SPACE CHARGE EFFECTS IN HIGH CURRENT LINEAR ELECTRON ACCELERATOR INJECTION SYSTEMS*

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Introduction

One of the simplest types of injection system used on the linear electron accelerator consists of an electron gun, a prebunching cavity or cavities and the first part of the accelerating waveguide itself. Additional components, such as inflectors and choppers are often incorporated but can be treated separately. One of the first problems encountered in the design of an injector is to determine the phase and energy spread of the beam injected into the accelerating waveguide. This problem has been treated by a number of authors but with certain limitations.[1, 2, 3, 4] For an accurate calculation it is necessary to include the effect of the beam pipe which functions as a grounded conducting cylinder surrounding the beam and also the longitudinal space charge defocusing forces which exist, in high current modes of operation. A number of simplifying assumptions have been made in order to clearly present these two particular effects. Only a single cavity system has been considered although the extension to a two or more cavity system can be readily accomplished.[3] Radial space charge defocusing forces are also present which can significantly affect the bunching process. In this presentation it has been assumed that a suitable magnetic field has been applied to confine the beam and resulting secondary effects have been neglected. A computer code has been developed which traces the electron beam parameters from the prebunching cavity through the drift space up to the critical pre-relativistic region of the accelerator itself. For an accurate calculation, one rf cycle of beam must be divided into at least fifteen axial spatial divisions. The electrons within these divisions interact nonlinearly with one another during the course of the bunching process, necessitating an accurate step-by-step procedure. A number of examples have been taken which illustrate the performance of this simple type of injection system in the high current mode of operation.

Calculation of the Longitudinal Space Charge Forces

The longitudinal space-charge defocusing forces present in a bunched electron beam of uniform radial charge density inside a hollow conducting cylinder are first calculated in the reference frame of the moving electrons. The solution to this problem will then allow us to determine the charge distribution in space of an electron beam moving under the effect of these longitudinal forces.

Consider an electron beam of uniform charge density and radius r_o inside a grounded conducting cylinder of radius a, and split into a number of small discs as shown in Fig. 1. From Green's function for the cylinder,[5] the potential for an elemental disc of charge density q can be calculated. It is then possible to determine the axial electric field exerted by this disc on any other disc a distance z away. The total longitudinal force existing on any disc is then the sum of the forces due to all other discs.

Green's function for the cylinder is given by

$$V = G(\rho, b, \varphi, \varphi_o, z) = \frac{1}{2\pi\epsilon a^2} \sum_{r=1}^{\infty} \sum_{s=0}^{\infty} \cdot$$

$$\cdot \frac{\left(2-\delta_s^o\right) e^{-\mu_r |z|} J_s(\mu_r b) J_s(\mu_r \rho) \cos s(\varphi - \varphi_o)}{\mu_r \left[J_{s+1}(\mu_r a)\right]^2}$$

where ϵ is the permittivity of free space and b and φ_o represent the position of the unit point charge, and μ_r is chosen so that $J_o(\mu_r a) = 0$. The field at ρ, φ, Fig. 1 due to the disc at z = o is

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$$E_z = \frac{-\partial V}{\partial z} = \frac{-\partial}{\partial z}\iiint G\, q\, b\, db\, dz\, d\varphi_o$$

$$E_z = -q\,\Delta z \int_o^{r_o}\int_o^{2\pi} \frac{\partial G}{\partial z}\, b\, db\, d\varphi_o$$

Now the average field on the second disc is given by,

$$\overline{E}_z = \frac{\int_o^{2\pi}\int_o^{r_o} E_z \rho\, d\rho\, d\varphi}{\int_o^{2\pi}\int_o^{r_o} \rho\, d\rho\, d\varphi}$$

Integrating over both the first and second discs gives

$$\overline{E}_z = \frac{-q\,\Delta z \int_o^{2\pi}\int_o^{r_o}\int_o^{2\pi}\int_o^{r_o} \frac{\partial G}{\partial z}\, b\, db\, d\varphi_o\, \rho\, d\rho\, d\varphi}{\pi r_o^2}$$

but

$$\frac{\partial G}{\partial z} = \frac{1}{2\pi\epsilon a^2}\sum_{r=1}^{\infty}\sum_{s=0}^{\infty} \cdot \frac{\left(2-\delta_s^o\right) e^{-\mu_r|z|} J_s(\mu_r b) J_s(\mu_r \rho)\cos s(\varphi-\varphi_o)}{\left[J_{s+1}(\mu_r a)\right]^2}$$

Integrating the φ term we see that only $s = 0$ will contribute if the summation over s includes only integral values.

Then,

$$\overline{E}_z = \frac{2q\Delta z}{r_o^2 a^2 \epsilon}\sum_{r=1}^{\infty}\frac{e^{-\mu_r|z|}}{\left[J_1(\mu_r a)\right]^2} \cdot \int_o^{r_o}\int_o^{r_o} J_o(\mu_r b) J_o(\mu_r \rho)\, b\, db\, \rho\, d\rho (\text{sign } z)$$

where the sign of z is incorporated to take into account the reversal of the direction of the field as the disc moves through the point of observation. Now carrying out the integration over db and setting $\gamma_r = \mu_r a$, where γ_r satisfies $J_o(\gamma_r) = 0$ we find the average axial electric field acting on one disc due to a second disc a distance z away to be,

$$\overline{E}_z = \frac{Q}{2\pi\epsilon r_o^2}\sum_{r=1}^{\infty} e^{-\mu_r|z|}\left[\frac{2 J_1\left(\gamma_r \frac{r_o}{a}\right)}{\gamma_r J_1(\gamma_r)}\right]^2 (\text{sign } z)$$

where Q is the total charge in the disc and in general the force acting on the i^{th} disc due to the j^{th} disc, each disc having charge Q is,

$$F_{ij} = \frac{Q^2}{2\pi\epsilon r_o^2}\sum_{r=1}^{\infty} e^{-\gamma_{ij}\mu_r|z_{ij}|}\left[\frac{2 J_1\left(\gamma_r \frac{r_o}{a}\right)}{\gamma_r J_1(\gamma_r)}\right]^2 (\text{sign } z_{ij})$$

where,

$z_{ij} = |z_i - z_j|$

$\gamma_{ij} = 1/2(\gamma_i + \gamma_j)$ is an approximate relativistic correction.

The total force acting on the i^{th} disc due to n discs is then,

$$F_T = \sum_{j=1}^{n} F_{ij} = \sum_{j=1}^{n}\frac{Q^2}{2\pi\epsilon r_o^2}\sum_{r=1}^{\infty} e^{-\gamma_{ij}\mu_r|z_{ij}|} \cdot \left[\frac{2 J_1\left(\gamma_r \frac{r_o}{a}\right)}{\gamma_r J_1(\gamma_r)}\right]^2 (\text{sign } z_{ij}) \qquad (1)$$

Equation (1) can now be used directly in determining the current distribution of an electron beam which has been subjected to an rf field as is encountered in an injection system for an electron linac which contains a single rf cavity.

The bunching produced by a single cavity buncher has been reported extensively in the literature and only the final equations will be reproduced here.

Consider a simple injection system as shown in Fig. 2 consisting of a cylindrical resonator and a drift distance of length L. Let a current I_o be injected into the cavity at a voltage V_o and with a gap voltage $V_g \sin\theta$. Consider a length of charge ℓ entering the cavity and let this be divided into n discs, the length ℓ being defined by $\ell = \beta\lambda_o$. This length of beam will pass through the cavity in a time equivalent to one rf cycle. Each disc on passing through the cavity is assumed to receive a voltage impulse* ΔT_i given by,

$$\Delta T_i = -V_g \sin \frac{(i-1)2\pi}{n}$$

Immediately after the cavity the i^{th} disc can be represented by the parameter γ_i defined by

$$\gamma_i = 1 + \frac{T_i}{m_o c^2} \qquad (2)$$

where T_i is the total kinetic energy of the disc and m_o is the mass of the disc. The calculation of the effective bunching after a drift distance L, neglecting space charge forces and using the above relativistic expressions is well known. Inclusion of the space charge effects necessitates a step-by-step procedure through the drift space which is described in the following section.

*In a practical buncher this expression must be modified to include transit time effects and the gap coupling factor. These can be readily incorporated into this expression as described by Haimson,[1] but for simplicity will not be carried through in this presentation.

Bunching in the Presence of Space-Charge Forces

The procedure to be followed is to determine for each disc the velocity, v_i and coordinate z_i through the drift space to the input of the accelerator. With the first disc at z = 0 (the center of the cavity) at time t = 0, a time interval Δt (determined by the number of discs and the gun voltage) is taken and γ_i and the coordinate of each disc is determined neglecting space charge effects. The parameter γ_i can be calculated simply from Eq. (2). The coordinate z_i is then given by $z_i + v_i \Delta t$. For all discs with coordinates greater than zero the space charge forces acting on the i^{th} disc due to all other discs are calculated from Eq. (1). Then the change in the energy of the i^{th} disc is given by,

$$\Delta T_i = \sum_{j=1}^{n} F_{ij} \Delta z_i$$

from which $\Delta\gamma_i$ and hence a new v_i and z_i can be calculated.

This process is repeated until the complete drift space has been traversed for all discs of interest. Three complete rf cycles have been found to establish the equilibrium condition. In determination of the space charge forces between two discs calculations have shown that for the examples given in this article only five terms of the summation need be considered.

One further simplification which can be made is to determine the distance over which the space charge forces are of influence. Preliminary hand calculations have indicated that $F_{ij} \simeq 0$ for $\gamma_{ij} Z_{ij} > 2a$ and this has been used as a cut-off. A program has been written in order to carry out the step-by-step procedure necessary in the drift space.

The output provides the time of arrival of each disc at the end of the drift space, the velocity of each disc and the parameter γ. The first two outputs enable the phase spread of the bunch to be determined. The necessary input parameters for determining the electron capture and energy gain in the linear accelerator have then been determined.

Sample results for the single cavity system of Fig. 2 are shown in Figs. 3 and 4. These illustrate the effect of space charge and the beam pipe on the bunching process.

Conclusions

The effect of neglecting space charge forces in bunching calculations is shown in Fig. 3 and in this particular example results in overestimating the current in a phase spread of $\pm 90^{\circ}$ by approximately 8% and in a phase spread of $\pm 45^{\circ}$ by approximately 20%. Figure 4 compares the current distribution (obtained from interval between discs in the phase diagram) at the end of the drift space with and without the beam pipe. Neglecting the beam pipe results in an overestimate of the current in a $\pm 90^{\circ}$ phase spread by approximately 6% and in a phase spread of $\pm 45^{\circ}$ by approximately 18%. The beam pipe and space charge also affect the electron velocity distribution as shown in Fig. 3 which is of importance in determining the capture and phase spread of the bunches in the accelerator itself.

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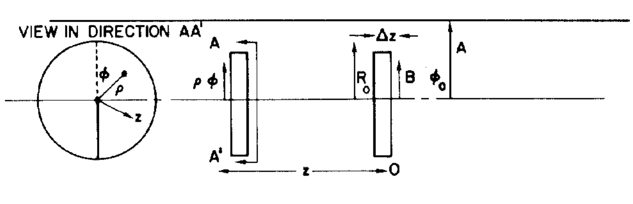

Fig. 1. Coordinate System for the Beam in a Conducting Cylinder.

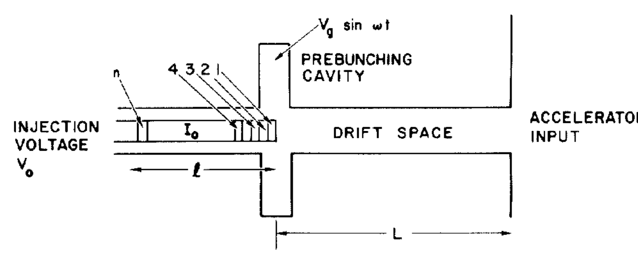

Fig. 2. Injection System Parameters.

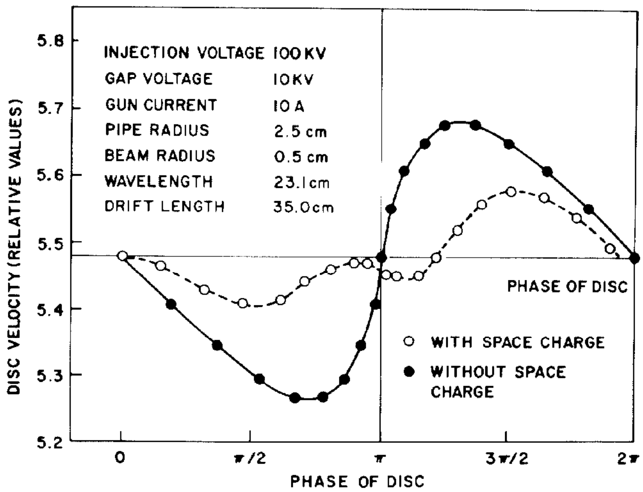

Fig. 3. Phase Diagrams with and without Space Charge.

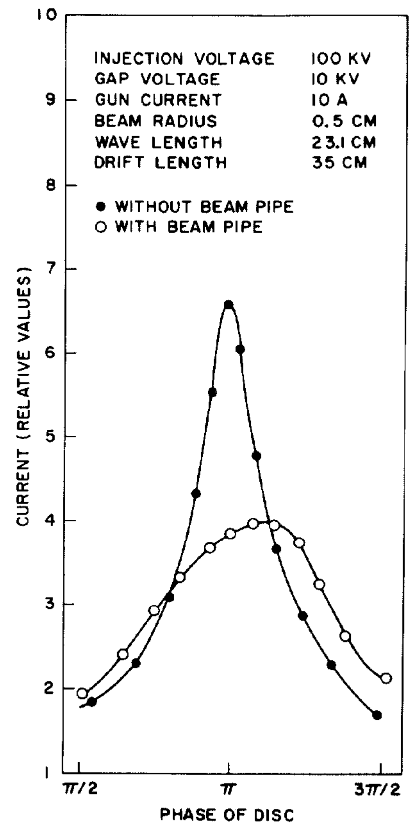

Fig. 4. Current Distribution with and without Beam Pipe.