COMPUTER STUDIES OF ORBITS IN HIGH ENERGY MICROTRONS

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Summary

This paper reports briefly several types of orbit calculations performed to study the feasibility and design features of a superconducting 600-MeV split microtron being planned for the University of Illinois. Programs have been written for the Illinois IBM7094 computer to study phase stability in the presence of magnetic field inhomogeneities, vertical defocusing produced by the magnet fringing fields, and axial and radial motion of electrons in the accelerating linac. Some of the calculations are described in a report which is available on request.[1]

Tentative Parameters

The configuration which we will consider is similar to that suggested by Wiik, Schwettman, and Wilson[2], a split or racetrack microtron in which two uniform-field d.c. magnets with parallel edges are separated by a distance large compared to the magnet dimensions, and a linear accelerator is placed in the common straight section. Tentative parameters being considered are frequency $f = 1.3$ GHz, magnet field strength $B = 13$--15 kG, harmonic gain per turn $\Delta N = 2$, linac gradient 2 MeV/ft, energy gain ~ 30 MeV/turn, straight section length 6--7 meters, and injection energy about 250 keV. Inflection of the injected beam into the common orbit is accomplished by a set of magnets located just in front of the linac and arranged to provide no net deflection of the recirculating beam.

Region of Phase Stability

An average energy gain per turn of approximately $\Delta E_{nom} = 1.43\,(B/f)\Delta N$ (in MeV) is required to increase the period of revolution by ΔN rf cycles per turn. If the peak energy of the linac E_{rf} is greater than ΔE_{nom}, a region of phase exists in which injected particles execute stable phase oscillations for a large number of revolutions. Initial phases outside this region lead to extremely rapid blowup, i.e., loss of the particle from the phase bucket. The size of the stable region is a strong function of E_{rf} and of ΔN.

Numerical calculation of the phase oscillations experienced by a particle injected along the central ray have been made for hard-edge, uniform magnetic fields. In these calculations the change of phase of the particle in passing through the linac is neglected. This makes an appreciable error only in the first passage, and does not affect the phase stability. Exact relations for electron velocity and radius of curvature have been used.

The results of a number of computer runs of this type are shown in Fig. 1. The abscissa is the peak linac energy divided by ΔN, and the ordinate is the initial phase at injection. For each value of ΔN the curves show the upper and lower boundaries of the stable phase region. Here the phase is expressed in radians and is measured from the following zero of the rf field, so that the peak linac field occurs at a phase of 1.5708.

The parameters assumed in Fig. 1 are $B = 15$ kG, $f = 1.3$ GHz. The linac length is $8.25\,\Delta N$ (in feet). An additional 6 feet has been allowed for clearance, inflection, and focusing, and the lengths are then adjusted slightly to obtain initial synchronization of the orbit. The conditions for the 3 curves in Fig. 1 are as follows:

Harmonic Change	Injection Energy	Straight Section Length
$\Delta N = 1$	$E_o = 17.5$ MeV	$L = 448.65$ cm.
$\Delta N = 2$	$E_o = 1.0$ MeV	$L = 702.3$ cm.
$\Delta N = 3$	$E_o = 1.0$ MeV	$L = 955.94$ cm.

The stable region exists only for E_{rf} greater than the minimum value, $16.5\Delta N$. The curve for $\Delta N = 1$ shows by far the largest stable region, with $\Delta N = 2$ and 3 giving greatly reduced stability. The curves for $\Delta N = 1$ and $\Delta N = 3$ have additional (solid) points which were computed for $L = 702.3$ cm, and which show that the stable region is not appreciably affected by a change in straight section length alone.

For small values of E_{rf}, the stable region extends on either side of 90°, but for sufficiently high linac energies the stable region is entirely on the falling side of the rf waveform and eventually becomes quite narrow. A wide stable region occurs down to 60° for $\Delta N = 1$, and to about 79° for $\Delta N = 2$.

An interesting feature of the curves in Fig. 1 is that for small values of E_{rf} close to the minimum value, the stability region is nearly the same for all values of ΔN. This region is of particular interest since it turns out that, as the result of the reduced width of the stable phase region, the final energy spread of the particles after a given number of turns becomes very small.

An example of the computed final energy of stable orbits is shown in Fig. 2. Curves of energy vs. ϕ_o for $E_o = 1.0$ MeV are shown for 6 closely spaced values of E_{rf}. Each curve covers the entire stable region of ϕ_o. It can be seen that the energy spread becomes appreciably smaller as we approach the minimum E_{rf}. We can see that extremely small energy spreads, down to 0.05%, can be obtained if we are willing to accept the reduced phase window ($\sim 7^o$) and to provide the very high stability required for E_{rf} ($\sim \pm 0.05\%$). This type of operation is very interesting in connection with experiments on inelastic electron scattering.

Effect of Nonuniform Magnetic Fields

Degree of Nonuniformity Expected in the Magnet

The high harmonic number and narrow phase window should make the tolerance on magnetic field uniformity in a high-energy microtron very critical and pose a major problem in design of the magnets. The magnet for a 600-MeV microtron would have a pole area of about 1.5 x 3.0 meters and a gap of 2 cm or less. This necessitates a large ratio of maximum to minimum path length for the flux in the iron, independently of the arrangement of the return yoke. (C-type magnets are preferred because of the better access to the orbit region.) The iron path length is as much as 9 meters at the edge furthest from the yoke, and about 1 meter at the edge nearest the yoke. Since the best iron forgings have a maximum permeability of only about 1500, this path length variation is equivalent to a 25% variation in the total reluctance of the magnetic circuit.

Fortunately, the flux tends to redistribute itself in such a way as to make the field in the gap more uniform. In some crude, preliminary tests with a 4-ton yoke, we had no trouble in obtaining a field variation of 3/4% or less at 8000 gauss in a 5/8" gap over a range of distances from the return yoke of about 18". The gap height in this test was uniform to within a total of about 0.002". It is also known that homogenization of the field by inserting auxiliary air gaps above and below the poles will improve the uniformity. At this stage it is difficult to assess accurately the uniformity achievable, but it seems reasonable to expect that, as the field strength is varied over a wide range (to obtain a continuous variation of the beam energy), we may expect field shape variations of the order of 1%, and that the variation should occur principally with distance from the return yoke.

Calculation of Orbits in a Nonuniform Field

A program has been written to trace electrons through a field with a uniform gradient at right angles to the entrance edge of the magnet gap. This program includes only the horizontal motion, but is highly accurate. This routine, after careful checking, was incorporated into a program to compute microtron phase oscillations. Fig. 3 shows a series of runs made for the conditions: B = 15 kG at the entrance edge, f = 1.3 GHz, L = 702.3 cm, $N_o = 61$, $\Delta N = 2$, $E_o = 1.0$ MeV, for eighteen turns (nominal energy gain per turn = 33.0 MeV). Runs were made for field gradients of -3, -1.5, -1, 0, 1, 2, 3, and 4 gauss/cm. The positive gradients refer to an increase of field as the particle goes further into the gap from the entrance edge. The data plotted for each value of B_1 are for the values of E_o and E_{rf} giving the widest phase window. It can be seen that the phase window, shown by the "corridor" at the top of the figure, does not change greatly over this range of B_1 which corresponds to field variations of up to 3.5%. It may be concluded that this particular form of field variation, which is one likely to be encountered, will not be a serious problem in the microtron.

Defocusing in the Fringing Field

One of the major difficulties of the split-magnet microtron is the large vertical defocusing effect experienced by the electrons in passing through the fringing field of the main magnets during the first few revolutions. A program has been written to compute the trajectory in detail for a number of energies.

Calculations for a 15-kG field in a 2-cm gap showed that a 30-MeV beam entering the fringing field normally at 20 cm from the gap edge and 3 mm from the median plane is 13 mm from the median plane and diverging at an angle of 40 milliradians as it leaves the magnet at the same distance from the gap after deflection through essentially 180 degrees. Although this is a very large defocusing effect it can be compensated by an ideal lens placed near the exit point of the beam. More complete studies of these defocusing effects and ways to reduce them is in progress.

Injection of Electrons into the Linac Section

Several possibilities for injection of electrons into the linac section of the microtron have been studied by means of a computer program which calculates and plots the beam trajectories as the electron proceeds through the section in small increments. The accelerator section considered was a set of 40 Los Alamos type L cavities resonating at 1300 MHz, with 180° phase difference between successive cavities. The phase velocity of the accelerating component was taken equal to the velocity of light. In a superconducting structure it would be inconvenient to introduce magnetic focusing fields into the linac region and it seemed reasonable to investigate the behavior of low energy electrons injected directly into the structure under different injection conditions.

A program was written for numerical solution of the equations of radial and axial motion. The axial field given to us by the Los Alamos group was expanded into harmonics, and components up to the seventh harmonic were used in the calculation. The coefficients of the expansion were $A_1 = 0.94$, $A_3 = -0.14$, $A_5 = -0.18$, $A_7 = 0.03$. Space charge effects and the radial dependence of the energy gain were neglected.

We present here some results for injection at an energy of 250 keV into a linac with a nominal energy gain of 2 MeV/ft. and which results in output energies and phases satisfying the conditions for stable acceleration in the microtron. Using cylindrical coordinates R, θ, Z, injection conditions were specified by initial values of distance R_o from the Z axis, the angle $(\alpha_R)_o = (dR/dZ)_o$ in the plane through the axis (axial plane), and the angle $(\alpha_\theta)_o = (R\, d\theta/dZ)_o$, where θ is the azimuth angle with respect to the axial plane.

The trajectories as plotted by computer for axial-plane rays with $\theta = 0$, $R_o = 2$ mm and $\alpha = -1$, 0, and +1 milliradian, are shown in Fig. 4 for two injection phases. The injected electrons entered the accelerator at a phase ϕ_o leading that of the crest of the accelerating field by 45° and 75°, corresponding to phases after passage through the linac of 11.4° lagging and 3.5° leading, roughly the limits for phase stability in a microtron with $\Delta N = 2$. It can be seen that, although there are small oscillations on the trajectories, the displacements and angles of the outgoing beams are in most cases reduced in passing through the linac section.

The trajectories for non-axial-plane rays for electrons injected at a leading phase of 45° are shown in Fig. 5, where the lower half of the figure shows the azimuthal angle θ as a function of the position in the cavity. The three curves correspond to beams with angles of -1, 0, and +1 milliradian in the initial axial plane but all have an additional initial velocity out of this plane corresponding to 1 milliradian. As might be expected for this case, the centrifugal force, together with conservation of angular momentum, prevents the electrons from coming close to the axis and forces them to spiral around it.

The results of these calculations are summarized in Table I, together with those for an injection energy of 1 MeV. The table also includes the output vectors for a case where the first 6 cavities were excited to a gain of 2 MeV/ft and the remaining 34 cavities to 1 MeV/ft. In Table I a leading phase is denoted by a negative sign.

Table I. Parameters specifying output electrons for selected input conditions resulting in the larger excursions. The kinetic energies E are in mc^2 units (0.511 MeV), the phases ϕ in degrees, R in millimeters and α in milliradians.

A. Energy gain G = 2 MeV/ft.

Input				Output			
T_o	ϕ_o	R_o	α_{R_o}	E	ϕ	R	α_R
.5	-75°	2	+1	58.7	- 3.5	+1.21	-.04
.5	-45°	2	-1	58.7	+11.4	-1.68	-.34
2.0	-25°	2	-1	60.8	- 3.6	-1.93	-.43
	-10°	2	-1	60.5	+10.4	-2.23	-.47

B. 6 cavities at 2 MeV/ft and 34 cavities at 1 MeV/ft

0.5	-75°	2	+1	33.7	+ 0.4	+2.66	+.26
0.5	-60°	2	+1	34.0	+ 6.0	+0.23	-.12

It is evident from these results that electrons of rather low energy can be injected directly into a standing-wave linac without loss or defocusing effects and would satisfy the requirements for operation of a microtron.

References

1. C. S. Robinson, The Design of High Energy Microtrons, Technical Report No. 156, Physics Research Laboratory, University of Illinois, January 19, 1967.
2. B. H. Wiik, H. A. Schwettman, and P. B. Wilson, a 200 MeV Superconducting Racetrack Microtron, Stanford University Report HEPL-396, December, 1965.

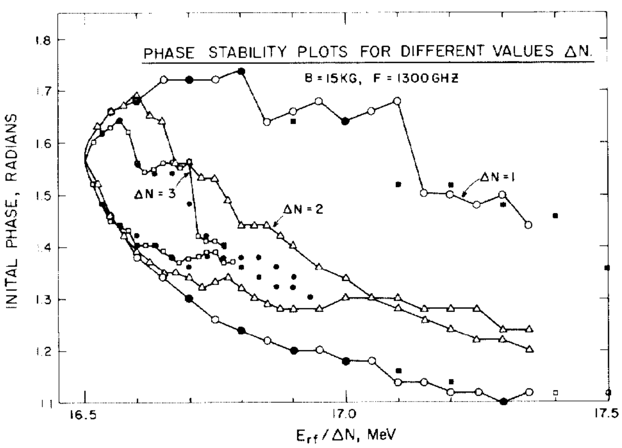

Fig. 1. Phase stability region for different values of harmonic gain ΔN. The upper and lower limits of the stable phase are plotted against the peak linac energy E_{rf}. For convenience in comparing different values of ΔN, the linac energy has been divided by ΔN. The solid points for $\Delta N = 1$ and $\Delta N = 3$ correspond to different values of straight-section length, as discussed in the text.

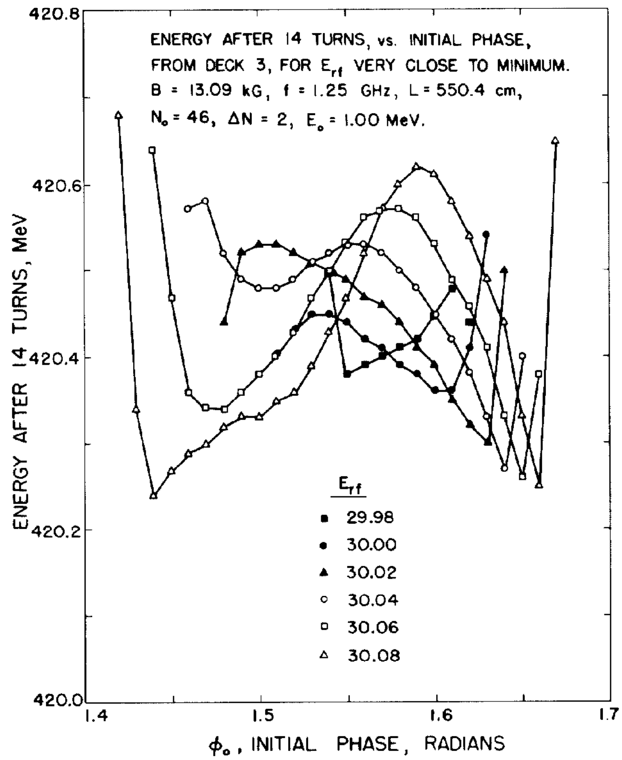

Fig. 2. Spread in final energy. The energy of the electron after 14 turns is plotted vs. the initial phase, for six different values of linac energy.

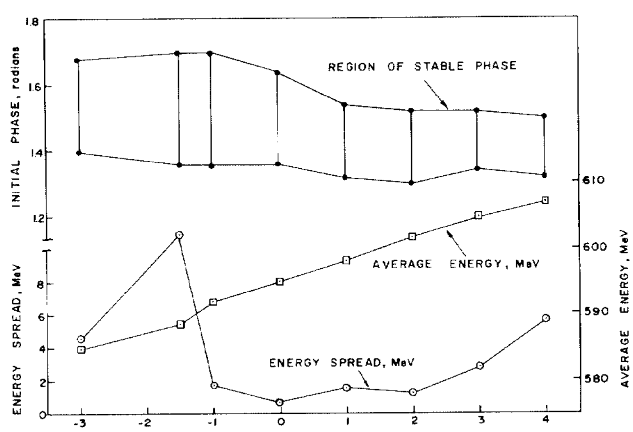

Fig. 3. Effect of Magnetic Field Gradient. The upper curves show the stable range of initial phase, and the lower curves show the extreme spread and average of the final energy. The abscissa is the gradient in a direction normal to the magnet edge.

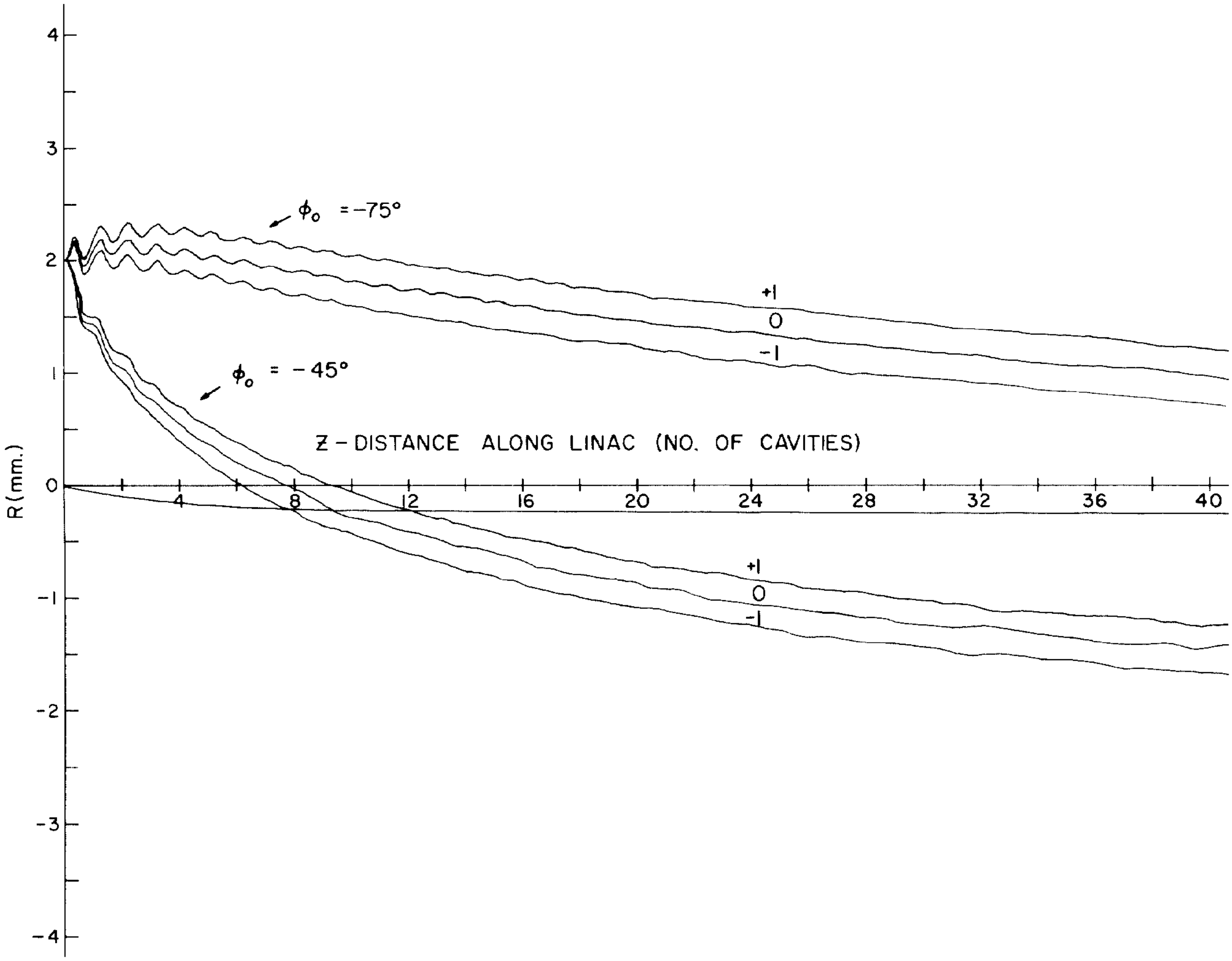

Fig. 4. Radial coordinate of electron in mm vs. distance along linac section in number of cavities. Initial conditions: kinetic energy E = 0.255 MeV, energy gain per unit length G = 2 MeV/ft, initial phase ϕ_0 = -75° and -45° with respect to peak, α_{R_0} = -1, 0, +1 mrad, α_{θ_0} = 0. Some of the small wiggles in the right hand part of the figure represent inaccuracies in drafting.

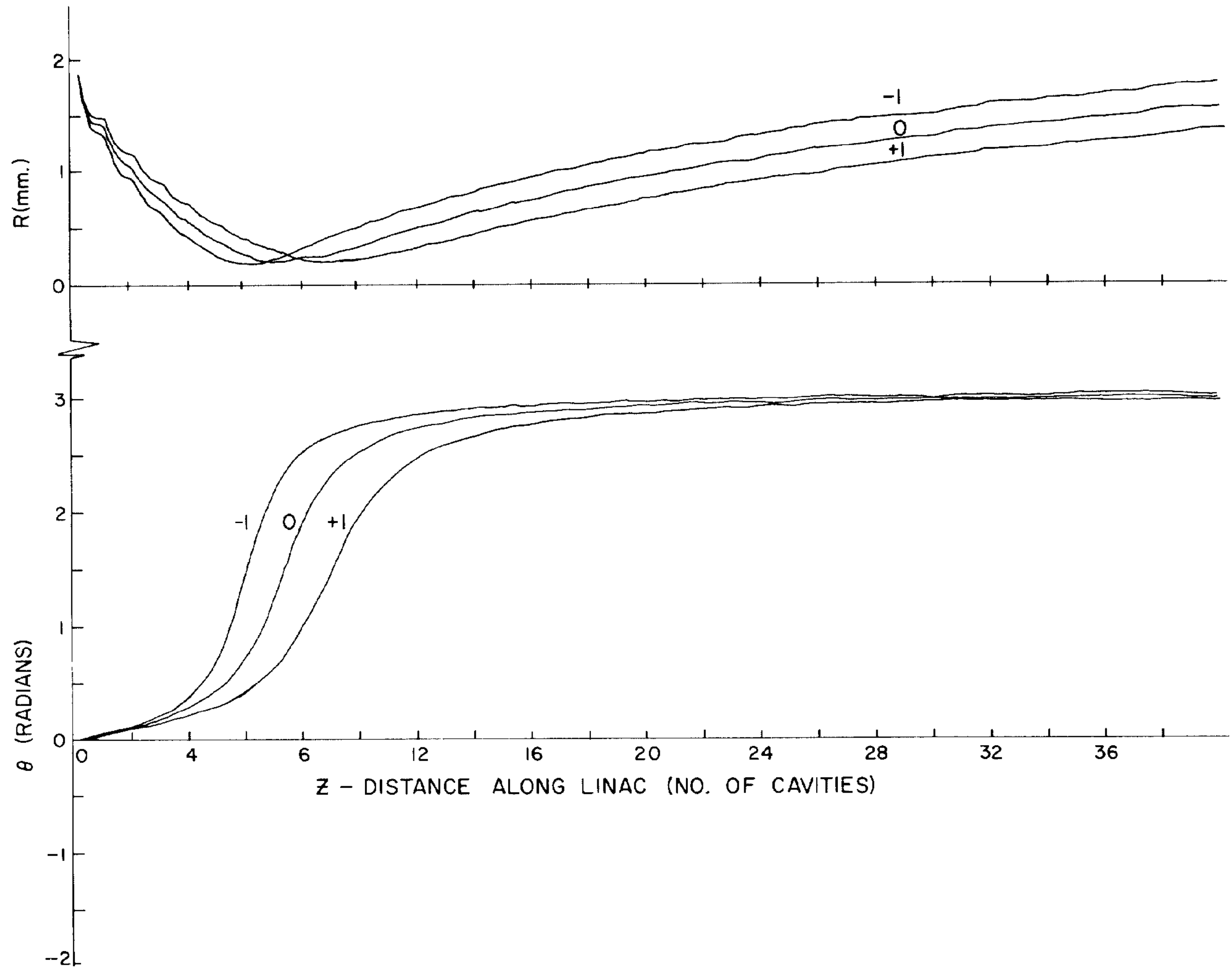

Fig. 5. Radial coordinate of electron in mm and azimuth angle in radians, vs. distance along linac section in number of cavities. Initial conditions: kinetic energy E = 0.255 MeV, energy gain per unit length G = 2 MeV/ft, initial phase ϕ_0 = -45° with respect to peak, α_{R_0} = -1, 0, +1 mrad, α_{θ_0} = 1 mrad.