METHODS OF COMPUTING THE TIME CONSTANT AND IMPEDANCE OF MAGNETS*

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Summary

For regulation and ripple studies of dc magnets, the essential parameters which must be known as a function of dc magnetization and/or frequency are the unit step time constant and impedance. An exact value for the unit step time constant of a laminated or solid core magnet can only be obtained by measurement. It can, however, be estimated from the dc time constant which is easily computed from the magnet geometry, number of coil turns, and the coil resistance. Measured unit step time constants and computed dc time constants are given for a number of magnets. Formulas to compute the impedance of laminated core magnets are developed. An empirical formula for calculating the impedance of solid core magnets is deduced from test data.

Introduction

For the design of closed loop regulators for magnet power supplies, the unit step time constant $\tau_{\sqsubset}$ of the magnets must be known. Its value cannot readily be computed; however, it can be estimated from the dc time constant τ_{dc}. Knowledge of the magnet impedance at ripple frequencies is essential for the design of economical power supply filters. For the ring magnet of synchrotrons, it is of interest to know the magnet impedance over a large portion of the audio range in order to estimate the effects of ripple flux and coil resonances on synchrotron oscillations.

The power supply regulators used at the Zero Gradient Synchrotron (ZGS) can be adjusted to match the time constants of magnets. This adjustment can be made in steps which are multiples of two; therefore, it is sufficient to know $\tau_{\sqsubset}$ within $\lesssim \pm 50\%$. Similar tolerances are acceptable for the determination of the impedance of magnets at audio frequencies.

The impedance of a laminated ring magnet octant of the ZGS will be calculated over the frequency range from 10 Hz to 9 kHz for dc magnetizations of zero and 21.5 kG. The impedance and unit step time constant of solid core bending and quadrupole magnets were determined experimentally. Most measurements were made with zero dc magnetization; a few were made at rated current. From these measurements, an empirical formula, based on magnet dimensions, will be deduced to enable the calculation of L and R for frequencies between 60 Hz and 360 Hz.

Equivalent Magnetic Circuit

The relationship between magnet excitation current I, field strength H, flux path ℓ, magnet coil turns n, and magnet flux ϕ, is as follows:

$$In = \oint H_\ell d\ell = H_g \ell_g + H_1 \ell_1 + H_2 \ell_2 + \ldots \quad , (1)$$

$$H = \frac{\phi}{\mu_o \mu A} \quad , \qquad (2)$$

where subscript g refers to the magnet air gap and subscripts 1, 2, ... refer to magnet iron sections.

$\mu_o = 4\pi \times 10^{-9} \frac{H}{cm}$ = permeability of air,
μ = relative permeability, and
A = area of flux path (cm^2) .

Substituting Eq. (2) into Eq. (1) and multiplying both sides by $1/n^2$ results in

$$\frac{I}{n\phi} = \frac{1}{L} = \frac{\ell_g}{\mu_o n^2 A_g} + \frac{\ell_1}{\mu_o \mu_1 n^2 A_1} + \ldots$$

$$= \frac{1}{L_g} + \frac{1}{L_1} + \frac{1}{L_2} + \ldots \quad . \qquad (3)$$

As illustrated by Eq. (3) the magnet inductance L can be thought of as being the parallel connection of the inductances of the different magnet sections along the flux path. Each of these different inductances has n turns and the magnetic properties of its path length. Such an equivalent magnetic circuit allows one to compute separately the inductances of the various magnet sections as a function of dc magnetization and/or frequency.

DC Inductance and Time Constant

For magnets which have equal areas for air gap A_g and iron core A_c, the inductance L in henrys is

$$L = \frac{L_g L_c}{L_g + L_c} = \mu_o n^2 \frac{A}{\ell_g + \frac{\ell_c}{\mu_{dc}}} \quad , \qquad (4)$$

where

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ℓ_c = length of flux path in iron (cm), and
μ_{dc} = relative dc permeability of iron under conditions of magnetization present in the core.

If the core area is changing along the flux path lengths ℓ_1, ℓ_2, $\ell_3 \ldots$, the term ℓ_c/μ_{dc} in Eq. (4) is given by

$$\frac{\ell_c}{\mu_{dc}} = A\left(\frac{\ell_1}{\mu_1 A_1} + \frac{\ell_2}{\mu_2 A_2} + \frac{\ell_3}{\mu_3 A_3} + \ldots\right) \quad . \qquad (5)$$

The dc time constant τ_{dc} is obtained by dividing the inductance L from Eq. (4) by the resistance R_{Cu} of the coil.

The magnets discussed in this paper use low carbon steel as core material. Their relative dc permeability, as a function of flux density, is shown in Fig. 1.

Picture Frame Bending Magnets

The cross section of a picture frame bending magnet is shown in Fig. 2a. From the magnet dimensions and number of coil turns given and the permeability curve of Fig. 1, the magnet inductance can be computed for any value of flux density with Eqs. (4) and (5). It is assumed that the flux density increases linearly from zero at the outside of the coil to a constant value at the inside of the coil. Therefore, the air gap width is taken as the sum of dimensions E + G. The air gap length is taken as core length M plus coil width G, and the air gap height is the physical gap height F. For the iron inductance, the core length M is used as effective magnet length. Other dimensions for the various iron sections were taken from the table of Fig. 2.

Quadrupole Magnets

Because of symmetry, the inductance of a quadrupole magnet may be computed from one quadrant. The total inductance is then four times the inductance of one quadrant, which has a fourth of the total number of turns. Figure 2b is a cross section through a quadrupole magnet; dimensions are given in the table.

Figure 3 illustrates the flux distribution in the gap assumed for calculating the gap inductance. The mean gap length of a quadrant is taken as

$$\ell_g = \frac{\ell' W' + \ell'' W''}{W' + W''} \quad , \qquad (6)$$

where
ℓ' = 0.36 x pole tip radius r (cm),
ℓ'' = 1/2 of dimension D in Fig. 2b (cm),
W' = difference of dimensions B and C in Fig. 2b (cm), and
W'' = dimension C in Fig. 2b (cm).

The gap area A_g is taken as dimension B multiplied by the effective magnet length ℓ_{eff}. ($\ell_{eff} \approx$ core length plus pole tip radius, cm.) Under these assumptions, the air gap inductance becomes

$$L_g = 4 \times \mu_o \left(\frac{n}{4}\right)^2 \frac{A_g}{\ell_g} = \frac{\pi n^2 A_g}{\ell_g 10^9} \quad . \qquad (7)$$

With reference to Fig. 3, the core inductance is computed from the average core area $A_c = W_c \times \ell_{eff}$ where

$$W_c = \frac{\ell_1 W_1 + \ell_2 W_2 + \ell_3 W_3}{\ell_c} \quad , \qquad (8)$$

and

$$\ell_c = \ell_1 + \ell_2 + \ell_3 = \text{average length of flux path},$$

as

$$L_c = \pi \mu_{dc} n^2 A_c / \ell_c 10^9 \quad . \qquad (9)$$

Finally, the quadrupole inductance is

$$L = L_g L_c / \left(L_g + L_c\right) \quad .$$

The inductance and time constant of some ZGS magnets are given in Table 1. Except for the ZGS octant values, there is less than 5% change between zero and rated field. Therefore, only one value is listed for all but the ZGS octant magnets.

Impedance of Laminated Core Magnets

Complex Permeability

Neglecting capacitive effects, the impedance of a coil containing a ferromagnetic core is $Z = R + j\omega L$. If we deduct the coil resistance R_{Cu} from R, we have left

$$Z = R_{Fe} + j\omega L \quad . \qquad (10)$$

Resistance R_{Fe} is due to core losses (hysteresis and eddy currents). Both the core losses and the inductance are dependent on the properties and the design of the core, and Eq. (10) can be written as

$$Z = j\omega \mu_o \bar{\mu} n^2 \frac{A}{\ell} \qquad (11)$$

where
$\bar{\mu}$ = $\mu_L - j\mu_R$ = relative complex permeability of core material, and
ω = $2\pi f$.

From Eqs. (10) and (11), the real and imaginary parts of the complex permeability are

$$\mu_o \bar{\mu} = \mu_o\left(\mu_L - j\mu_R\right) = \frac{R_{Fe} + j\omega L}{j\omega n^2 \frac{A}{\ell}} =$$

$$\mu_o \bar{\mu} = \frac{L}{n^2 \frac{A}{\ell}} - j \frac{R_{Fe}}{\omega n^2 \frac{A}{\ell}} . \qquad (12)$$

For small amplitude alternating fields such as magnet ripple fields (B < 1 G), hysteresis losses become negligible and only eddy currents and the reversible permeability[1] have to be considered. Figure 1 shows the relative reversible permeability μ_r of low carbon steel as a function of dc flux density. The complex permeability has been computed for homogeneous infinitely large sheets having constant permeability and resistivity over their cross sections. Its real part is given by

$$\mathrm{Re}\frac{\bar{\mu}}{\mu_r} = \frac{\mu_L}{\mu_r} = \frac{\delta}{d} \frac{\sinh\frac{d}{\delta} + \sin\frac{d}{\delta}}{\cosh\frac{d}{\delta} + \cos\frac{d}{\delta}} , \qquad (13)$$

and the imaginary part by

$$\mathrm{Im}\frac{\bar{\mu}}{\mu_r} = \frac{\mu_R}{\mu_r} = -\frac{\delta}{d} \frac{\sinh\frac{d}{\delta} - \sin\frac{d}{\delta}}{\cosh\frac{d}{\delta} + \cos\frac{d}{\delta}} , \qquad (14)$$

where

μ_r = relative reversible permeability at given dc field for $\omega \to 0$,

d = thickness of sheet (cm),

δ = $\sqrt{\frac{\rho}{\pi f \mu_o \mu_r}}$ = skin depth (cm), and

ρ = resistivity of sheet; for low carbon steel $\rho \approx 15 \times 10^{-6}\ \Omega$cm

For $d \gg 2\delta$, Eqs. (13) and (14) simplify to

$$\frac{\mu_L}{\mu_r} = -j \frac{\mu_R}{\mu_r} = \frac{\delta}{d} . \qquad (15)$$

Picture Frame Bending Magnets

If we multiply Eq. (3) by $1/j\omega$ and use $\bar{\mu}$ instead of μ for the iron core sections, we obtain

$$\frac{I}{j\omega n \phi} = \frac{1}{Z} = \frac{1}{Z_g} + \frac{1}{Z_1} + \frac{1}{Z_2} + \ldots . \qquad (16)$$

The air gap impedance is

$$Z_g = j\omega \mu_o n^2 \frac{A_g}{\ell_g} ,$$

and for a core section we have from Eqs. (11) and (12),

$$Z_{1,2,\ldots} = \omega \mu_o n^2 \frac{A_{1,2,\ldots}}{\ell_{1,2,\ldots}} \times \left(\mu_{R_{1,2,\ldots}} + j\mu_{L_{1,2,\ldots}}\right) . \qquad (17)$$

For an octant of the ZGS ring magnet, the inductance L and the resistance R_{Fe} were computed[2] and are shown in Fig. 4. If we add the coil resistance R_{Cu} in series with R_{Fe} and L and connect the equivalent coil capacitance C in parallel, we obtain the octant impedance as shown in Fig. 5. A few of the computed points have been checked on an octant and found to agree well with the plots shown. It was found that the increase in copper resistance R_{Cu}, due to skin effect, was small compared with R_{Fe}.

Impedance of Solid Core Magnets

A number of picture frame bending magnets and quadrupole magnets were measured with zero dc magnetization and small amplitude sine waves of 60 Hz and 360 Hz. From these measurements, an empirical formula for the computation of the iron contribution Z_{Fe} to the magnet impedance Z was deduced. With

$$Z = Z_g Z_{Fe} / (Z_g + Z_{Fe})$$

measured and Z_g computed from the gap dimensions, the corresponding $Z_{Fe} = R_{Fe} + j\omega L_c$ can be computed. One then can compute μ_R and μ_L for picture frame magnets from

$$\mu_R + j\mu_L = \frac{R_{Fe} + j\omega L_c}{\omega 4 \pi n^2 A_c / \ell_c 10^9} , \qquad (17a)$$

and for quadrupole magnets from

$$\mu_R + j\mu_L = \frac{R_{Fe} + j\omega L_c}{\omega \pi n^2 A_c / \ell_c 10^9} . \qquad (18)$$

At very low frequencies, the field penetrates the whole core area; while at high frequencies, it is concentrated in a thin surface layer of depth δ. Therefore, it appears that Eq. (15) should approximate μ_L and μ_R. A correction factor K, shown in Fig. 6, was obtained by dividing measured values by the approximate values calculated with Eq. (15). This leads to the empirical formula

$$\mu_L = -j\mu_R = \mu_r \frac{\delta}{d} K . \qquad (19)$$

In Eq. (19), d is equal to W_c for quadrupole magnets and equal to $W_c/2$ for picture frame magnets.

Equation (19) also applies when the magnets are excited at different dc levels. This was checked by measuring the impedance of bending magnet BM-110 and quadrupole magnet QM-104 at 60 Hz and 360 Hz for various dc excitations up to rated current. The measured and computed values were within ± 25% of each other.

Inductance and resistance values for some ZGS magnets at 60 Hz and 360 Hz are given in Table 1.

Time Constant for Step Change in Supply Voltage

As long as the inductance of the air gap is very much smaller than the parallel connected inductance of the solid or laminated core, the unit step time constant $\tau_{\_\lceil}$ will change little with current. It, therefore, can be measured with a voltage step that produces a peak current of only a few amperes instead of rated current.

The unit step time constant will decrease with magnet saturation. The $\tau_{\_\lceil}$ values given in Table 1 for zero dc fields were measured by applying 2 V to the magnets. The values given for BM-110 and QM-104 at rated field strength were obtained by suddenly applying rated voltage. In both cases the unit step was applied when the magnet field was zero. Eddy current effects will always make $\tau_{\_\lceil} < \tau_{dc}$, however, $\tau_{\_\lceil}$ might be estimated from computed values of τ_{dc} as indicated by Table 1.

References

[1] Bozorth, R. M., Ferromagnetism, New York, N. Y., D. Van Nostrand Co., 1951.

[2] Praeg, W. F., "Impedance, Time Constant, and Ripple Flux of a Ring Magnet Octant of the ZGS at Audio Frequencies," Particle Accelerator Division, Argonne National Laboratory, Internal Report WFP-3, November 27, 1963.

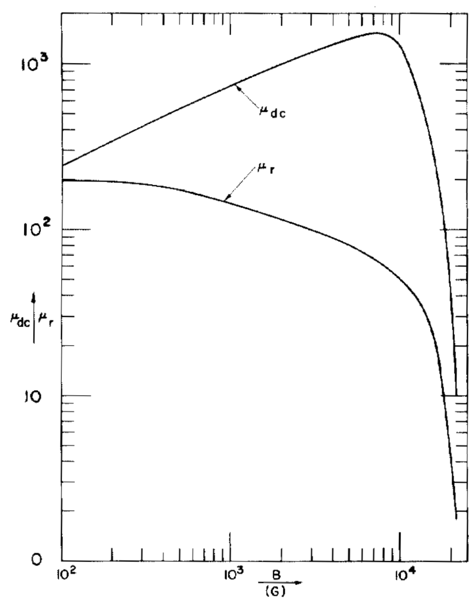
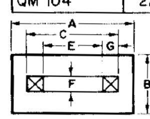


Fig. 1 Relative values of dc permeability μ_{dc} and reversible permeability μ_r of low carbon steel as function of dc flux density.

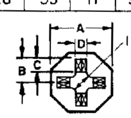
MAGNET	NO. OF TURNS	DIMENSIONS (INCHES) A	B	C	D	E	F	G	H	I	*M
ZGS OCTANT	30	102.5	55	53.75	—	35.5	5.75	8.12	—	—	648
BM 105	96	81	42	45.6	—	18	6	13.8	—	—	72
BM 107	192	81	42	43.1	—	15	6	14			30
BM 110	132	77	44	41.6	—	24	8	8.7	—	—	36
QM 102	228	53	16.3	8.7	3.8	—	—	—	—	10.1	29
QM 103	120	27.5	8.2	4.1	2	—	—	—	—	5.3	33
QM 104	228	53	17	9.3	3.8	—	—	—	—	10.1	10.5

*M = CORE LENGTH



BENDING MAGNET

Fig. 2a



QUADRUPOLE MAGNET

Fig. 2b

NOTES:
ALL MAGNETS ARE SOLID CORE EXCEPT THE ZGS OCTANT WHICH IS MADE FROM INSULATED 1/2" LAMINATIONS.

Fig. 2 Cross sections and dimensions of some ZGS bending and quadrupole magnets

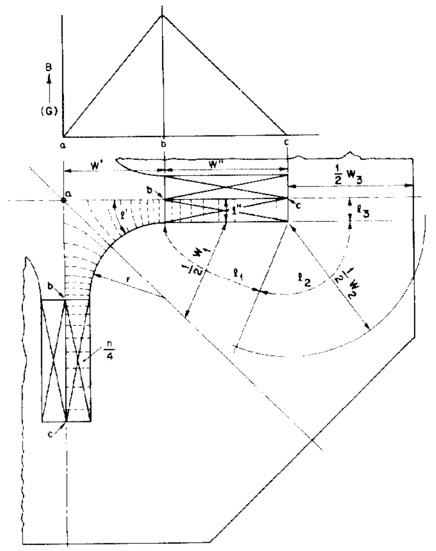


Fig. 3 Flux density along lines a-c and dimensions used to compute impedance of quadrupole magnets

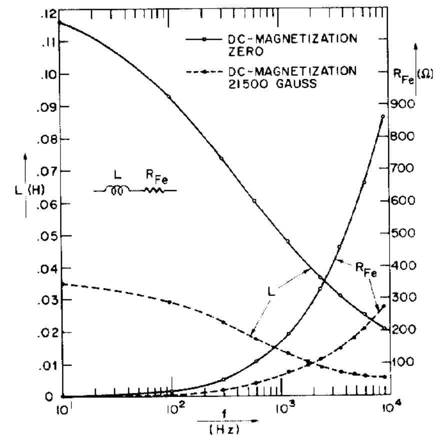


Fig. 4 Inductance and resistance of ZGS ring magnet octant as a function of frequency

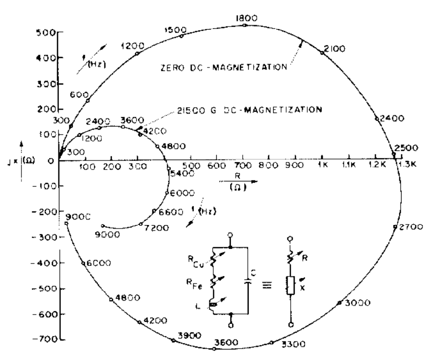


Fig. 5 Impedance of ZGS ring magnet octant at various frequencies

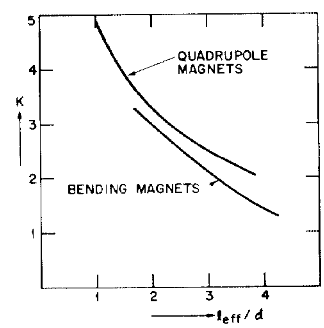


Fig. 6 Correction factor K for solid core magnets as a function of ℓ_{eff}/d

Table 1 Inductance, resistance and time constant values of some ZGS Magnets

MAGNET	DC FIELD STRENGTH KG	D C L	D C R_{Cu}	D C T_{dc}	UNIT STEP $T_{\sqcup}$	60 Hz L	60 Hz R_{Fe}	360 Hz L	360 Hz R_{Fe}
ZGS* OCTANT	0	0.130	0.033	3.94	3.20	0.10	7.7	0.070	64
	21.5	0.040		1.21	—	0.031	2.8	0.022	24
BM 105	0	0.113	0.046	2.45	2.10	0.0098	3.6	0.0036	7.56
	18				—	0.0029	1.1	0.0012	2.67
BM 107	0	0.191	0.073	2.61	2.40	0.036	10.7	0.014	25.9
	18				—	0.013	4.94	0.0054	12.1
BM 110	0	0.095	0.046	2.07	1.75	0.019	5.18	0.0085	11.8
	18				1.35	0.0052	1.96	0.0021	4.8
QM 102	0	0.124	0.093	1.33	1.19	0.031	7.28	0.0122	21.8
	12.7				—	0.0099	3.75	0.0041	9.18
QM 103	0	0.032	0.090	0.36	0.35	0.0125	2.98	0.0048	11.1
	12.7				—	0.0037	1.41	0.0015	3.47
QM 104	0	0.065	0.057	1.14	1.13	0.023	4.89	0.0094	16
	12.7				0.95	0.0074	2.80	0.0030	6.85

L IN HENRIES, R IN OHMS, T IN SECONDS

* MADE FROM 1/2" LAMINATIONS, ALL OTHER MAGNETS HAVE SOLID CORES.

— NO MEASURED VALUE AVAILABLE