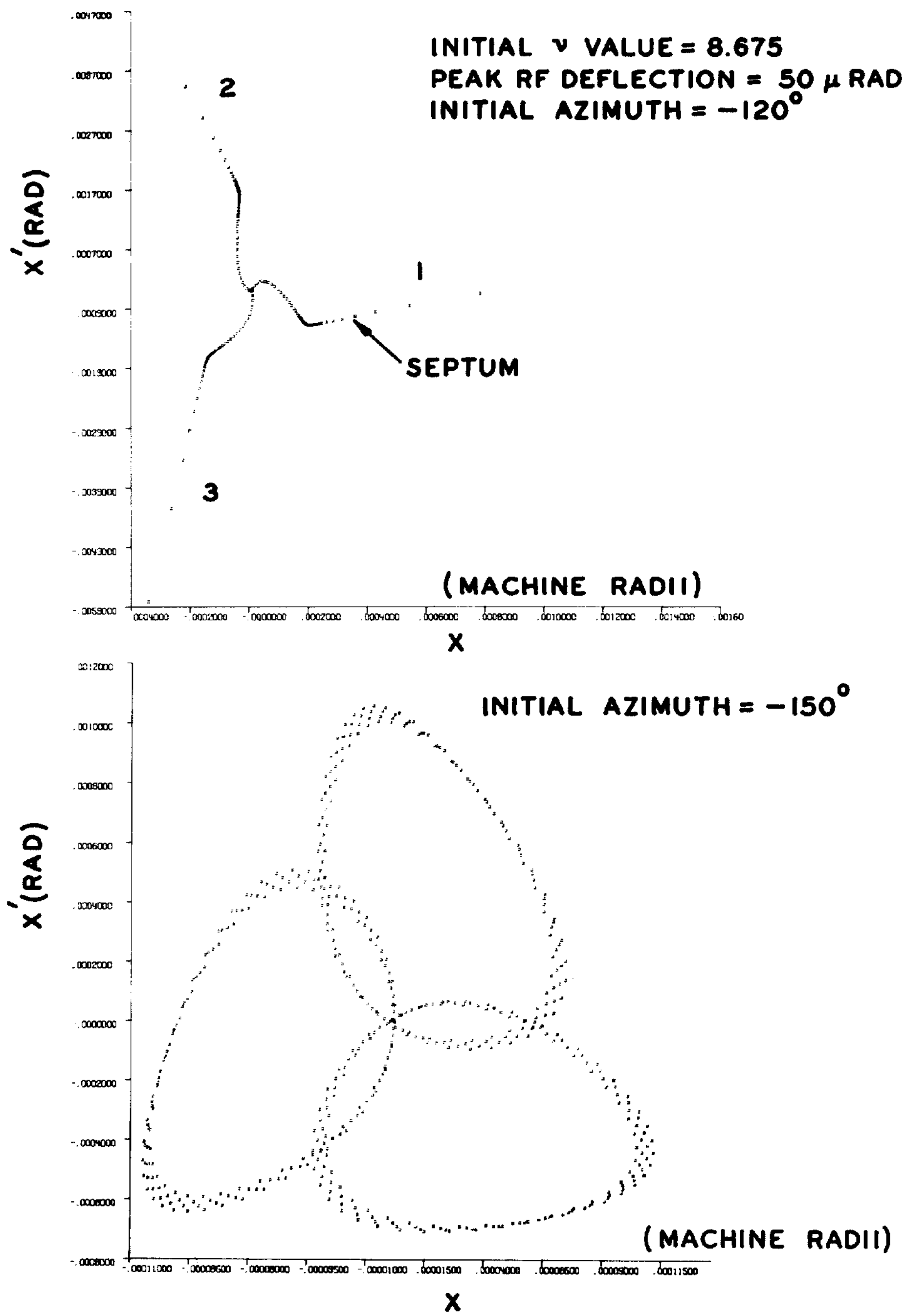

Fig. 1. Phase plots for RF deflection frequency = 0.6667 ω_o.

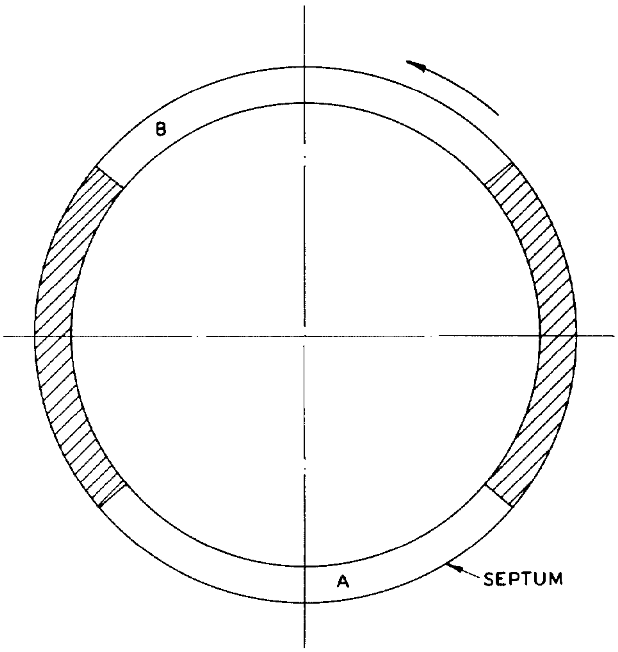

Fig. 2. Trapping regions (shaded) of initial azimuth.

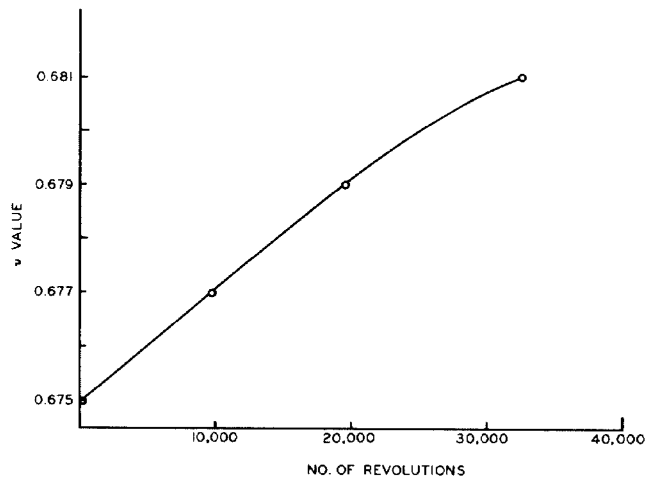

Fig. 4. Time of emission vs. initial ν value for particles initially on their equilibrium orbits. Peak RF deflection amplitude is 20 μrad.

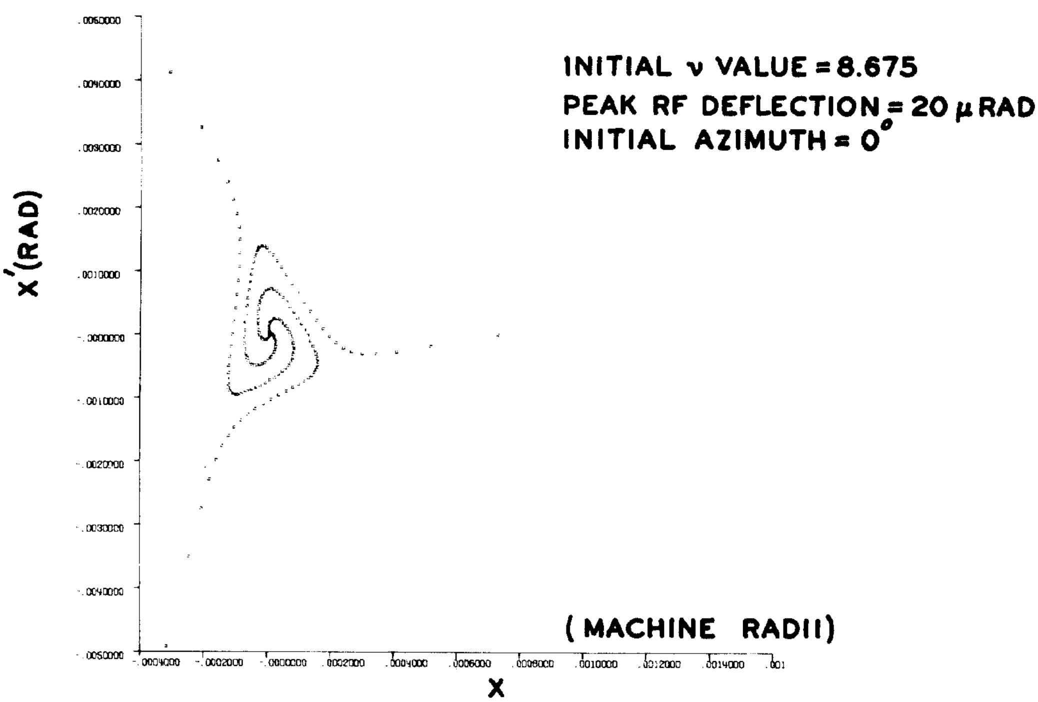

Fig. 3. Phase plots for RF deflection frequency = 0.6717 ω_0.