COMPUTER CALCULATIONS OF EFFECT OF SPACE CHARGE ON LONGITUDINAL BEAM DYNAMICS IN PROTON LINEAR ACCELERATORS*

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Introduction

A machine program for calculating the effect of space charge on the longitudinal motion in a proton linac was presented at the 1966 Los Alamos Linear Accelerator Conference.[1] This program assumes a uniformly charged ellipsoidal bunch[2] of constant transverse semi-axes throughout the calculation. Since then, numerical results for transmitted current and longitudinal beam quality have been obtained with this program. These results showed that with increasing current, the assumption of a uniformly charged ellipsoidal bunch becomes less and less justified. It was, therefore, decided to also try other space charge force models which would allow for any longitudinal charge distribution. Three additional models, assuming cylindrical symmetry and a fixed transverse beam radius, were programmed. In all of these, the charge density is assumed to vary only longitudinally and the bunch can, therefore, be represented by a succession of thin disks, each uniformly charged. In the point-disk (PD) program, the contributions of the individual disks to the force at a point on the axis are summed; the point-disk-image (PDI) (model used by Morton[3]) is obtained in the same way but includes the effects of image charges induced on drift tubes and the influence of neighboring bunches; the disk-disk (DD) calculation starts from the force between two coaxial charged disks and sums the contribution from all other disks to the force on any one of them.

All computations were done on the CDC 6600 computer at the Brookhaven National Laboratory. The machine parameters used are those of the existing 50-MeV injector linac for the AGS: $E_oT = 1.6$ MV/m at input, $\lambda = 1.5$ m, $\varphi_s = -26^\circ$, injection energy = 0.75 MeV.

Numerical Results

Using each of the four potentials described above, results for transmitted current and beam quality were obtained for input currents of 30, 85, 170, 255 and 340 mA, using 120 particles. The beam radius in all runs was taken to be 0.32 cm and the bunch initially occupied the phase region between -0.45 and +0.45 rad. The initial distribution in longitudinal phase space was selected to conform to a uniformly charged ellipsoid in XYZ space and to maintain this distribution as long as possible (it cannot be maintained when space charge forces are comparable to RF focussing forces). Figure 1 and Table 1 show the output current at 50 MeV as a function of input current for each of the four force models. Similar results have been obtained recently by Swenson and Crandall using rings.[4] Table 1 also lists energies beyond which no particles are lost (a particle is considered lost when its phase differs from that of the synchronous particle by more than 3 rad).

The beam quality was estimated by an output routine which calculates the rms values of χ ($= \varphi - \varphi_s$) and ΔW ($= W - W_s$) for the transmitted particles, excluding stragglers. The three cylindrical potentials gave results so similar that only those for PDI are plotted in Fig. 2 along with those for the ellipsoid and, for purposes of comparison, for a longitudinally matched beam with zero space charge. It can be seen from Fig. 2 that there is excellent agreement for the rms values of χ and ΔW obtained from the two potentials for 30 mA. At this current, the space charge force is only one-third the linear part of the RF force and the ellipsoidal charge distribution is preserved throughout the linac. This is no longer the case for 85 mA and higher currents where the space charge forces are comparable to, or in some cases greater than, the RF focussing forces. In these circumstances, the potential, which is the sum of the RF and space charge contributions is very sensitive to the model used in calculating the latter. This accounts for the large discrepancy shown in Fig. 2 between the rms values of ΔW obtained from the ellipsoidal and cylindrical charge distributions.

Figure 3 shows χ_{max} (half-width of bunch in radians) as a function of β plotted on log-log scales for various values of the current using the ellipsoidal distribution. It can be seen from this graph that for currents of 170 and 255 mA the bunch expands in χ (or Z) and particles are lost until the space charge forces are reduced to values slightly smaller than the RF focussing forces. Thereafter, the average phase damping as approximated by the straight lines goes as β^{-P} where $1/2 \leq P \leq 3/4$. Theory predicts $P = 3/4$ for zero space charge and $P = 1/2$ for dense space charge.[5,8] Similar results were obtained for the cylindrical distributions. A few other initial distributions in longitudinal phase space were tried and the transmitted currents were within ± 10% of those given in Table 1. See note following references.

Comparison with Theory

Values for space charge limiting currents in proton linacs have been predicted theoretically by several investigators.[3,5-10] In these theoretical

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*Work performed under the auspices of the U.S. Atomic Energy Commission.

treatments static calculations alone were made, i.e. the effects of acceleration were not considered. Using the assumption of a uniformly charged ellipsoid,[5-8,10] theory shows that limiting currents occur for $\mu = 0.3 - 0.4$ at injection, where μ is the ratio of space charge force to the linear part of RF restoring force. Using $\mu = 0.4$ yields approximately 43 mA for machine and beam parameters given above. In the case of a cylindrical charge distribution[3,9] theoretically predicted limiting currents are somewhat higher. A value of 85 mA was obtained with the aid of the computer program described in Ref. 3.

To investigate the possibility that the discrepancy between theoretically predicted limiting currents and computer calculated transmitted currents arises from the omission of the effects of acceleration in the theoretical treatments, computer runs were repeated for a simulated non-accelerating linac. Limiting currents of 49 and 79 mA were now found for the ellipsoidal and cylindrical charge distributions respectively. These are in good agreement with theoretically predicted values of 43 and 85 mA.

Conclusions

1. Longitudinal dynamics computations through a linac seem to indicate that currents can be transmitted which are higher by a factor of 2.5 - 5 than those predicted theoretically on the basis of static calculations.

For currents with space charge forces initially smaller than the RF restoring force, the bunch starts damping nearly as $\beta^{-3/4}$. As the damping continues the space charge forces increase in importance and the damping proceeds as β^{-P}, $\frac{1}{2} < P < 3/4$. For space charge forces initially larger than the RF restoring force, the bunch grows rapidly, particles may be lost and space charge forces are reduced until they become comparable with the RF restoring forces. Thereafter the phase damping goes as β^{-P}, $\frac{1}{2} < P < 3/4$.

2. In the presence of strong space charge forces a charge distribution which is initially symmetric with respect to the center of the bunch soon becomes highly asymmetric. This results from a flow of particles out of one side of the bucket and a bunching of particles at the other side. Under these conditions, the bunch cannot accurately be described as a uniformly charged ellipsoid and use of the ellipsoidal model results in large longitudinal phase space dilution. However, for low currents (~ 30 mA and lower in this work) the assumption of a uniformly charged ellipsoid seems to be satisfactory. Particle dynamics programs based on this model can be used for both the longitudinal and transverse motion and require far less computer time than the other programs described above.

3. For high currents the longitudinal motion becomes very nonlinear, and a simple analytical model for coupling between the longitudinal and transverse motion[11,12] cannot be used. Further numerical work is required to investigate this effect.

Acknowledgments

The theoretical guidance provided by Dr. R.L. Gluckstern during the entire course of this work is gratefully acknowledged. The authors are indebted to Drs. P.M. Lapostolle, T. Nishikawa and J. Claus for helpful discussions and suggestions.

Table 1*

Case	Input Current (mA)	Output Current (mA)	Energy above which no particles lost (MeV)
DD	170	154	4.22
	255	198	10.29
	340	218	6.52
PD	170	145	5.78
	255	172	6.03
	340	187	11.96
PDI	170	146	6.78
	255	181	6.25
	340	191	6.78
Ellipsoid	170	145	3.0
	255	179	6.52
	340	210	10.27

*Transmission was 100% for 30 and 85 mA.

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Note

The ellipsoidal program calculates the image charge effects in accelerating gaps by assuming the bunch to be between infinite parallel conducting plates. This model is not a good one when the beam length is nearly equal to or larger than the gap length (which is the case for the runs at 170 mA and higher currents in the early drift tubes) and also contributes to the discrepancy between results for ΔW obtained from the ellipsoidal and cylindrical charge distributions.

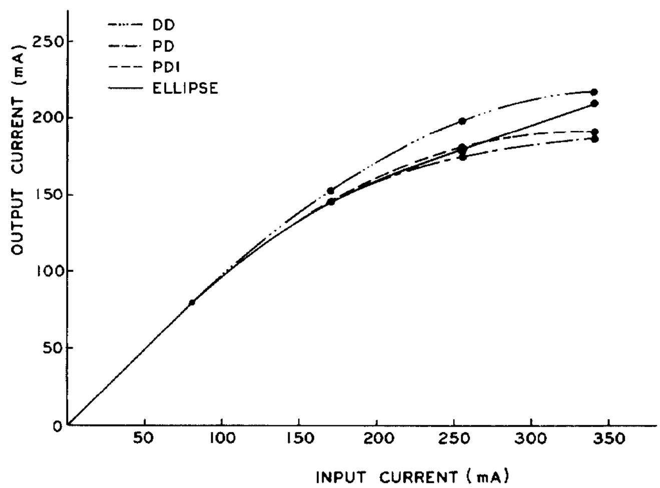

Fig. 1. Output current vs. input current.

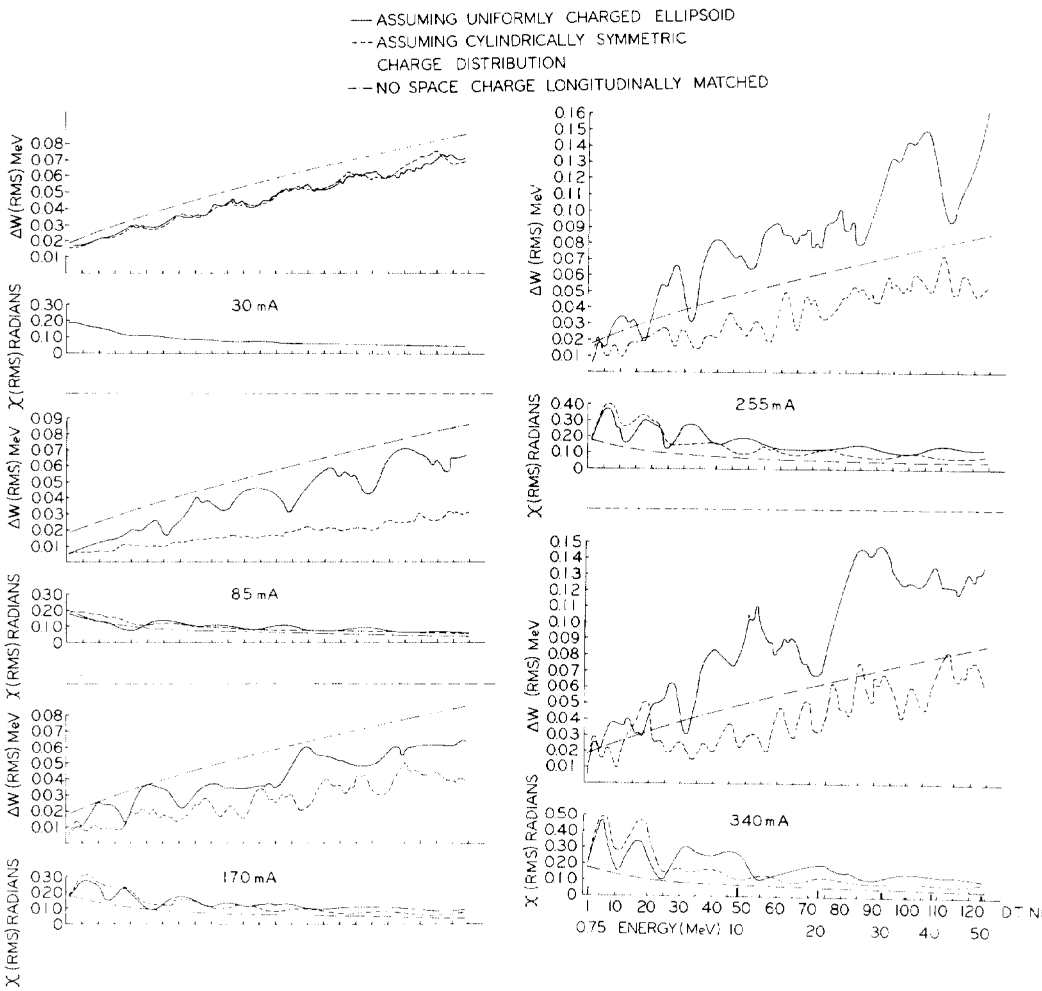

Fig. 2. Longitudinal beam quality as a function of energy for different input currents.

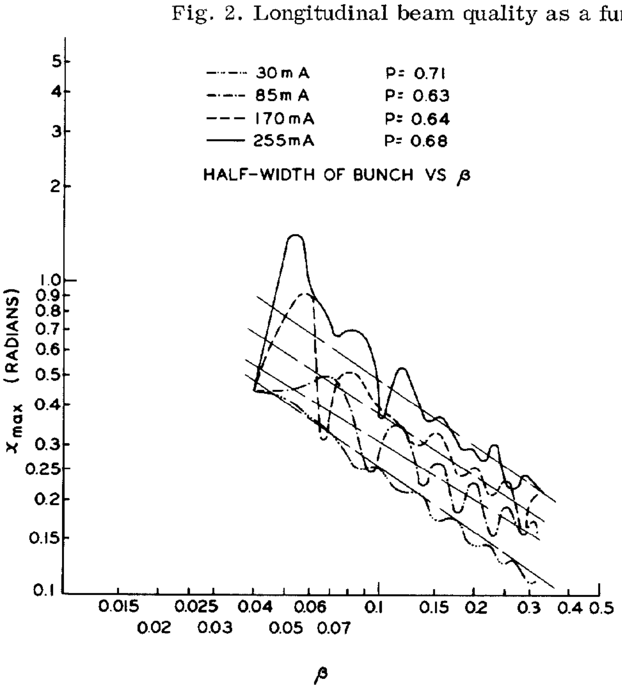

Fig. 3. Half-width of bunch vs. β.